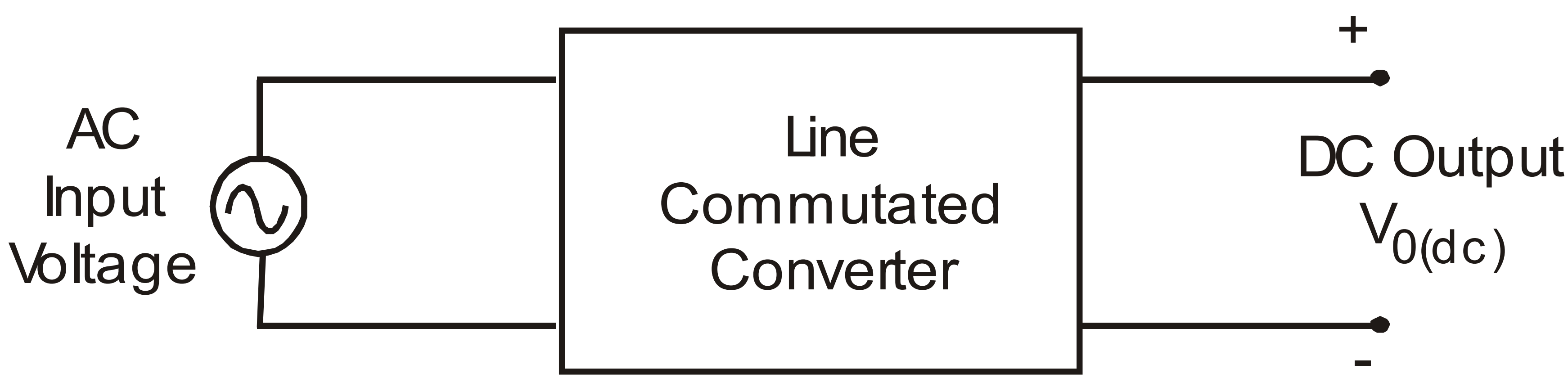
Controlled Rectifiers

(Line Commutated AC to DC converters)



- Type of input: Fixed voltage, fixed frequency ac power supply.
- Type of output: Variable dc output voltage
- Type of commutation: Natural / AC line commutation

Different types of Line Commutated Converters

- AC to DC Converters (Phase controlled rectifiers)
- AC to AC converters (AC voltage controllers)
- AC to AC converters (Cyclo converters) at low output frequency.

Differences Between Diode Rectifiers & Phase Controlled Rectifiers

- The diode rectifiers are referred to as uncontrolled rectifiers .
- The diode rectifiers give a fixed dc output voltage .
- Each diode conducts for one half cycle.
- Diode conduction angle = 180^0 or π radians.
- We can not control the dc output voltage or the average dc load current in a diode rectifier circuit.

Single phase half wave diode rectifier gives an

Average dc output voltage $V_{O(dc)} = \frac{V_m}{\pi}$

Single phase full wave diode rectifier gives an

Average dc output voltage $V_{O(dc)} = \frac{2V_m}{\pi}$

Applications of Phase Controlled Rectifiers

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Portable hand tool drives.

Classification of Phase Controlled Rectifiers

- Single Phase Controlled Rectifiers.
- Three Phase Controlled Rectifiers.

Different types of Single Phase Controlled Rectifiers.

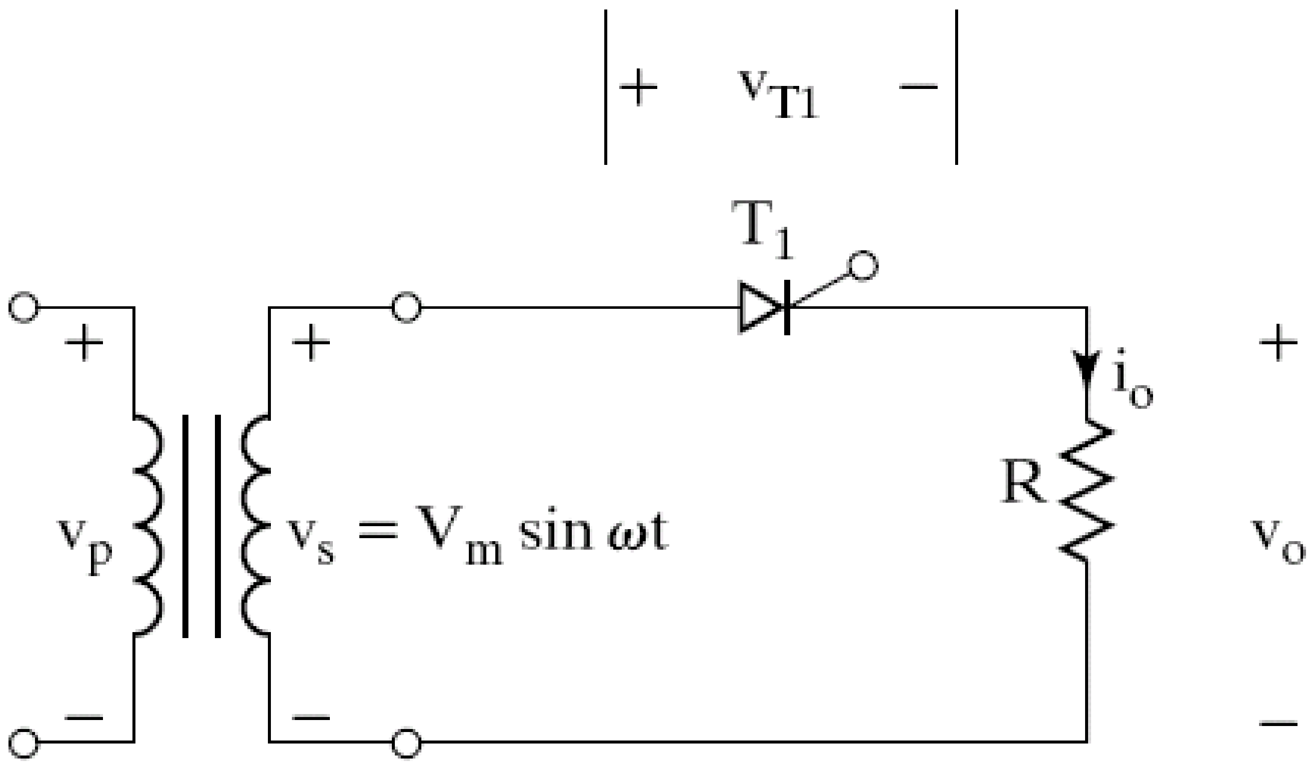
- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
  - Using a center tapped transformer.
  - Full wave bridge circuit.
    - Semi converter.
    - Full converter.

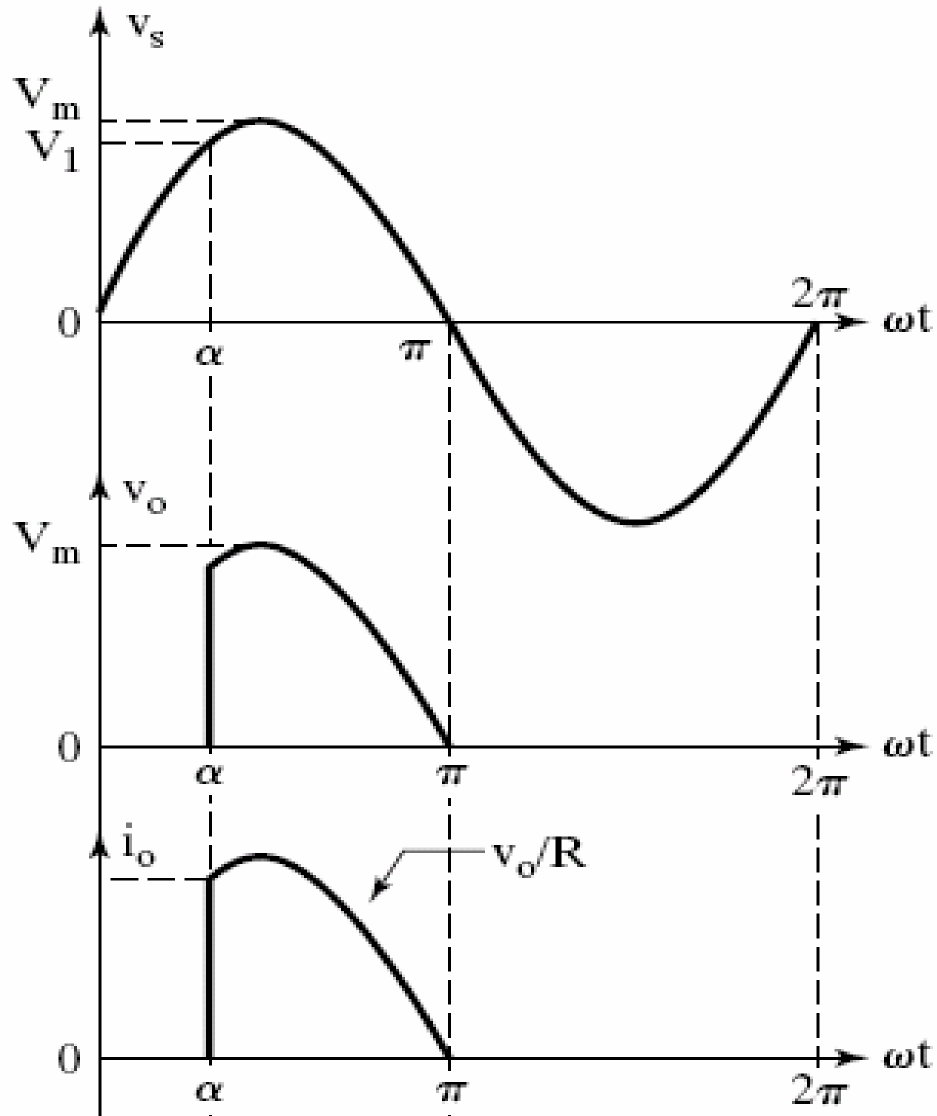
Different Types of Three Phase Controlled Rectifiers

- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
  - Semi converter (half controlled bridge converter).
  - Full converter (fully controlled bridge converter).

Principle of Phase Controlled Rectifier Operation

Single Phase Half-Wave Thyristor Converter with a Resistive Load

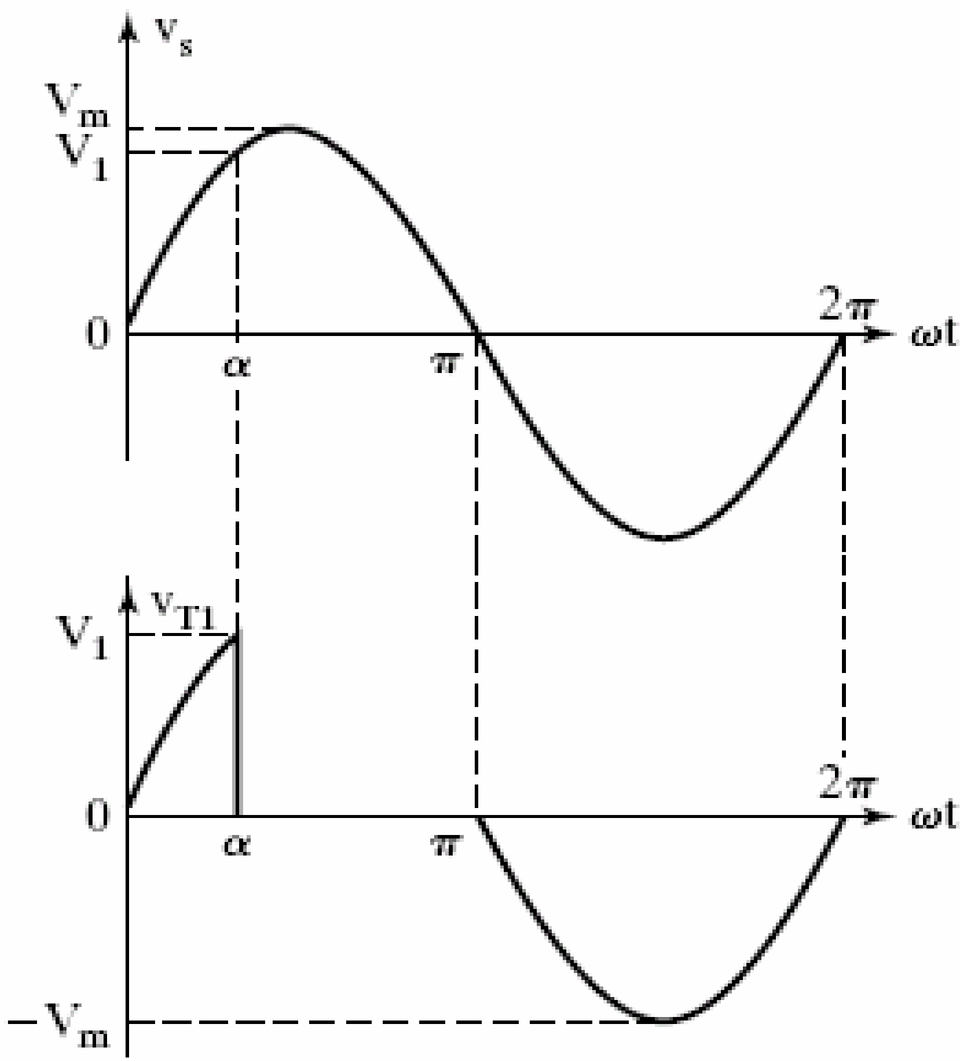




Supply Voltage

Output Voltage

Output (load)
Current



Supply Voltage

Thyristor Voltage

Equations

$$v_s = V_m \sin \omega t = \text{ i/p ac supply voltage}$$

$$V_m = \text{max. value of i/p ac supply voltage}$$

$$V_S = \frac{V_m}{\sqrt{2}} = \text{RMS value of i/p ac supply voltage}$$

$$v_O = v_L = \text{ output voltage across the load}$$

When the thyristor is triggered at $\omega t = \alpha$

$$v_O = v_L = V_m \sin \omega t; \;\; \omega t = \alpha \text{ to } \pi$$

$$i_O = i_L = \frac{v_O}{R} = \text{Load current}; \;\; \omega t = \alpha \text{ to } \pi$$

$$i_O = i_L = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t; \omega t = \alpha \text{ to } \pi$$

Where $I_m = \frac{V_m}{R} =$ max. value of load current

To Derive an Expression for the Average (DC) Output Voltage Across The Load

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{0}^{2\pi} v_O.d(\omega t);$$

$$v_O = V_m \sin\omega t \ for \ \omega t = \alpha \ to \ \pi$$

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\pi + \cos\alpha\right]; \quad \cos\pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]\ ;\ V_m = \sqrt{2}V_S$$

Maximum average (dc) o/p voltage is obtained when $\alpha = 0$ and the maximum dc output voltage

$$V_{dc(\max)} = V_{dm} = \frac{V_m}{2\pi}\left(1+\cos 0\right);\ \ \cos\left(0\right)=1$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{V_m}{\pi}$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right] \; ; \; V_m = \sqrt{2}V_S$$

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of $180^0 \left(\pi \text{ radians}\right)$

We can plot the control characteristic $\left(V_{O(dc)} \text{ vs } \alpha\right)$ by using the equation for $V_{O(dc)}$

Control Characteristic of Single Phase Half Wave Phase Controlled Rectifier with Resistive Load

The average dc output voltage is given by the expression

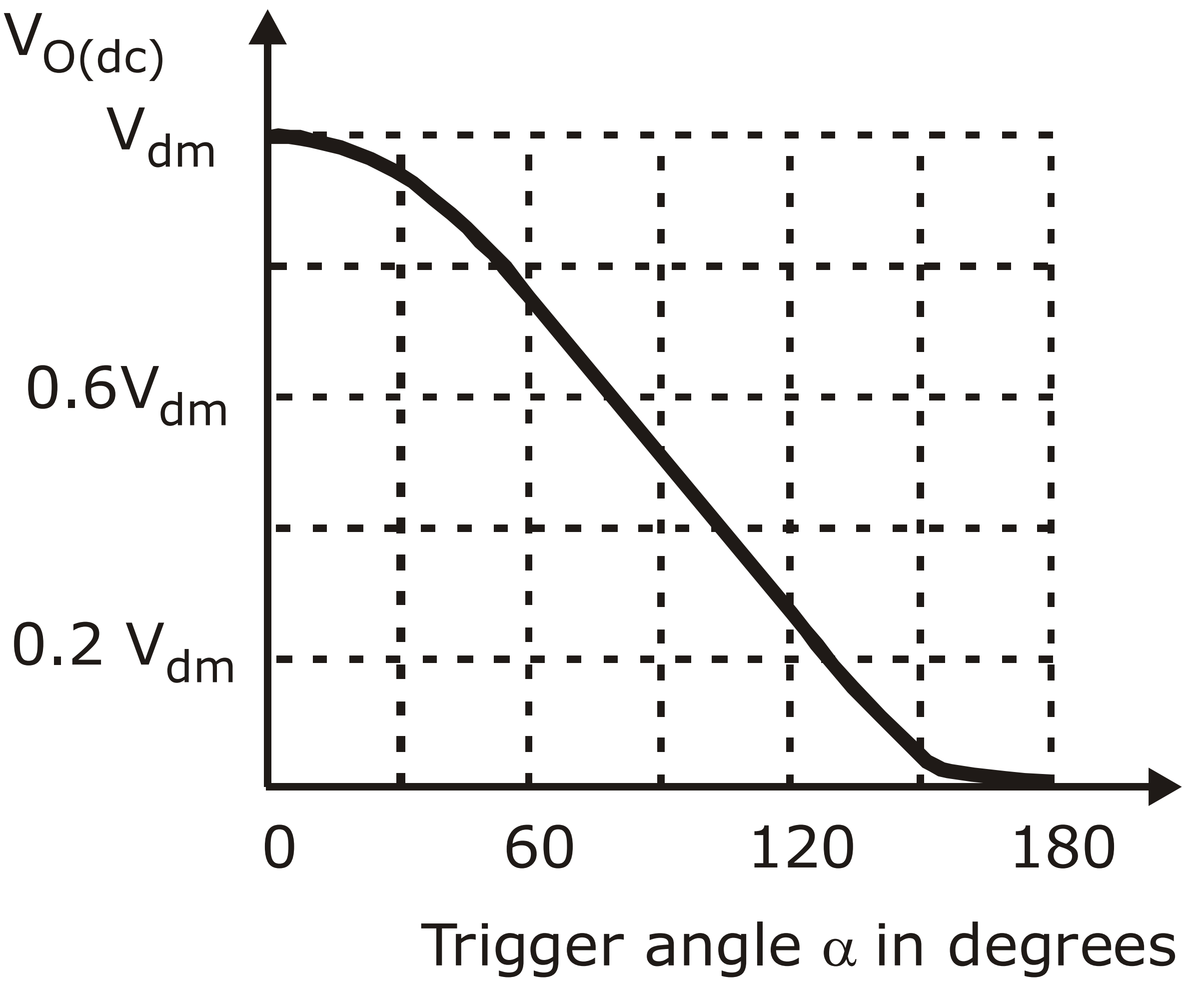
$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]$$

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle α

Trigger angle α in degrees	$V_{O(dc)}$	%
0	$V_{dm} = \frac{V_m}{\pi}$	100% V_{dm}
30^0	0.933 V_{dm}	93.3 % V_{dm}
60^0	0.75 V_{dm}	75 % V_{dm}
90^0	0.5 V_{dm}	50 % V_{dm}
120^0	0.25 V_{dm}	25 % V_{dm}
150^0	0.06698 V_{dm}	6.69 % V_{dm}
180^0	0	0

$$V_{dm} = \frac{V_m}{\pi} = V_{dc(\max)}$$

Control Characteristic



Normalizing the dc output voltage with respect to V_{dm}, the Normalized output voltage

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{\dfrac{V_m}{2\pi}\left(1+\cos\alpha\right)}{\dfrac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2}\left(1+\cos\alpha\right) = V_{dcn}$$

To Derive An Expression for the RMS Value of Output Voltage of a Single Phase Half Wave Controlled Rectifier With Resistive Load

The RMS output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_0^{2\pi} v_O^2 . d(\omega t) \right]$$

Output voltage $v_O = V_m \sin \omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_\alpha^{\pi} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

By substituting $\sin^2 \omega t = \frac{1-\cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \frac{(1-\cos 2\omega t)}{2} .d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \int_{\alpha}^{\pi} (1-\cos 2\omega t) d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \left\{ \int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t \, d(\omega t) \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left\{(\omega t)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi-\alpha) - \frac{(\sin 2\pi - \sin 2\alpha)}{2}\right)\right]^{\frac{1}{2}} ; \sin 2\pi = 0$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi-\alpha) + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\left((\pi-\alpha) + \frac{\sin 2\alpha}{2}\right)^{\frac{1}{2}}$$

Performance Parameters Of Phase Controlled Rectifiers

Output dc power (avg. or dc o/p power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \ ; \ i.e., \ P_{dc} = V_{dc} \times I_{dc}$$

Where

$V_{O(dc)} = V_{dc} =$ avg./ dc value of o/p voltage.

$I_{O(dc)} = I_{dc} =$ avg./dc value of o/p current

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

Efficiency $\eta = \dfrac{P_{O(dc)}}{P_{O(ac)}}$; % Efficiency $\eta = \dfrac{P_{O(dc)}}{P_{O(ac)}} \times 100$

The o/p voltage consists of two components

The dc component $V_{O(dc)}$

The ac /ripple component $V_{ac} = V_{r(rms)}$

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

$$\therefore \qquad V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

Form Factor (FF) which is a measure of the shape of the output voltage is given by

$$FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output (load) voltage}}{\text{DC load output (load) voltage}}$$

The Ripple Factor (RF) w.r.t. o/p voltage w/f

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

$$\therefore \quad r_v = \sqrt{FF^2 - 1}$$

Current Ripple Factor $r_i = \frac{I_{r(rms)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}$

Where $I_{r(rms)} = I_{ac} = \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}$

$V_{r(pp)}$ = peak to peak ac ripple output voltage

$$V_{r(pp)} = V_{O(\max)} - V_{O(\min)}$$

$I_{r(pp)}$ = peak to peak ac ripple load current

$$I_{r(pp)} = I_{O(\max)} - I_{O(\min)}$$

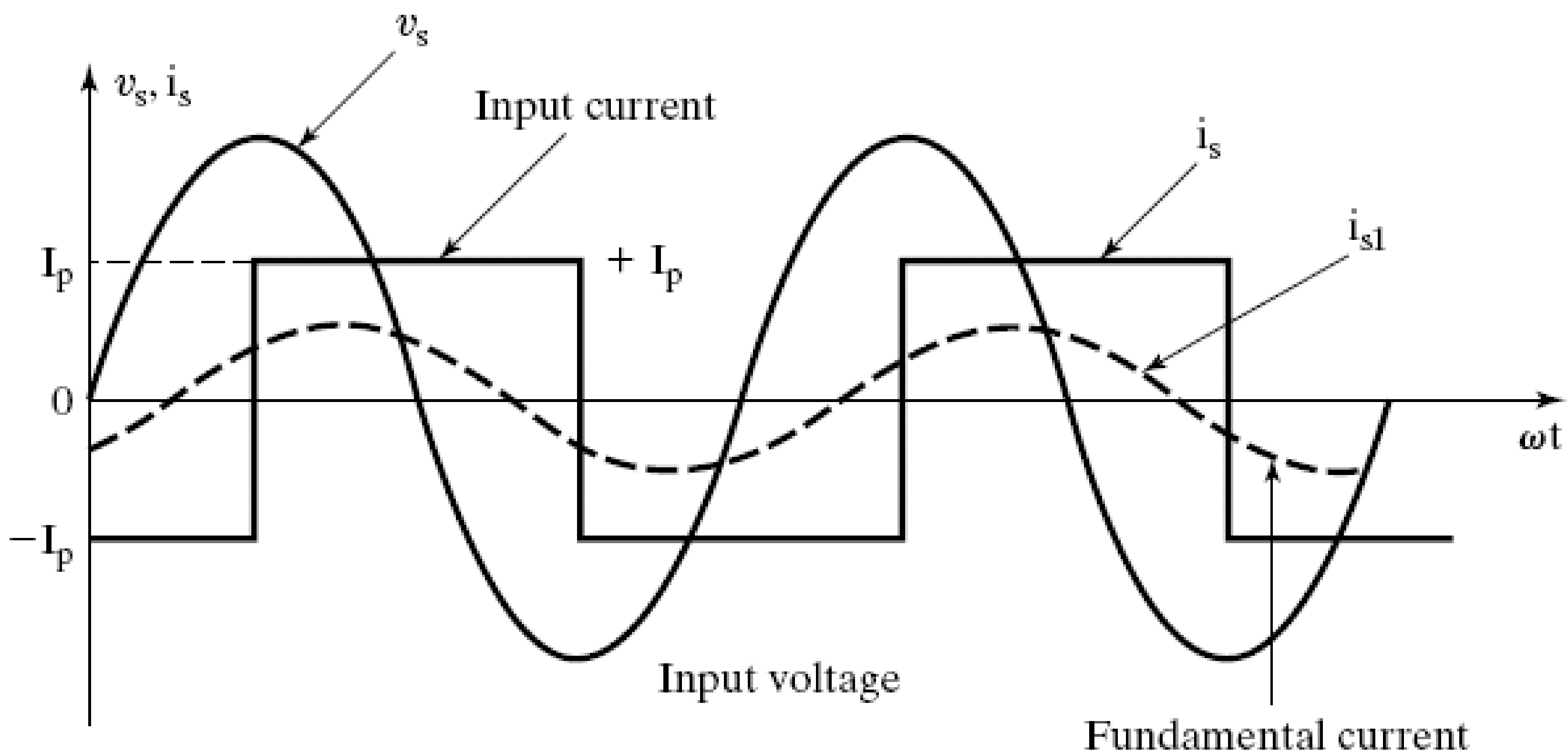
Transformer Utilization Factor (TUF)

$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

V_S = RMS supply (secondary) voltage

I_S = RMS supply (secondary) current


v_s
v_s, i_s
Input current
i_s
i_{s1}
I_p
$+ I_p$
0
ωt
$-I_p$
Input voltage
Fundamental current

Where

$v_S =$ Supply voltage at the transformer secondary side

$i_S =$ i/p supply current

(transformer secondary winding current)

$i_{S1} =$ Fundamental component of the i/p supply current

$I_P =$ Peak value of the input supply current

$\phi =$ Phase angle difference between (sine wave components) the fundamental components of i/p supply current & the input supply voltage.

$\phi =$ Displacement angle (phase angle)

For an RL load

$\phi =$ Displacement angle = Load impedance angle

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) or

Fundamental Power Factor

$$DF = Cos\phi$$

Harmonic Factor (HF) or

Total Harmonic Distortion Factor ; THD

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2} \right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}} \right)^2 - 1 \right]^{\frac{1}{2}}$$

Where

$I_S =$ RMS value of input supply current.

$I_{S1} =$ RMS value of fundamental component of the i/p supply current.

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S} \cos\phi = \frac{I_{S1}}{I_S} \cos\phi$$

The Crest Factor (CF)

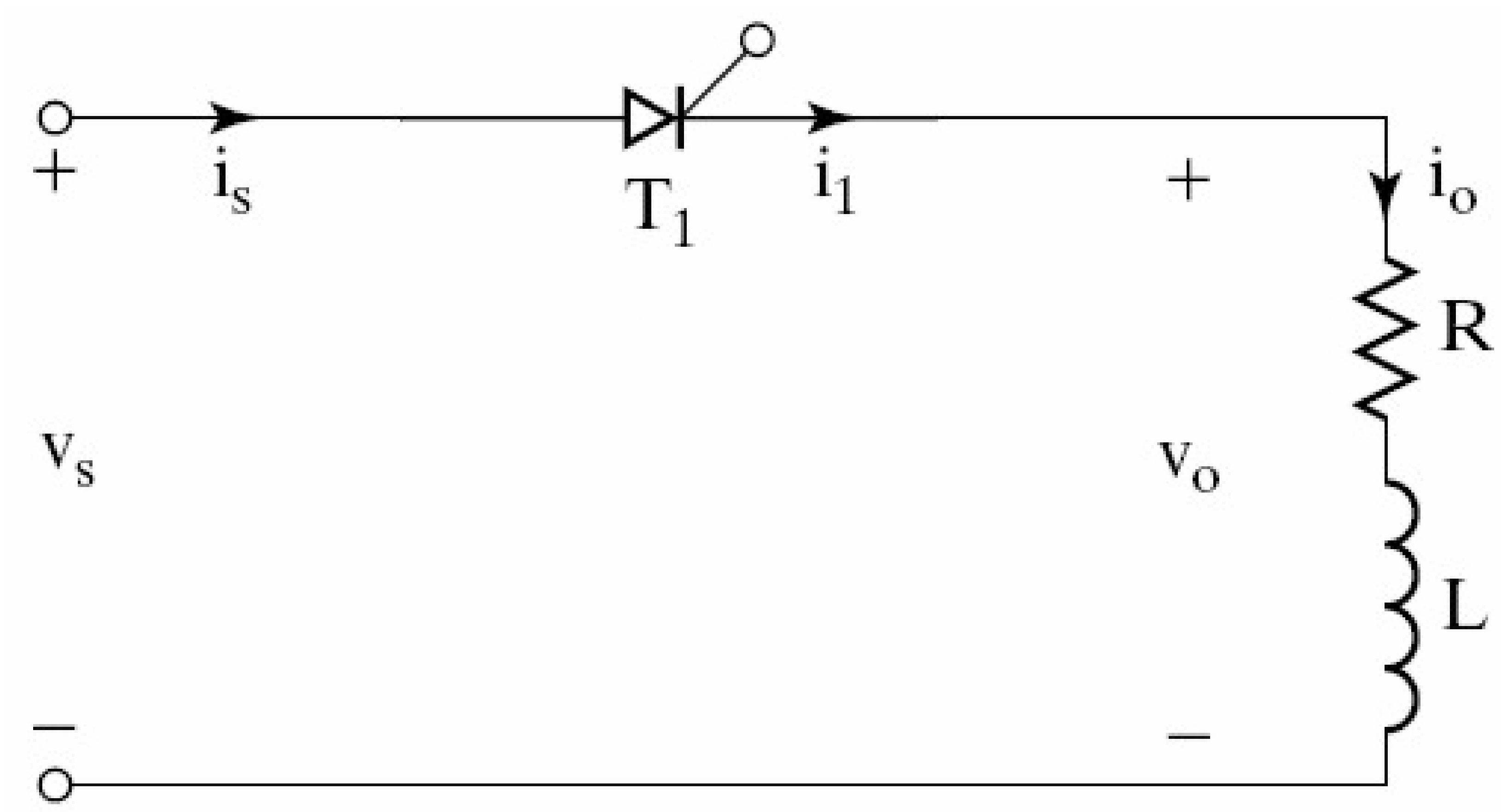
$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

For an Ideal Controlled Rectifier

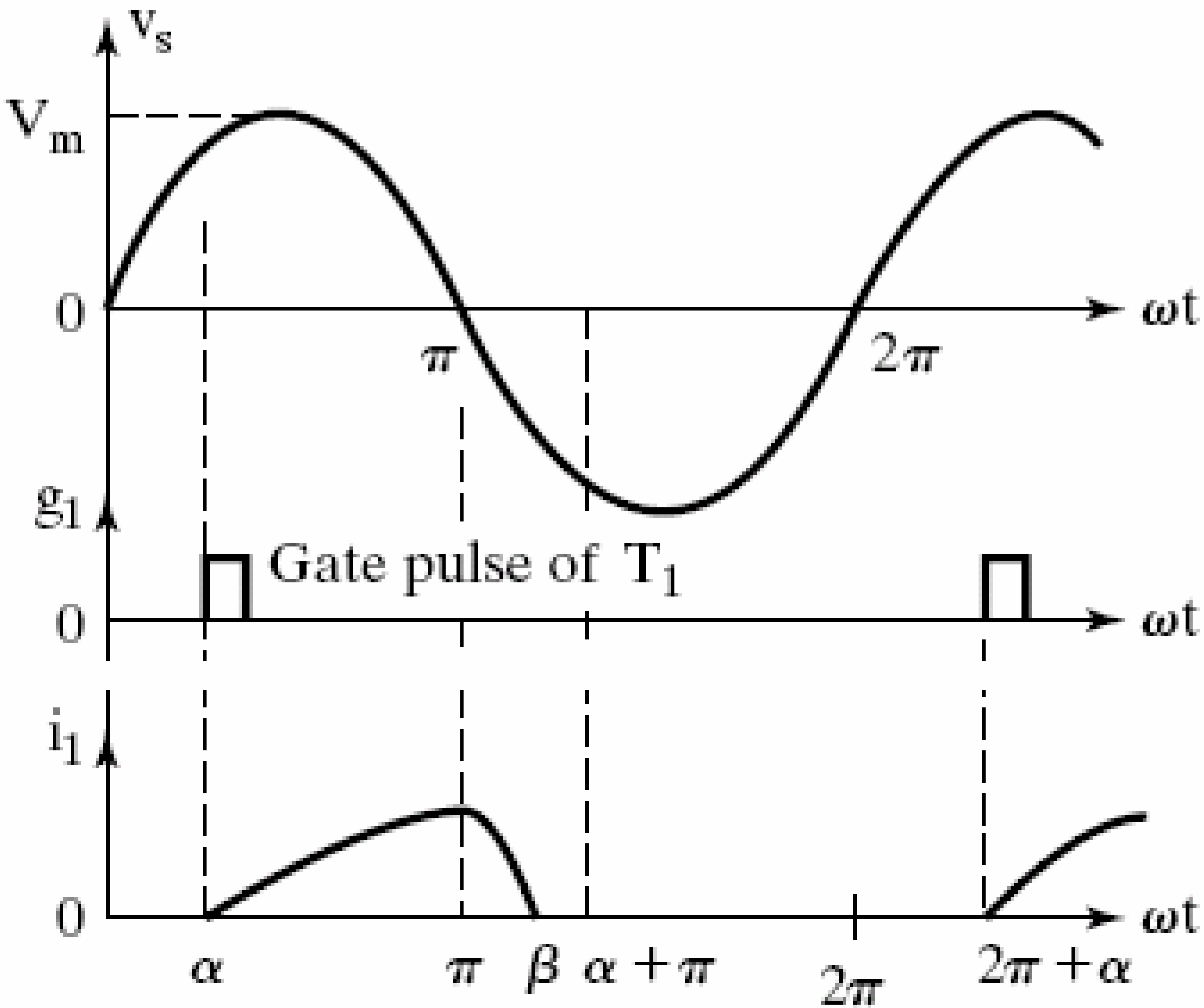
$$FF = 1; \;\; \eta = 100\% \; ; \;\; V_{ac} = V_{r(rms)} = 0 \; ; \;\; TUF = 1;$$

$$RF = r_v = 0 \; ; \;\; HF = THD = 0; \;\;\; PF = DPF = 1$$

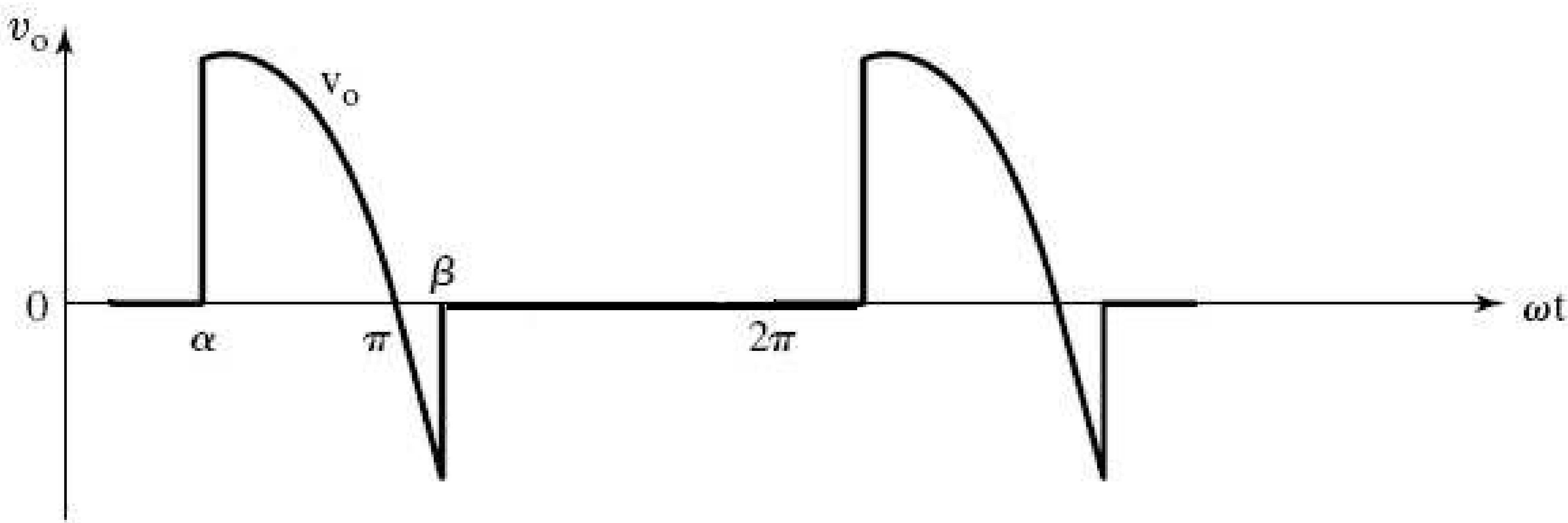
Single Phase Half Wave Controlled Rectifier With An RL Load


+
i_S
T_1
i_1
+
i_O
R
v_S
v_O
L
–
–

Input Supply Voltage (V_s) & Thyristor (Output) Current Waveforms


v_s
V_m
0
ωt
π
2π
g_1
Gate pulse of T_1
0
ωt
i_1
0
ωt
α
π
β
$\alpha + \pi$
2π
$2\pi + \alpha$

Output (Load) Voltage Waveform



To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin\omega t \;\; ; \; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

$V_m = \sqrt{2}V_S$ = maximum supply voltage.

$Z = \sqrt{R^2 + (\omega L)^2}$ =Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle.

$\tau = \frac{L}{R}$ = Load circuit time constant.

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\frac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

Extinction angle β can be calculated by using the condition that $i_O = 0 \ at \ \omega t = \beta$

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\right] = 0$$

$$\therefore \sin\left(\beta - \phi\right) = e^{\frac{-R}{\omega L}\left(\beta - \alpha\right)} \times \sin\left(\alpha - \phi\right)$$

β can be calculated by solving the above eqn.

To Derive An Expression For Average (DC) Load Voltage of a Single Half Wave Controlled Rectifier with RL Load

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right]$$

$$v_O = 0 \text{ for } \omega t = 0 \text{ to } \alpha \text{ \& for } \omega t = \beta \text{ to } 2\pi$$

$$\therefore V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O.d(\omega t)\right];$$

$$v_O = V_m \sin\omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\beta}\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

$$\therefore V_{O(dc)} = V_L = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

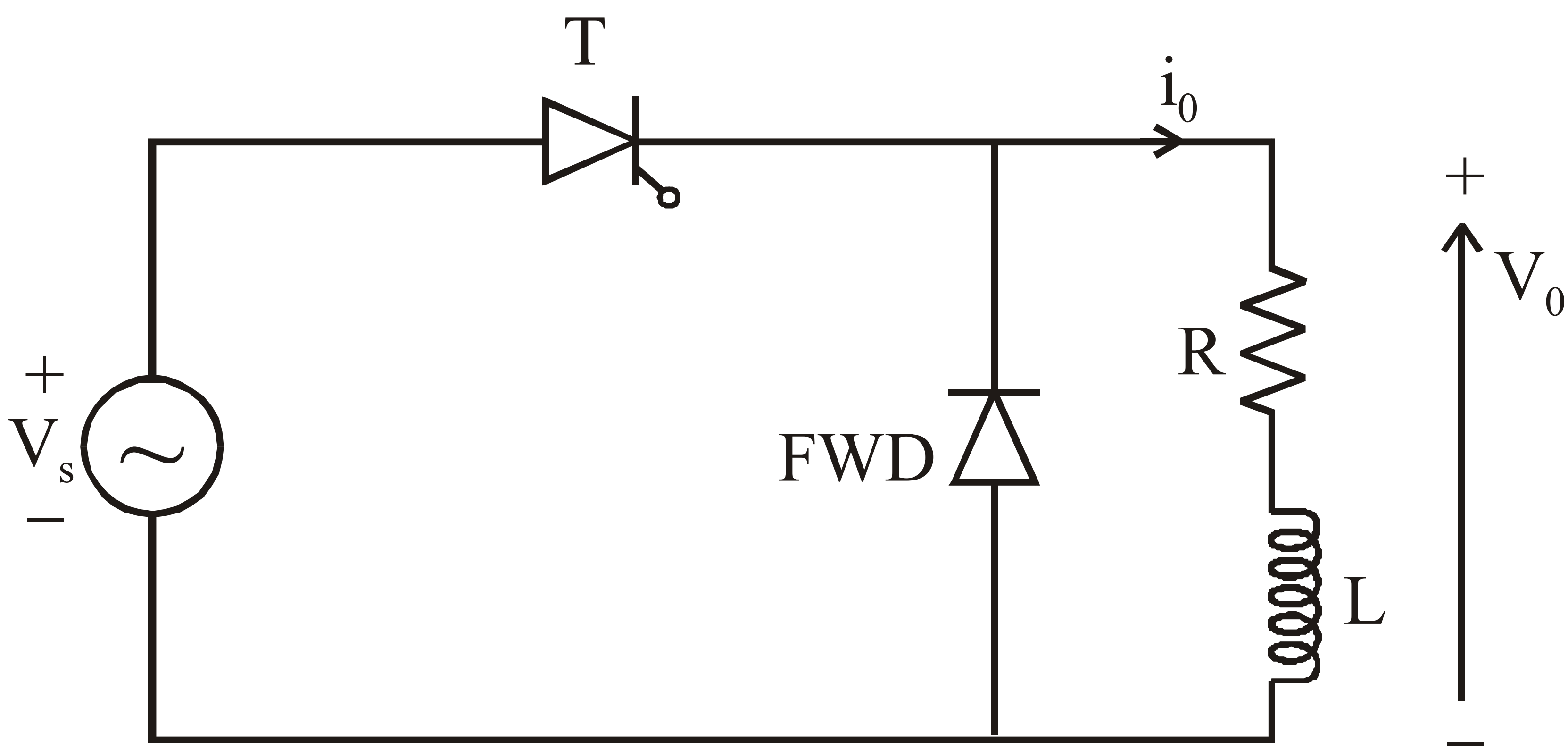
Effect of Load Inductance on the Output

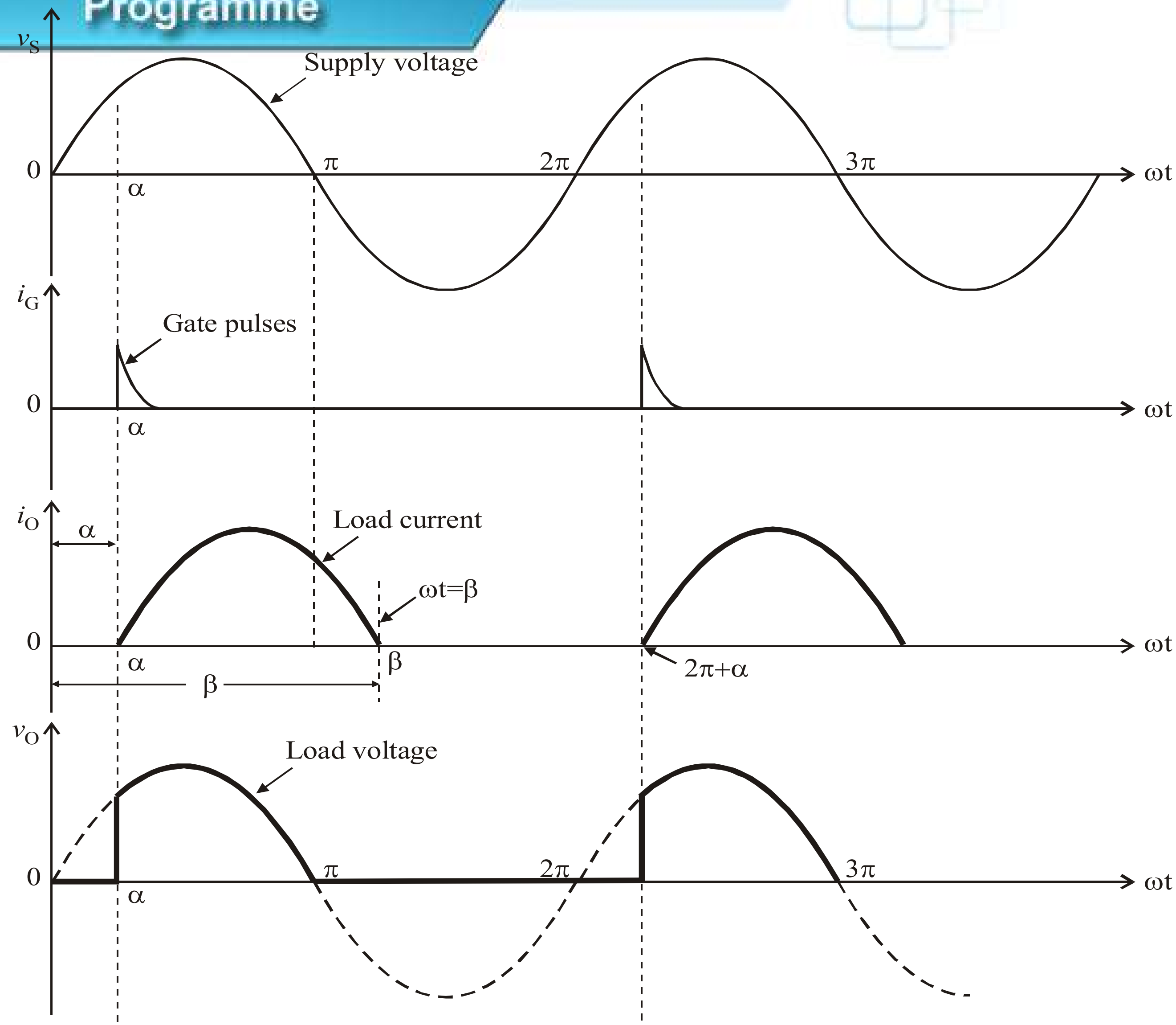
During the period $\omega t = \pi$ to β the instantaneous o/p voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L}\left(\cos\alpha - \cos\beta\right)$$

Single Phase Half Wave Controlled Rectifier
With RL Load
&
Free Wheeling Diode


T
i_0
+
V_0
R
FWD
+
V_s
−
L
−


v_S
Supply voltage
0
π
2π
3π
ωt
α
i_G
Gate pulses
0
ωt
α
i_O
α
Load current
ωt=β
0
ωt
α
β
β
2π+α
v_O
Load voltage
0
π
2π
3π
ωt
α

The average output voltage

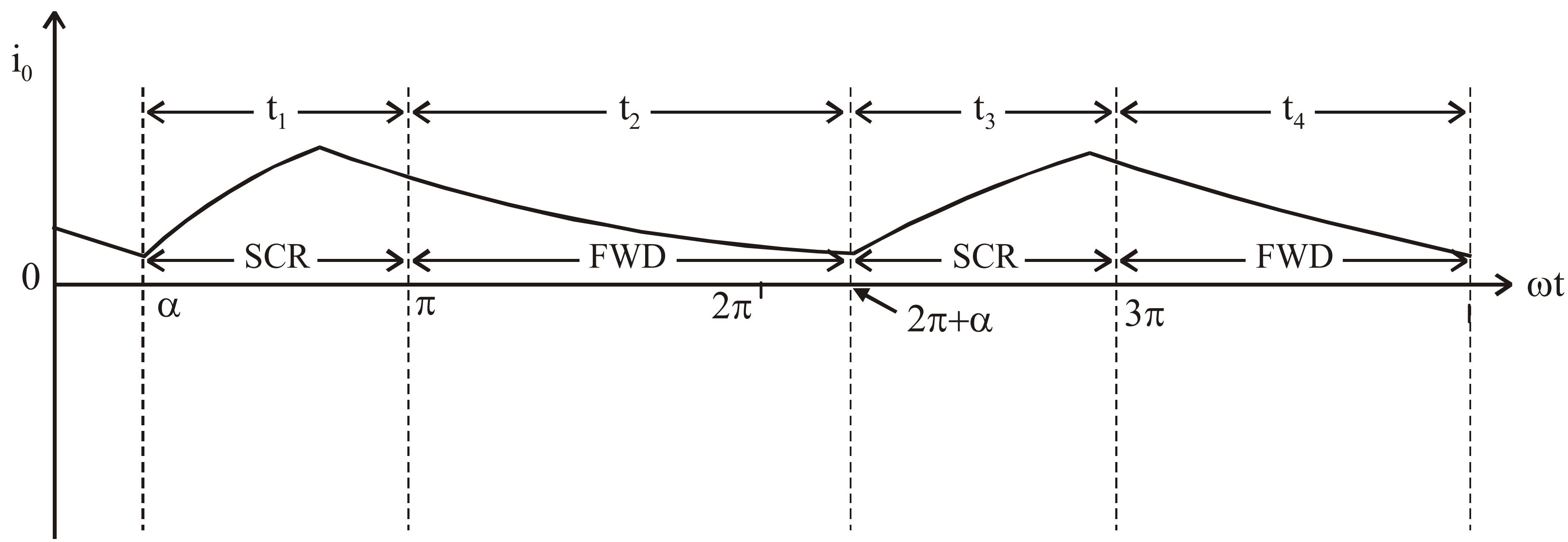
$V_{dc} = \frac{V_m}{2\pi}[1 + \cos\alpha]$ which is the same as that of a purely resistive load.

The following points are to be noted

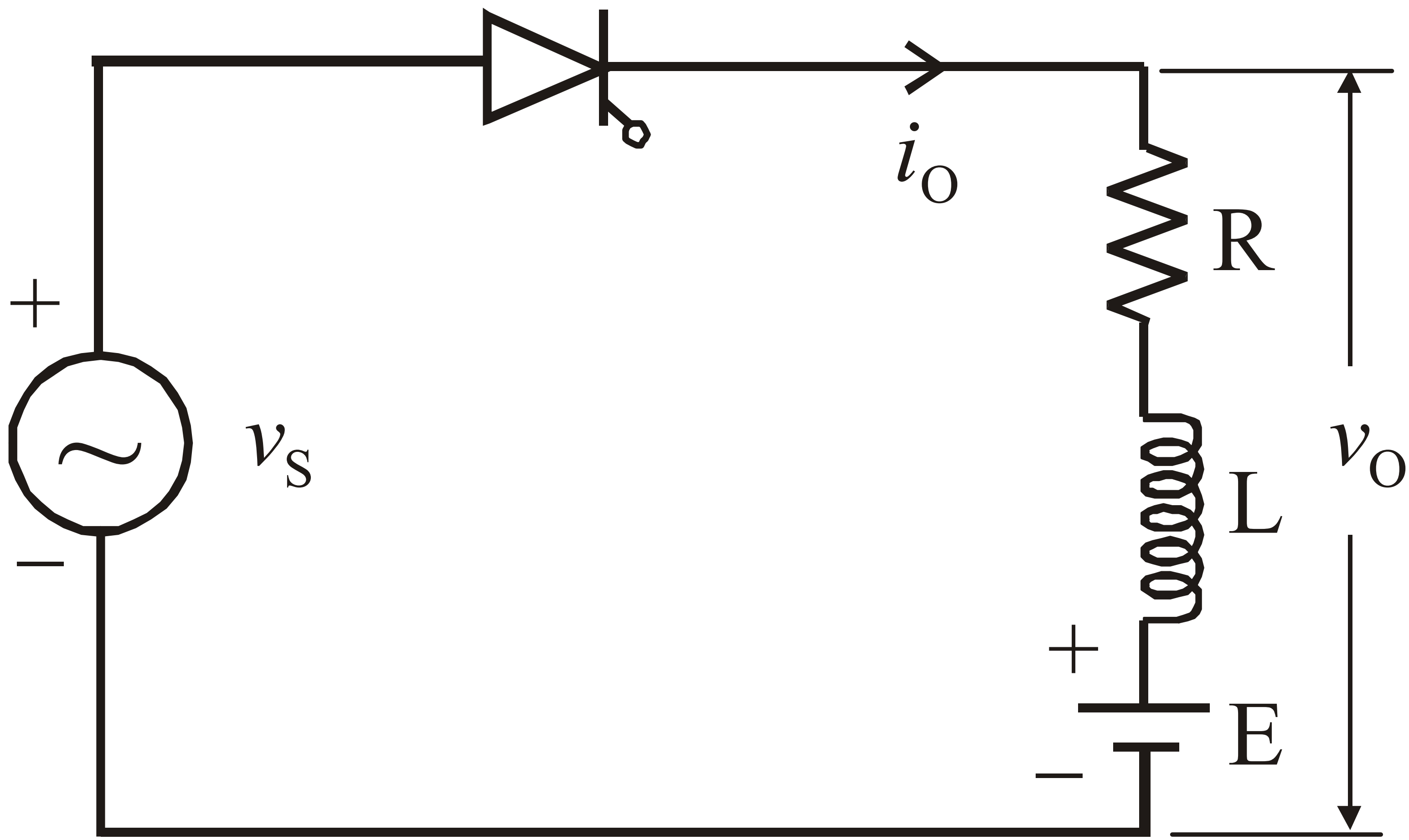
For low value of inductance, the load current tends to become discontinuous.

During the period α to π
the load current is carried by the SCR.
During the period π to β load current is carried by the free wheeling diode.
The value of β depends on the value of R and L and the forward resistance of the FWD.

For Large Load Inductance
the load current does not reach zero, & we obtain continuous load current



Single Phase Half Wave Controlled Rectifier With A General Load


+
−
v_S
i_O
R
L
+
−
E
v_O

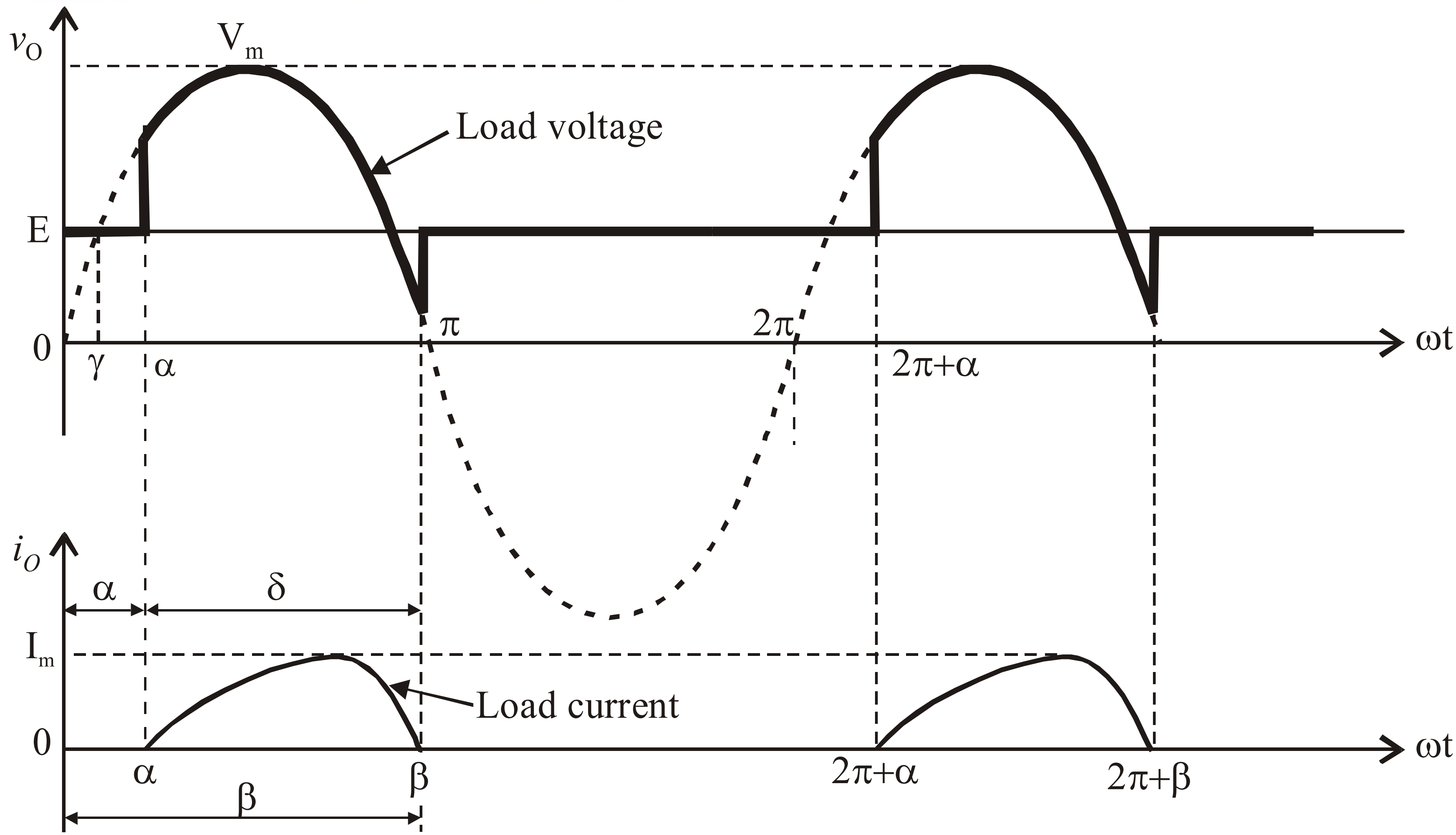
$$\gamma = \sin^{-1}\left(\frac{E}{V_m}\right)$$

For trigger angle $\alpha < \gamma$,

the Thyristor conducts from $\omega t = \gamma$ to β

For trigger angle $\alpha > \gamma$,

the Thyristor conducts from $\omega t = \alpha$ to β


v_O
V_m
Load voltage
E
0
γ
α
π
2π
ωt
2π+α
i_O
α
δ
I_m
Load current
0
α
β
β
2π+α
2π+β
ωt

Equations

$$v_S = V_m \sin\omega t = \text{ Input supply voltage.}$$

$$v_O = V_m \sin\omega t = \text{ o/p } \left(\text{load}\right) \text{ voltage}$$

$$\text{for } \omega t = \alpha \text{ to } \beta.$$

$$v_O = E \text{ for } \omega t = 0 \text{ to } \alpha \text{ \&}$$

$$\text{for } \omega t = \beta \text{ to } 2\pi.$$

Expression for the Load Current

When the thyristor is triggered at a delay angle of $\alpha > \gamma$, the eqn. for the circuit can be written as

$$V_m \sin \omega t = i_O \times R + L\left(\frac{di_O}{dt}\right) + \mathrm{E} \; ; \; \alpha \le \omega t \le \beta$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) - \frac{E}{R} + Ae^{\frac{-t}{\tau}}$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load Impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{ Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{ Load circuit time constant.}$$

The general expression for the o/p current can be written as $i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}t}$

To find the value of the constant 'A' apply the initial conditions at $\omega t = \alpha$, load current $i_O = 0$, Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}\times\frac{\alpha}{\omega}}$$

We obtain the value of constant 'A' as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right] e^{\frac{R}{\omega L}\alpha}$$

Substituting the value of the constant 'A' in the expression for the load current; we get the complete expression for the output load current as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right] e^{\frac{-R}{\omega L}(\omega t - \alpha)}$$

To Derive
An
Expression For The Average
Or
DC Load Voltage

$$V_{O(dc)} = \frac{1}{2\pi} \int_0^{2\pi} v_O . d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi} \left[\int_0^{\alpha} v_O . d(\omega t) + \int_{\alpha}^{\beta} v_O . d(\omega t) + \int_{\beta}^{2\pi} v_O . d(\omega t) \right]$$

$v_O = V_m \sin \omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi} \left[\int_0^{\alpha} E . d(\omega t) + \int_{\alpha}^{\beta} V_m \sin \omega t + \int_{\beta}^{2\pi} E . d(\omega t) \right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\omega t)\Big/_{0}^{\alpha} + V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} + E(\omega t)\Big/_{\beta}^{2\pi}\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\alpha - 0) - V_m(\cos\beta - \cos\alpha) + E(2\pi - \beta)\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[(\cos\alpha - \cos\beta)\right] + \frac{E}{2\pi}\left[(2\pi - \beta + \alpha)\right]$$

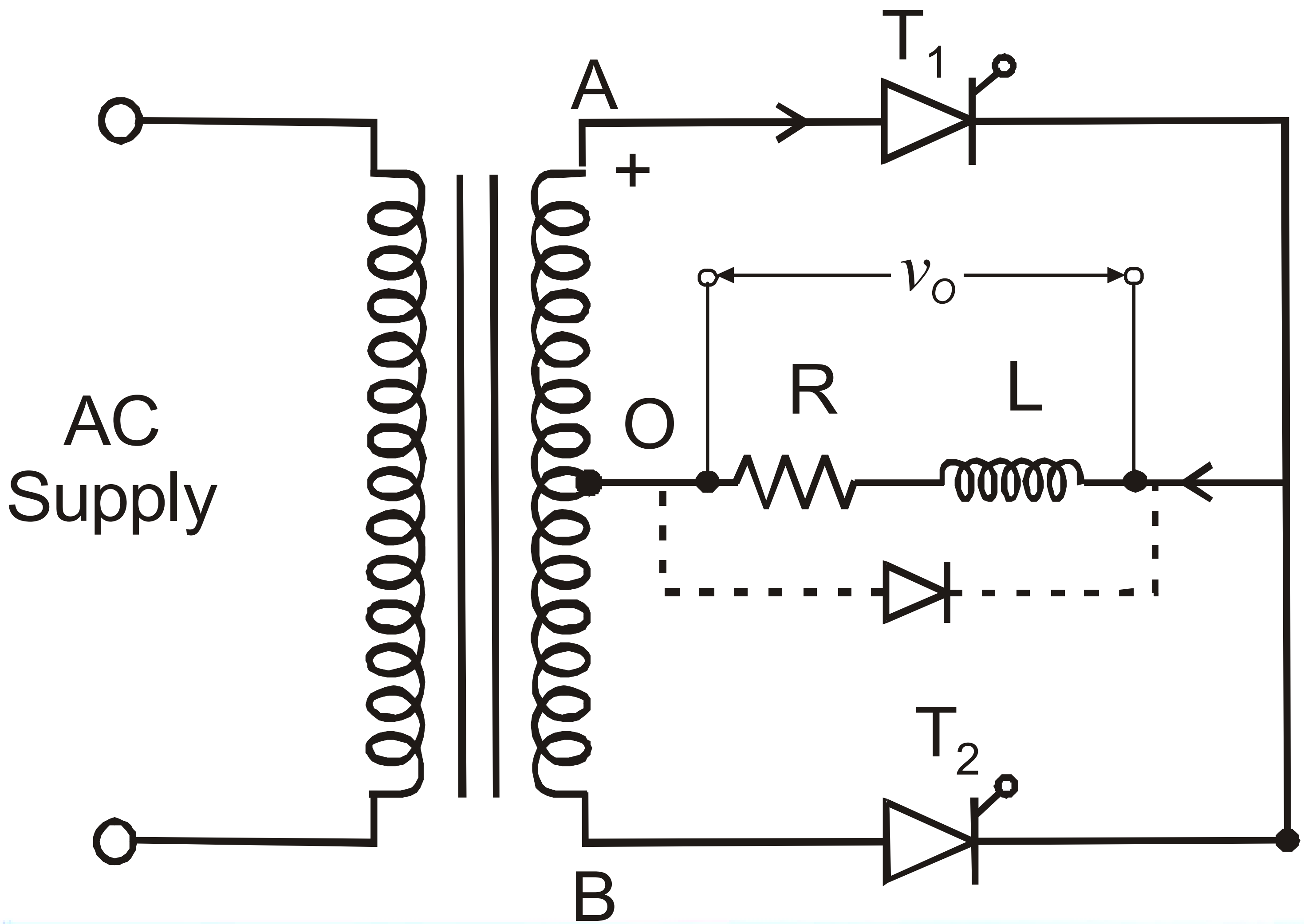
$$V_{O(dc)} = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + \left[\frac{2\pi - (\beta - \alpha)}{2\pi}\right]E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated by using the expression

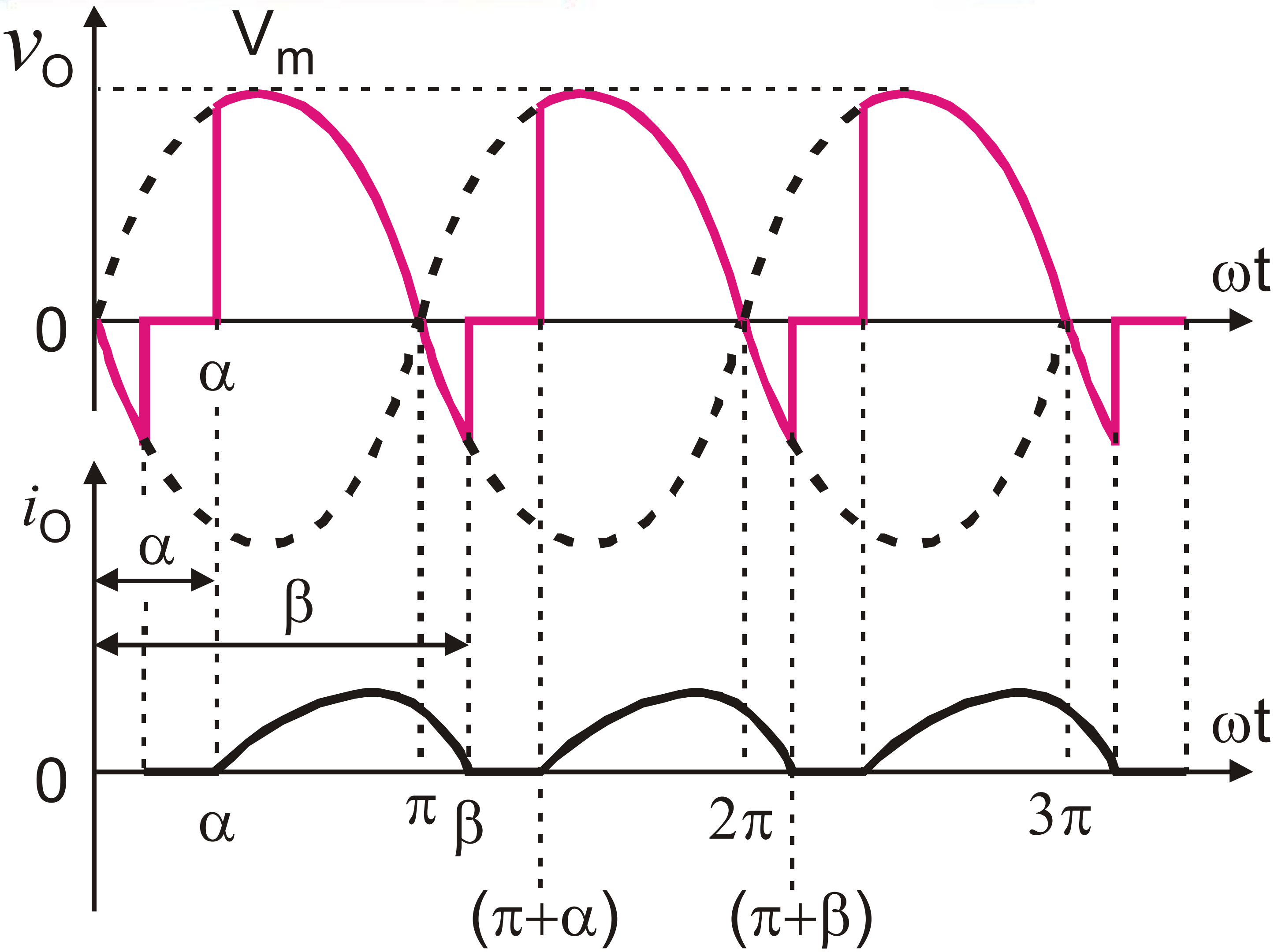
$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{0}^{2\pi} v_O^2 .d(\omega t)\right]}$$

Single Phase
Full Wave Controlled Rectifier
Using A
Center Tapped Transformer


T1
A
+
vo
AC
Supply
R
L
O
T2
B

Discontinuous Load Current Operation without FWD for

$$\pi < \beta < (\pi + \alpha)$$


v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \;;\; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

$V_m = \sqrt{2}V_S$ = maximum supply voltage.

$Z = \sqrt{R^2 + (\omega L)^2}$ =Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle.

$\tau = \frac{L}{R}$ = Load circuit time constant.

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\dfrac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha-\phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha-\phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha-\phi)\right]$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin(\omega t-\phi)+e^{\frac{-R}{\omega L}(\omega t-\alpha)}\left[\frac{-V_m}{Z}\sin(\alpha-\phi)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t-\phi)-\sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

Extinction angle β can be calculated by using the condition that $i_O = 0 \; at \;\; \omega t = \beta$

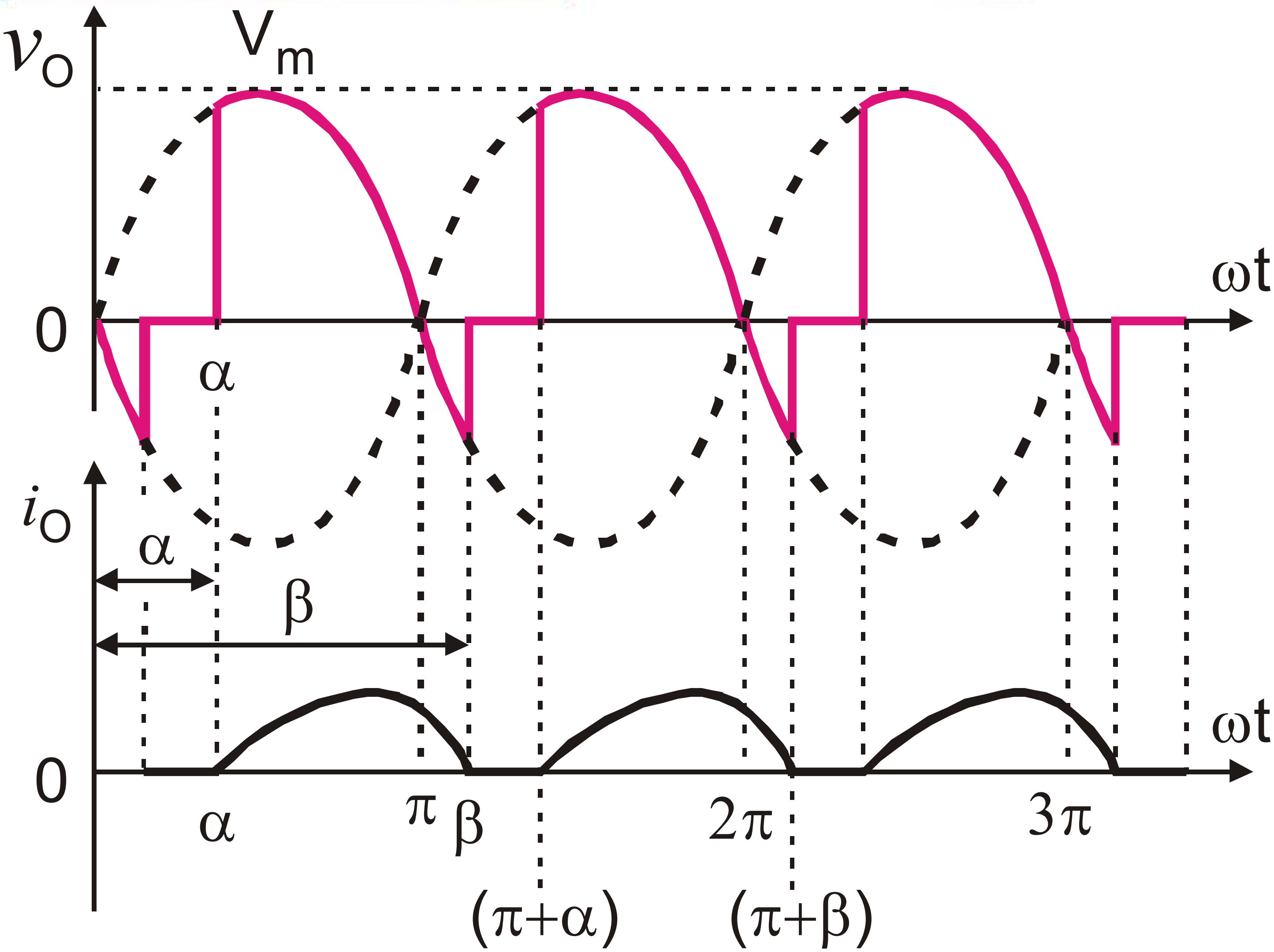
$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\right] = 0$$

$$\therefore \sin\left(\beta - \phi\right) = e^{\frac{-R}{\omega L}\left(\beta - \alpha\right)} \times \sin\left(\alpha - \phi\right)$$

β can be calculated by solving the above eqn.

To Derive An Expression For The DC Output Voltage Of A Single Phase Full Wave Controlled Rectifier With RL Load

(Without FWD)


v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

When the load inductance is negligible$\left(\text{ i.e., } L \approx 0\right)$

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \cos\pi\right)$$

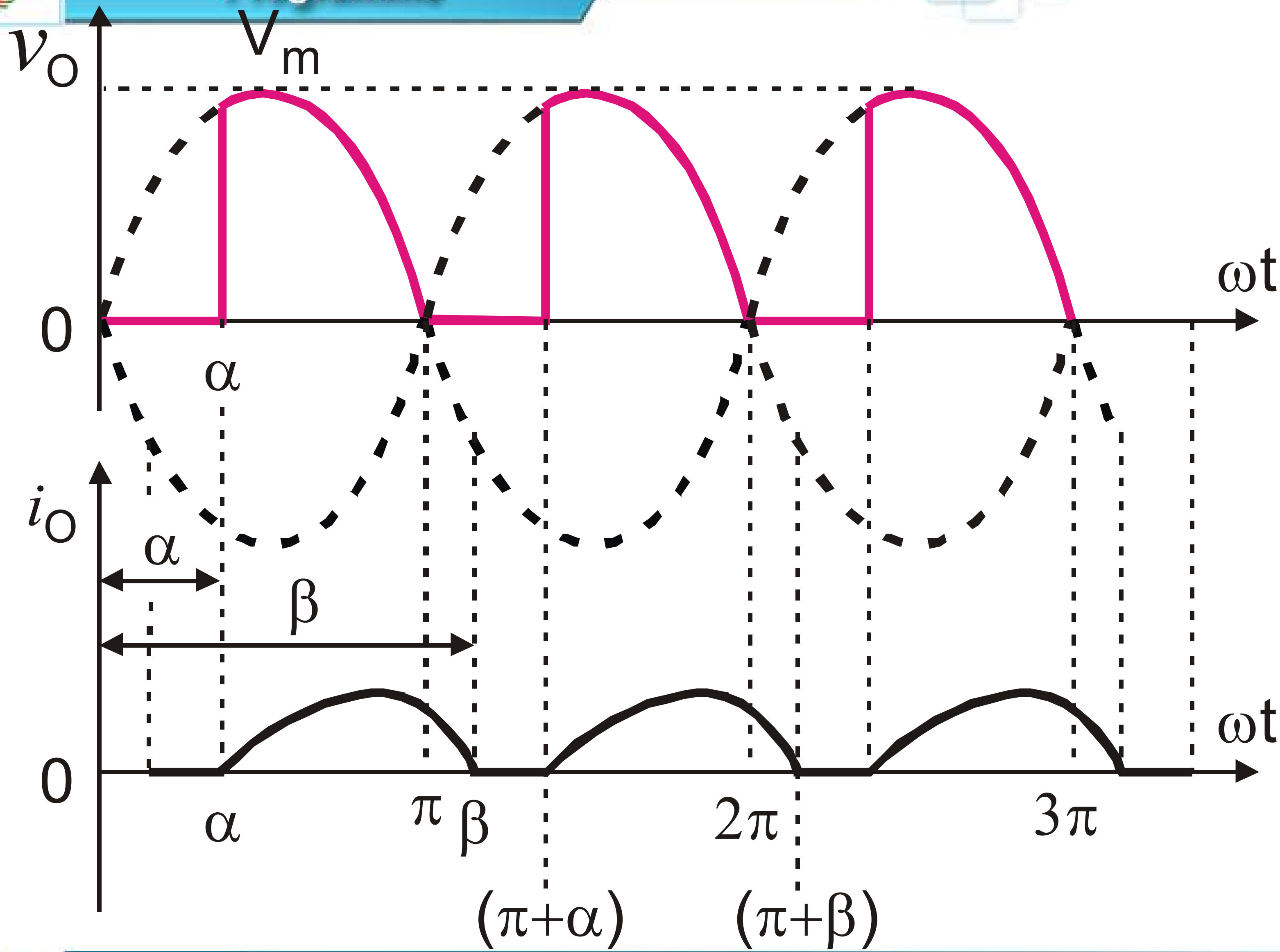
$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \left(-1\right)\right)$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(1 + \cos\alpha\right);\text{ for R load, when } \beta = \pi$$

To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

Discontinuous Load Current Operation with FWD


v
O
V
m
ωt
0
α
i
O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_O \approx 0$ during discontinuous load current.

To Derive an Expression For The DC Output Voltage For A Single Phase Full Wave Controlled Rectifier With RL Load & FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

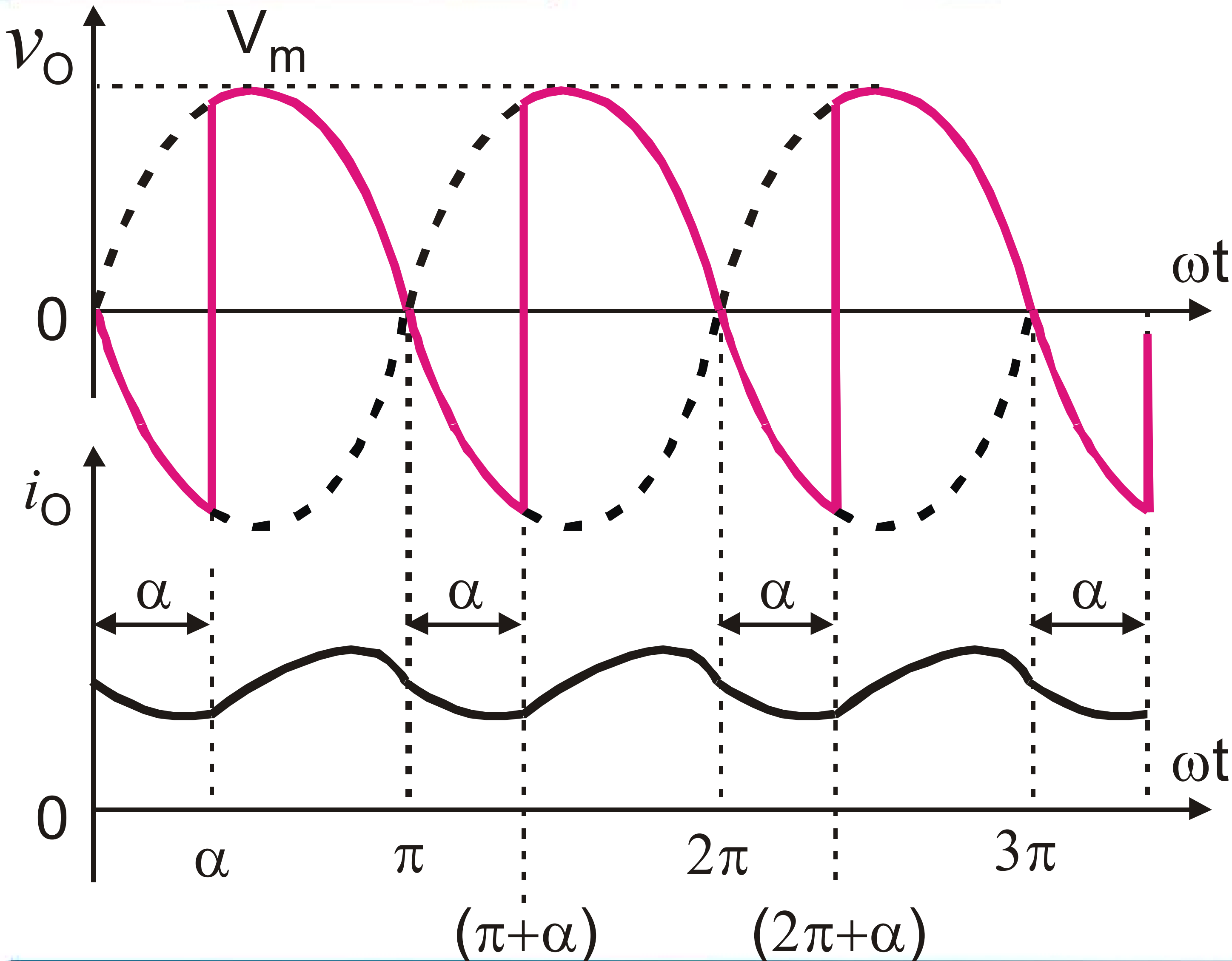
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \quad ; \; \cos \pi = -1$$

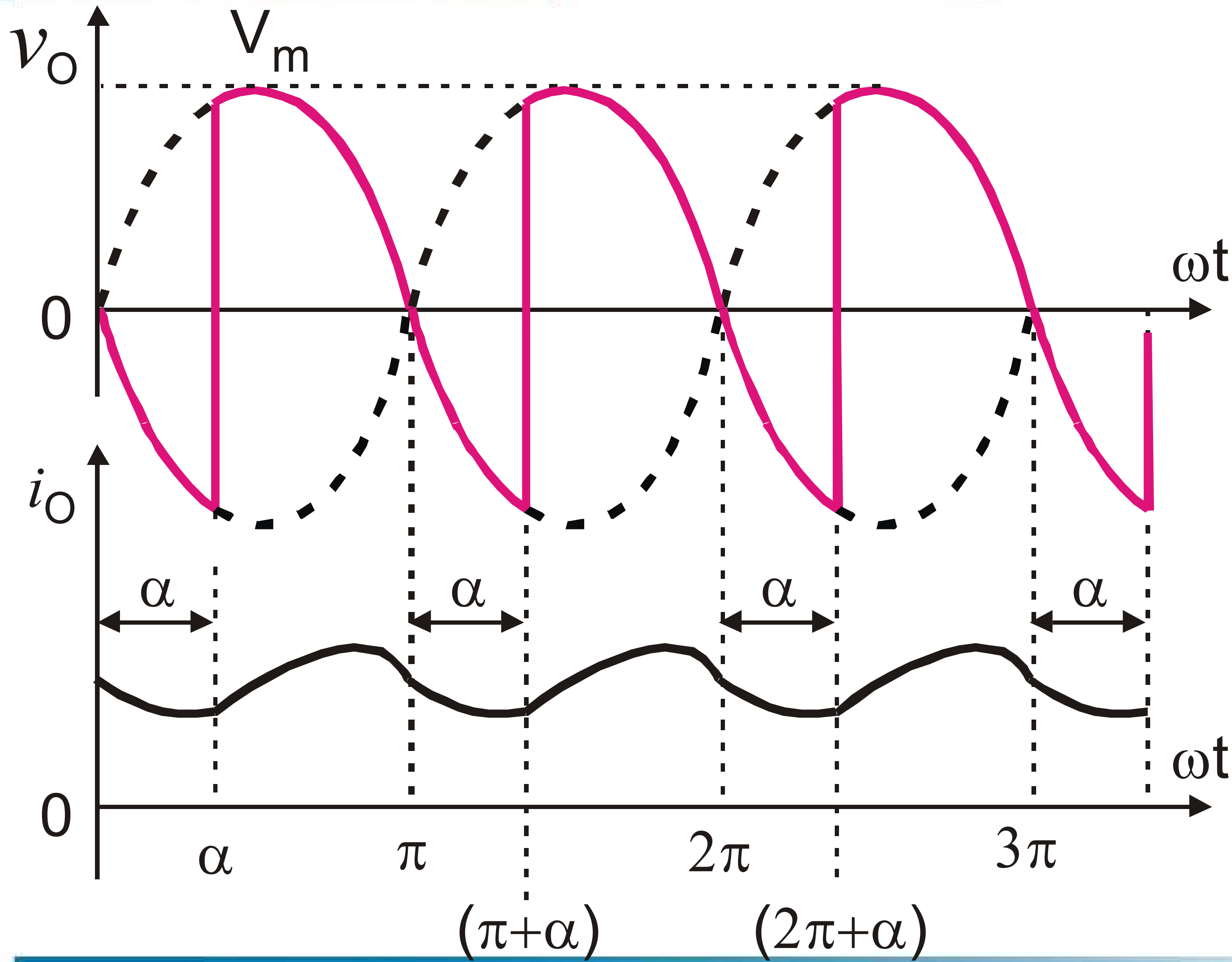
$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left(1 + \cos \alpha \right)$$

- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.

Continuous Load Current Operation (Without FWD)


v_O
V_m
0
ωt
i_O
α
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

To Derive
An Expression For
Average / DC Output Voltage
Of
Single Phase Full Wave Controlled Rectifier For Continuous Current Operation without FWD


v_O
V_m
0
ωt
i_O
α
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi+\alpha)} \right]$$

$$V_{O(dc)} = V_{dc}$$

$$= \frac{V_m}{\pi}\left[\cos\alpha - \cos\left(\pi + \alpha\right)\right] \ ;$$

$$\cos\left(\pi + \alpha\right) = -\cos\alpha$$

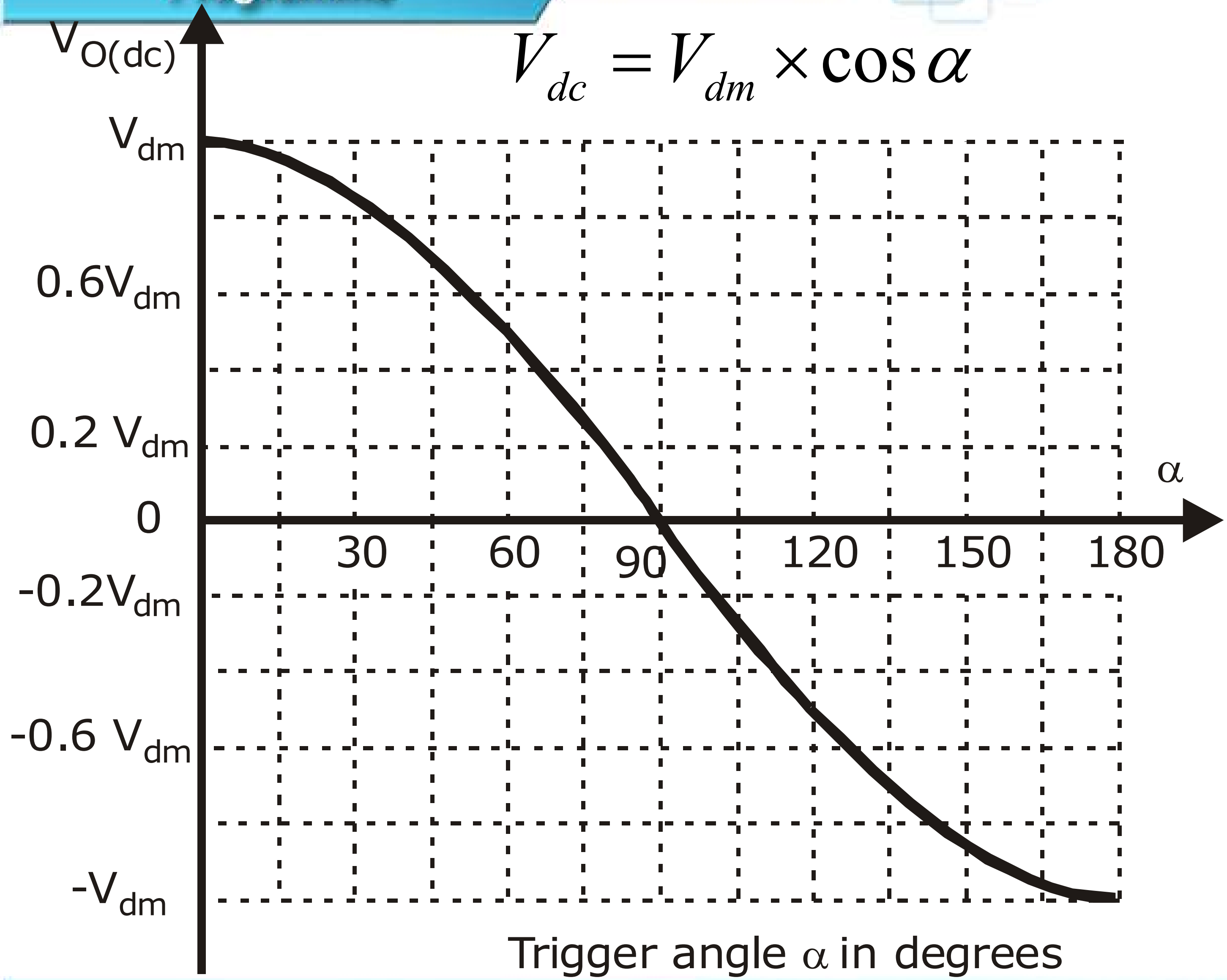
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\cos\alpha + \cos\alpha\right]$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

- By plotting $V_{O(dc)}$ *versus* α, we obtain the control characteristic of a single phase full wave controlled rectifier with RL load for continuous load current operation without FWD

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	$V_{dc} = V_{dm} \times \cos\alpha$
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	

$$V_{dc} = V_{dm} \times \cos\alpha$$



By varying the trigger angle we can vary the output dc voltage across the load. Hence we can control the dc output power flow to the load.

For trigger angle α, 0 to 90^0 $\left(i.e.,\ 0 \le \alpha \le 90^0\right)$;

$\cos\alpha$ is positive and hence V_{dc} is positive

V_{dc} & I_{dc} are positive ; $P_{dc} = \left(V_{dc} \times I_{dc}\right)$ is positive

Converter operates as a Controlled Rectifier.

Power flow is from the ac source to the load.

For trigger angle α, 90^0 to 180^0

$$\left(i.e.,\ 90^0 \le \alpha \le 180^0\right),$$

$\cos\alpha$ is negative and hence

V_{dc} is negative; I_{dc} is positive ;

$$P_{dc} = \left(V_{dc} \times I_{dc}\right) \text{ is negative.}$$

In this case the converter operates as a Line Commutated Inverter.

Power flows from the load ckt. to the i/p ac source.

The inductive load energy is fed back to the i/p source.

Drawbacks Of Full Wave Controlled Rectifier With Centre Tapped Transformer

- We require a centre tapped transformer which is quite heavier and bulky.
- Cost of the transformer is higher for the required dc output voltage & output power.
- Hence full wave bridge converters are preferred.

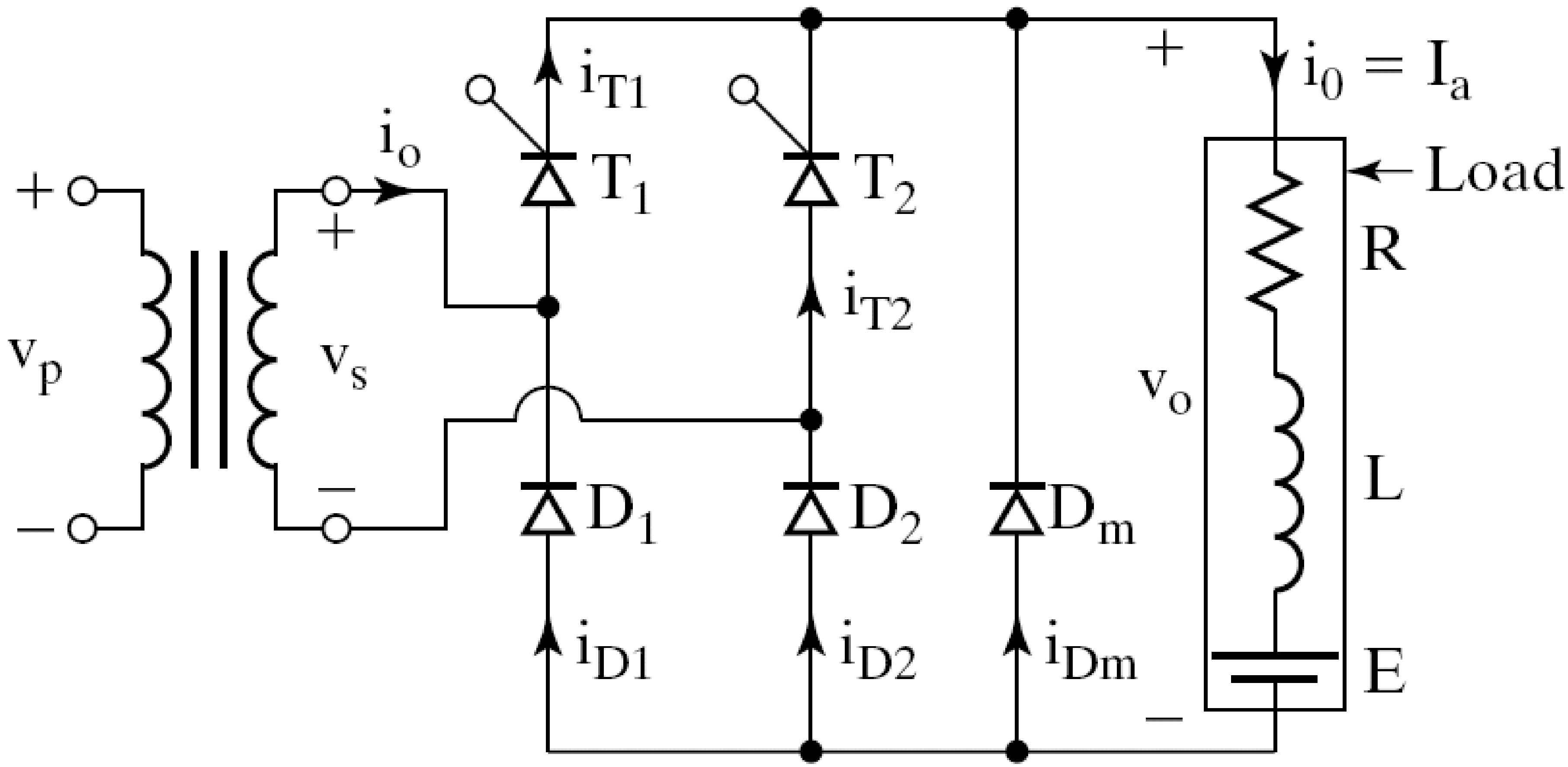
Single Phase
Full Wave Bridge Controlled Rectifier

2 types of FW Bridge Controlled Rectifiers are

- Half Controlled Bridge Converter
  (Semi-Converter)
- Fully Controlled Bridge Converter
  (Full Converter)

The bridge full wave controlled rectifier does not require a centre tapped transformer

Single Phase
Full Wave Half Controlled Bridge Converter
(Single Phase Semi Converter)


i_{T1}
i_o
T_1
T_2
i_{T2}
+
−
v_p
v_s
D_1
D_2
D_m
i_{D1}
i_{D2}
i_{Dm}
$i_0 = I_a$
Load
R
v_o
L
E

Trigger Pattern of Thyristors

Thyristor T_1 *is triggered at*

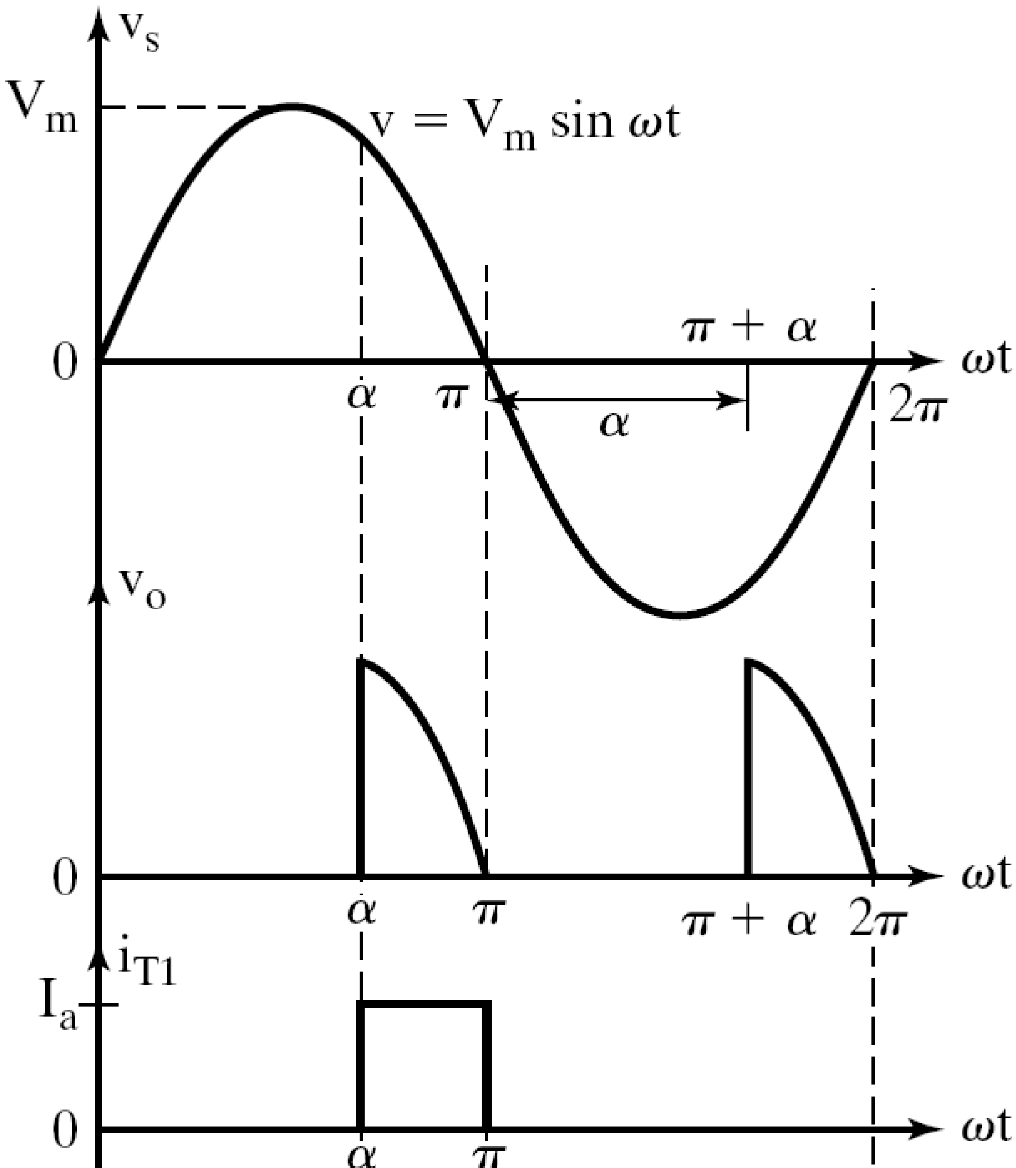
$$\omega t = \alpha,\ at\ \omega t = (2\pi + \alpha), \ldots$$

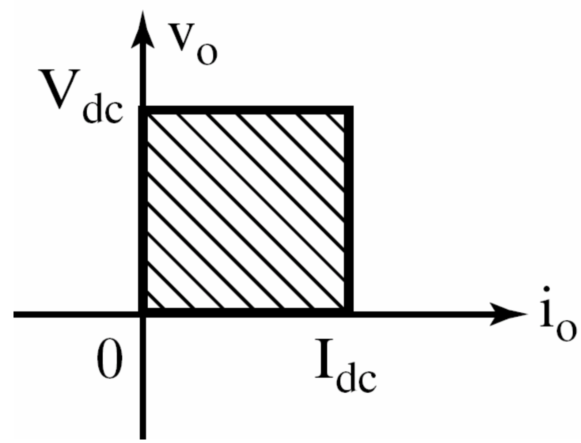
Thyristor T_2 *is triggered at*

$$\omega t = (\pi + \alpha),\ at\ \omega t = (3\pi + \alpha), \ldots$$

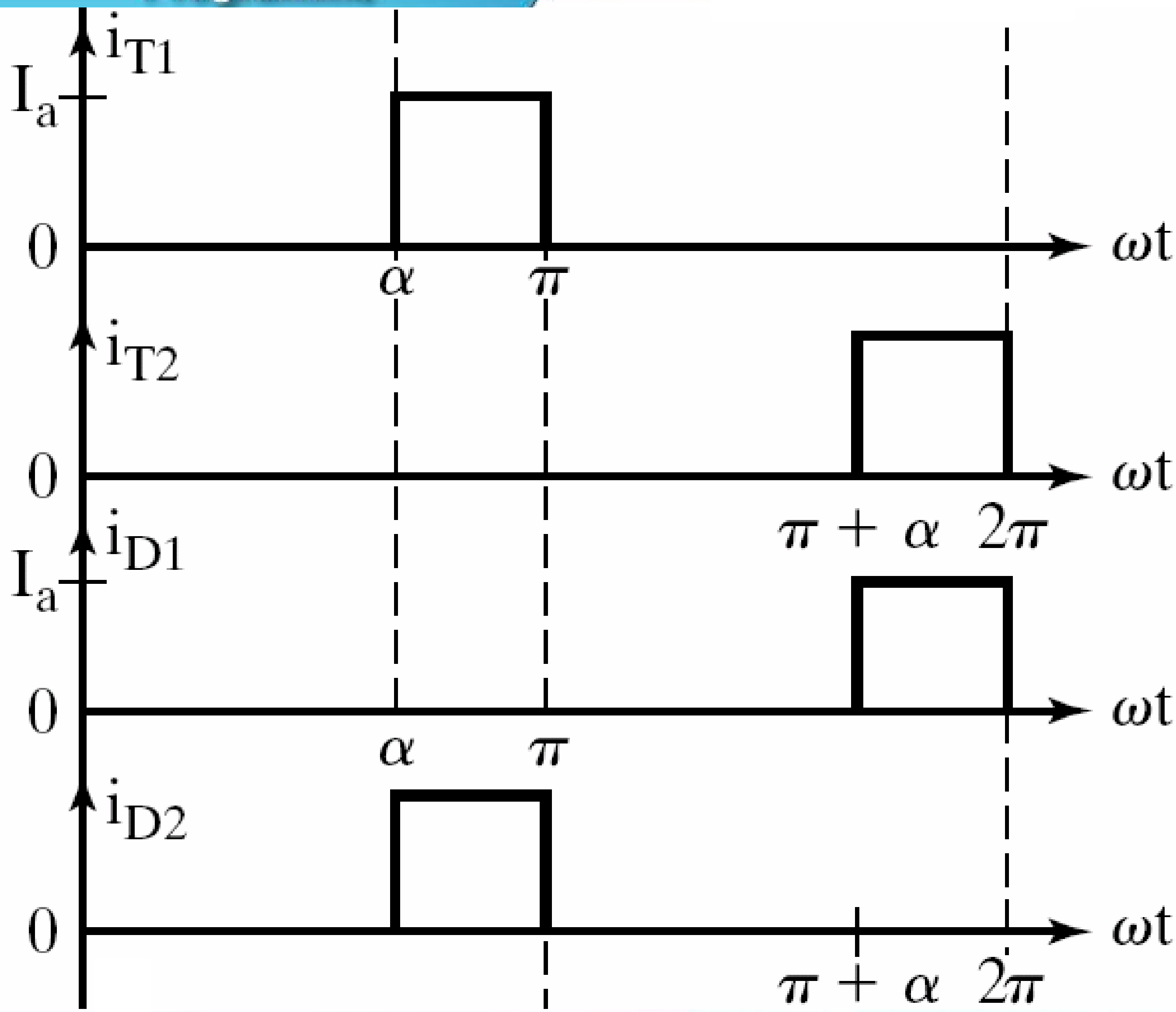
The time delay between the gating signals of T_1 & $T_2 = \pi$ *radians or* 180^0

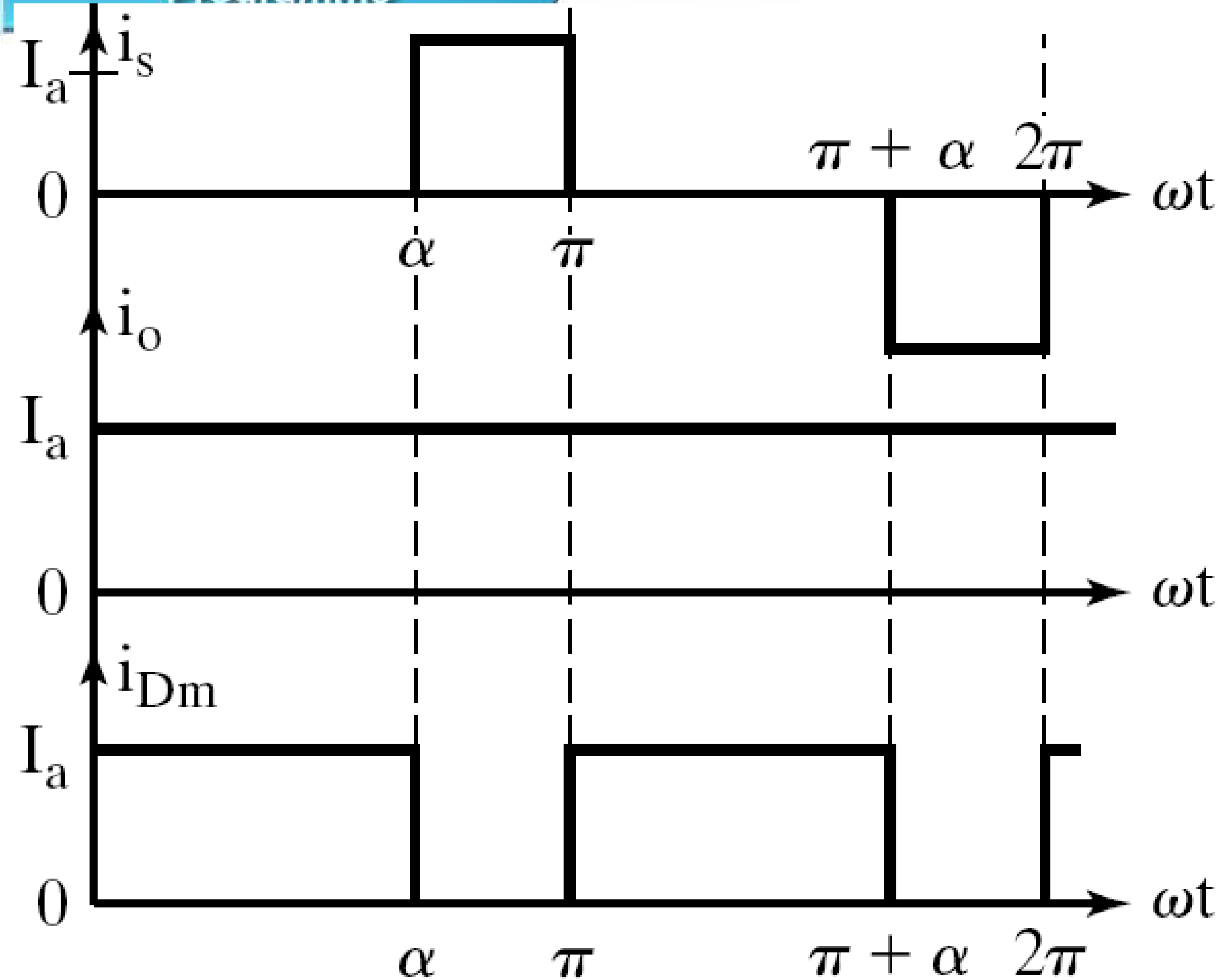
Waveforms of single phase semi-converter with general load & FWD for $\alpha > 90^0$





Single Quadrant Operation


i_{T1}
I_a
0
α
π
ωt
i_{T2}
0
ωt
π + α
2π
i_{D1}
I_a
0
ωt
α
π
i_{D2}
0
ωt
π + α
2π



I_a
i_s
0
α
π
$\pi + \alpha$
2π
ωt
i_o
I_a
0
ωt
i_{Dm}
I_a
0
ωt
α
π
$\pi + \alpha$
2π

Thyristor T_1 & D_1 conduct

from $\omega t = \alpha \ to \ \pi$

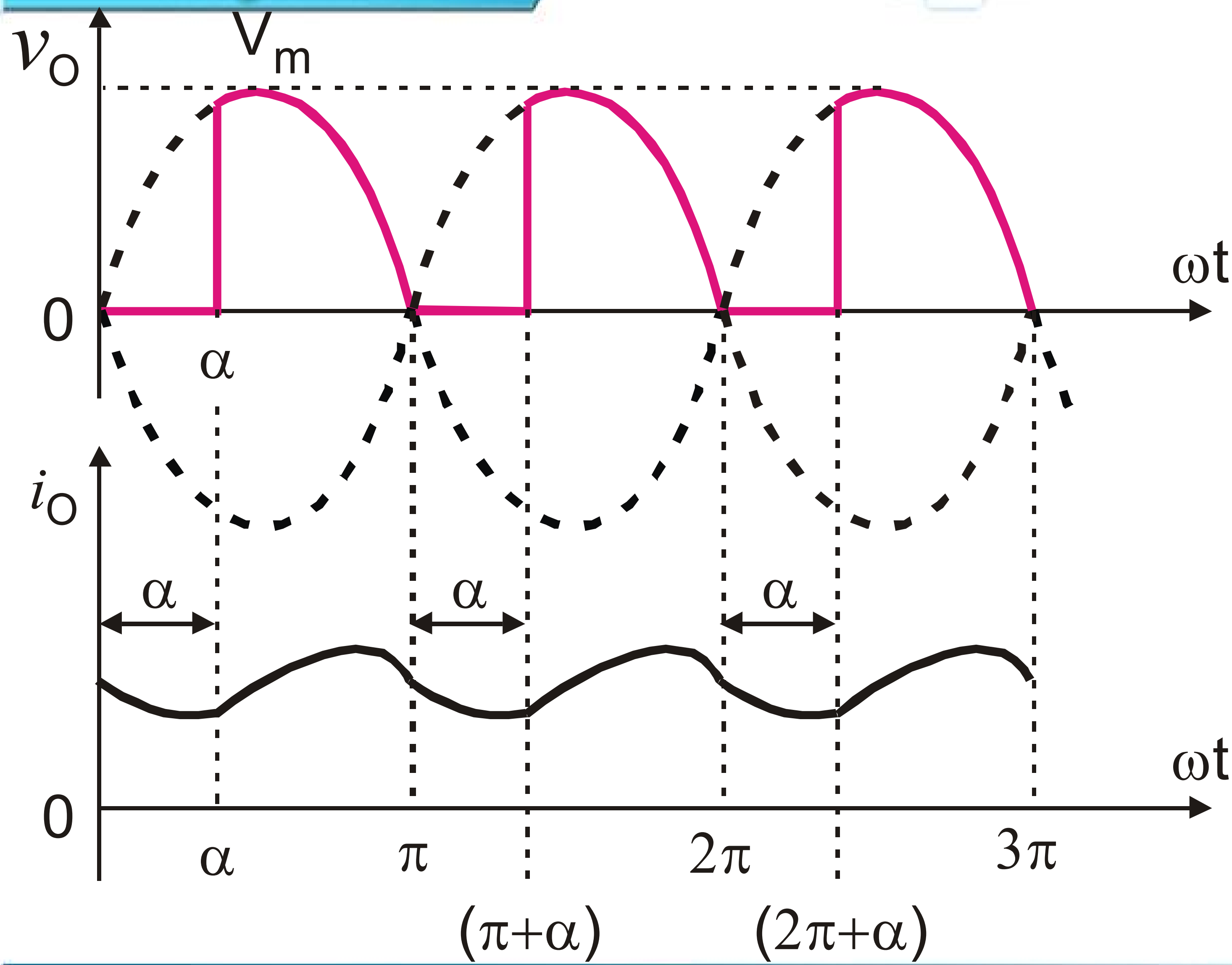
Thyristor T_2 & D_2 conduct

from $\omega t = \left(\pi + \alpha\right) \ to \ 2\pi$

FWD conducts during

$\omega t = 0$ to $\alpha, \ \pi \ to \ \left(\pi + \alpha\right), \ldots$

Load Voltage & Load Current Waveform of Single Phase Semi Converter for $\alpha < 90^0$ & Continuous load current operation


v_O
V_m
ωt
0
α
i_O
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

To Derive an Expression For The DC Output Voltage of A Single Phase Semi-Converter With R,L, & E Load & FWD For Continuous, Ripple Free Load Current Operation

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \quad ; \; \cos \pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

V_{dc} can be varied from a max.

value of $\frac{2V_m}{\pi}$ *to* 0 by varying α from 0 to π.

For $\alpha = 0$, The max. dc o/p voltage obtained is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalized dc o/p voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dn}} = \frac{\frac{V_m}{\pi}(1+\cos\alpha)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}(1+\cos\alpha)$$

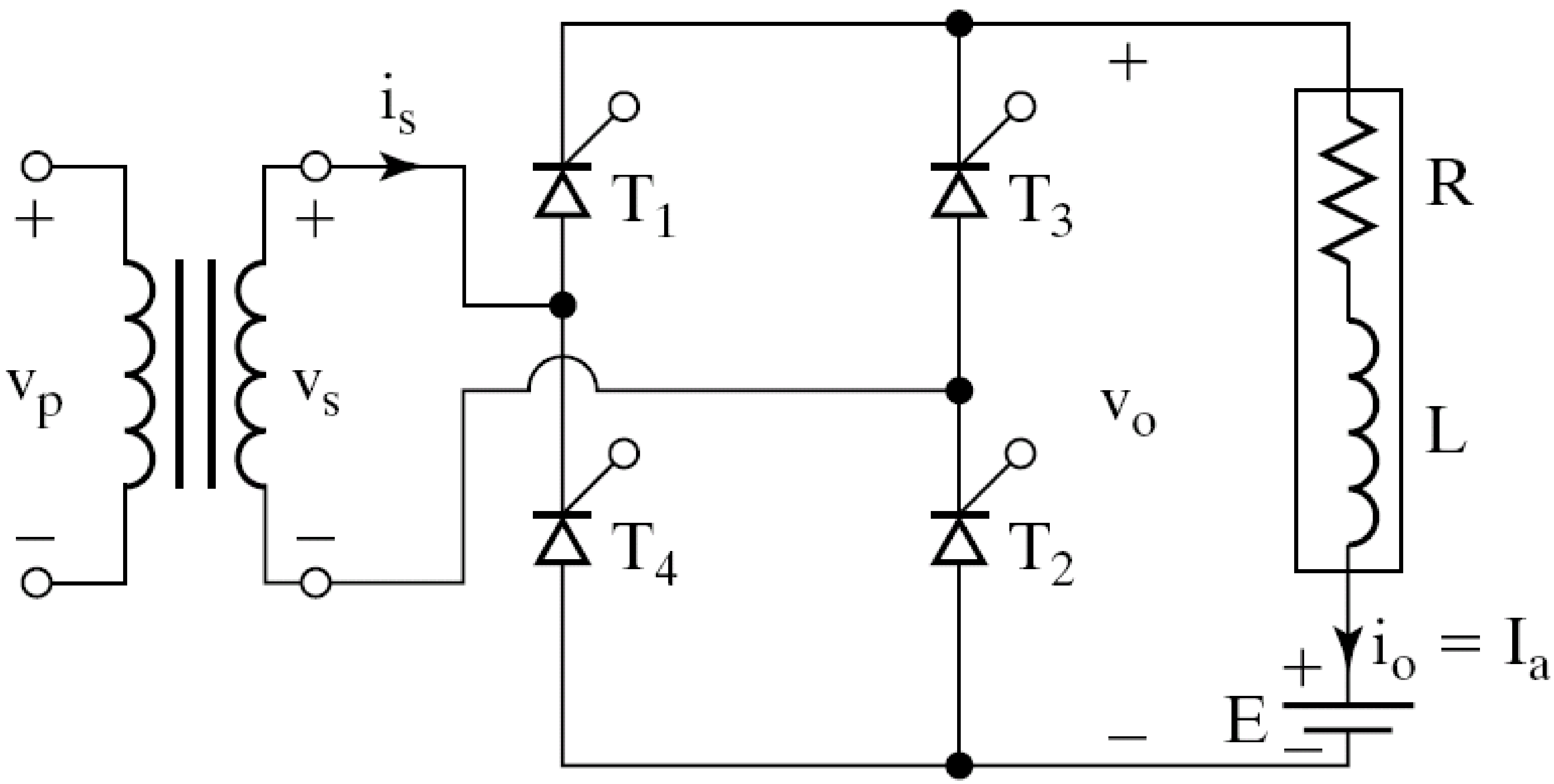
RMS O/P Voltage $V_{O(RMS)}$

$$V_{O(RMS)} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d\left(\omega t\right)\right]^{\frac{1}{2}}$$

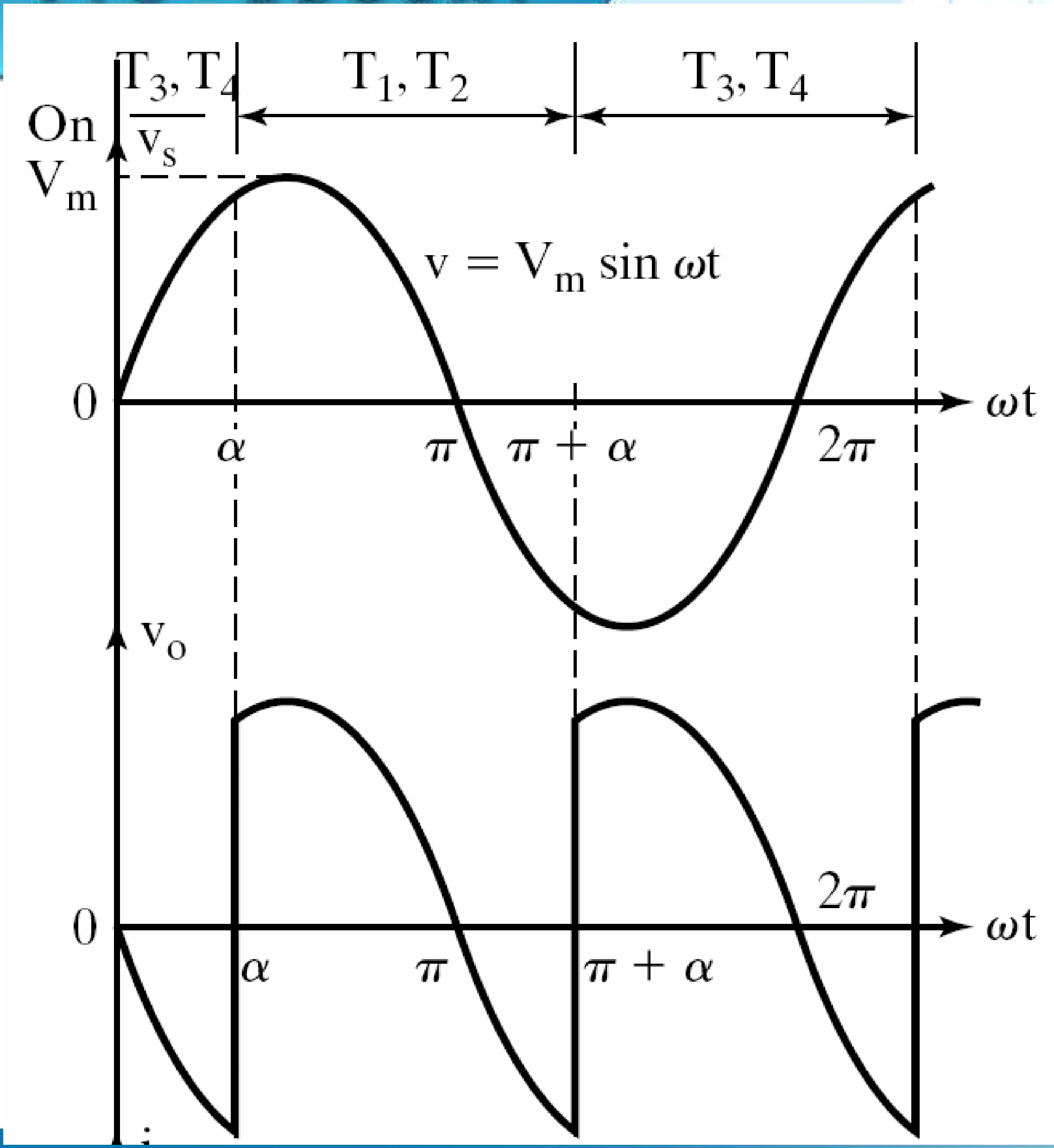
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi} \left(1-\cos 2\omega t\right) . d\left(\omega t\right)\right]^{\frac{1}{2}}$$

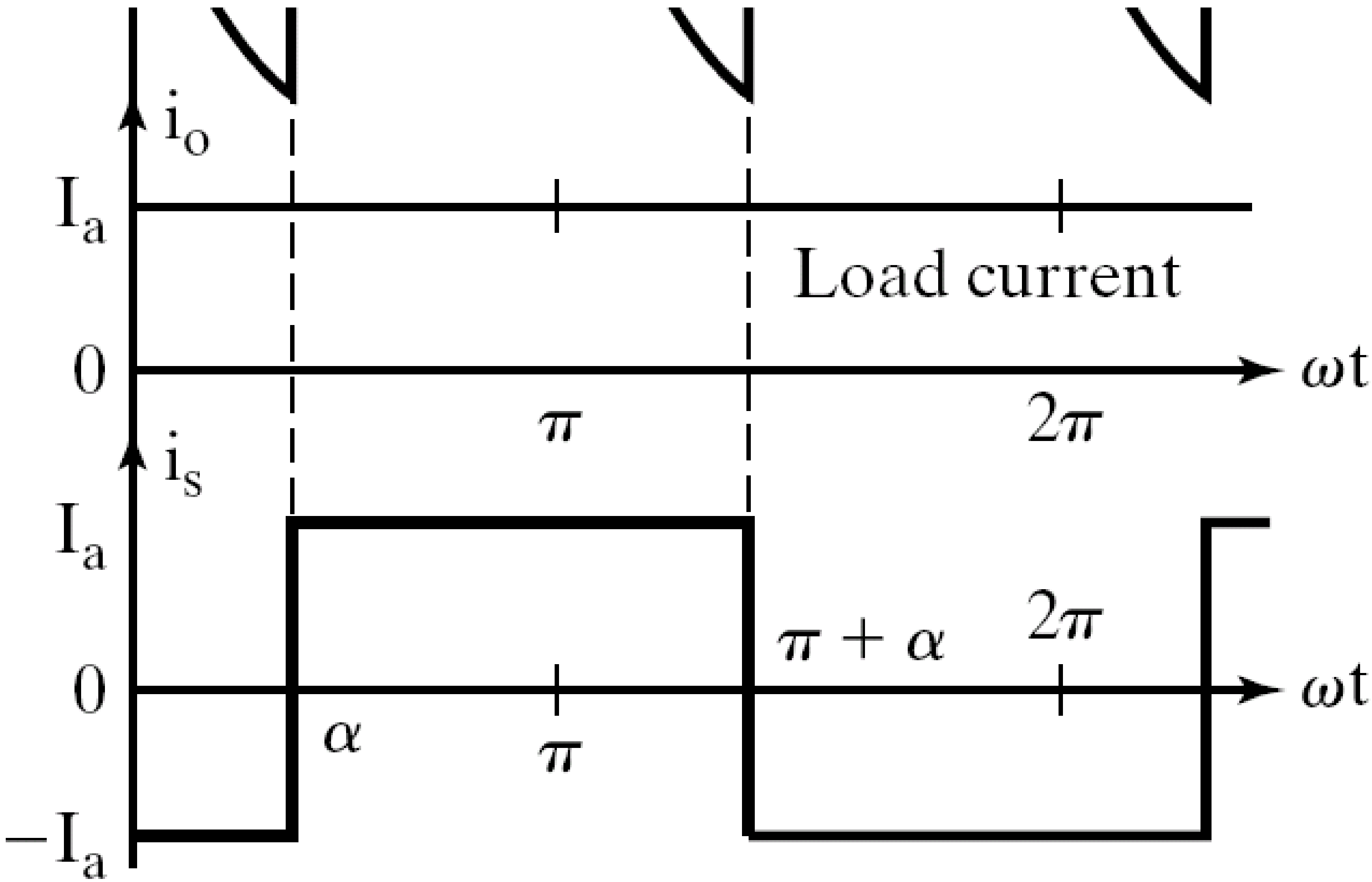
$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

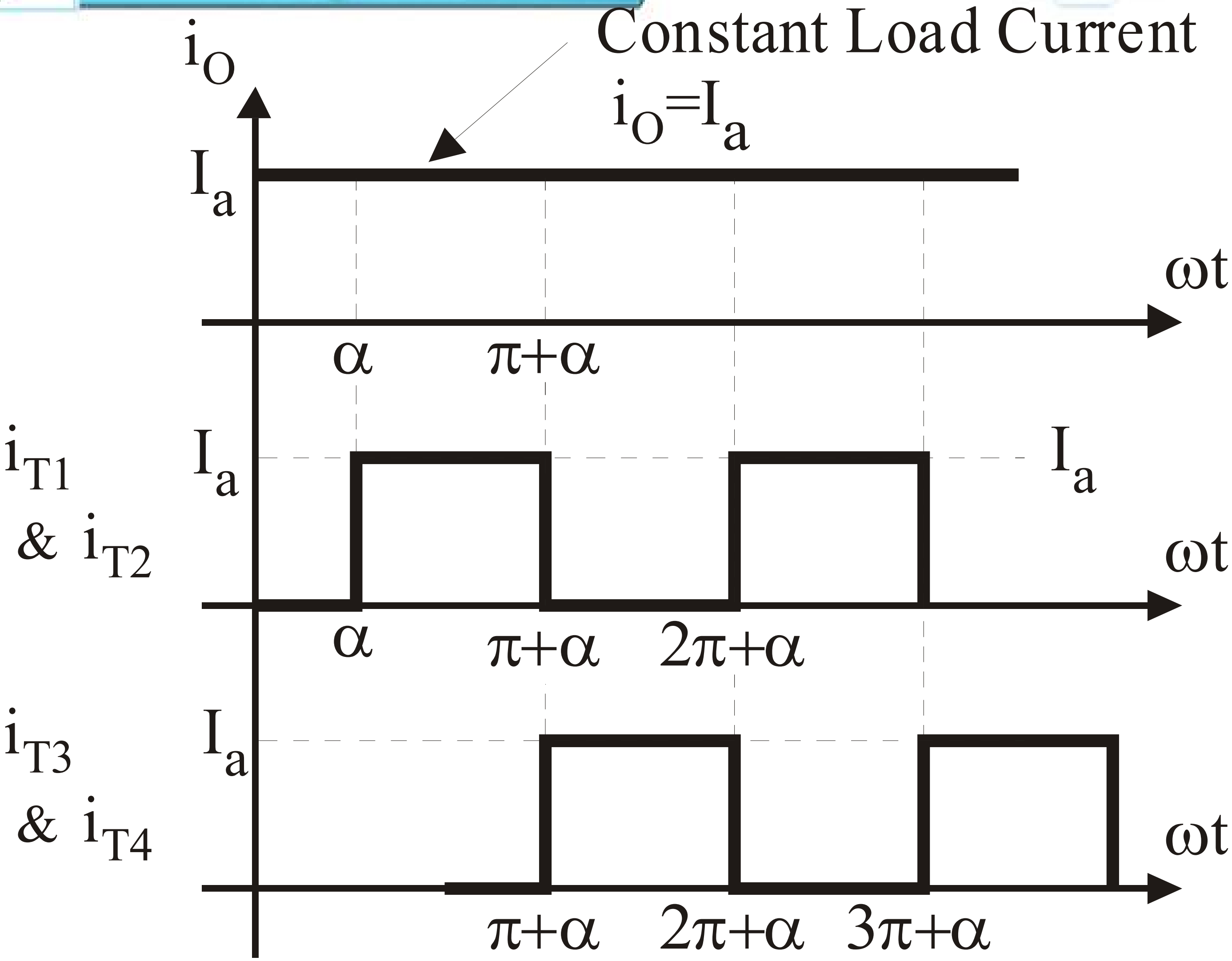
Single Phase Full Wave Full Converter (Fully Controlled Bridge Converter) With R,L, & E Load


i_s
+
v_p
−
+
v_s
−
T_1
T_3
T_4
T_2
+
v_o
−
R
L
i_o = I_a
+
E
−

Waveforms of Single Phase Full Converter Assuming Continuous (Constant Load Current) & Ripple Free Load Current


T3, T4
T1, T2
T3, T4
On
vs
Vm
v = Vm sin ωt
0
ωt
α
π
π + α
2π
vo
0
2π
ωt
α
π
π + α


i_o
I_a
Load current
0
ωt
π
2π
i_s
I_a
$\pi + \alpha$
2π
0
ωt
α
π
$-I_a$


Constant Load Current
$i_O = I_a$
i_O
I_a
ωt
α
π+α
i_{T1} & i_{T2}
I_a
I_a
ωt
α
π+α
2π+α
i_{T3} & i_{T4}
I_a
ωt
π+α
2π+α
3π+α

To Derive
An Expression For
The Average DC Output Voltage of a Single Phase Full Converter
assuming
Continuous & Constant Load Current

The average dc output voltage

can be determined by using the expression

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\left[\int_{0}^{2\pi} v_O . d\left(\omega t\right)\right];$$

The o/p voltage waveform consists of two o/p pulses during the input supply time period of 0 to 2π radians. Hence the Average or dc o/p voltage can be calculated as

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t . d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Maximum average dc output voltage is calculated for a trigger angle $\alpha = 0^0$ and is obtained as

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi} \times \cos(0) = \frac{2V_m}{\pi}$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

The normalized average output voltage is given by

$$V_{dcn} = V_n = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$\therefore V_{dcn} = V_n = \frac{\dfrac{2V_m}{\pi}\cos\alpha}{\dfrac{2V_m}{\pi}} = \cos\alpha$$

By plotting $V_{O(dc)}$ *versus* α,

we obtain the control characteristic of a single phase full wave fully controlled bridge converter

(single phase full converter)

for constant & continuous

load current operation.

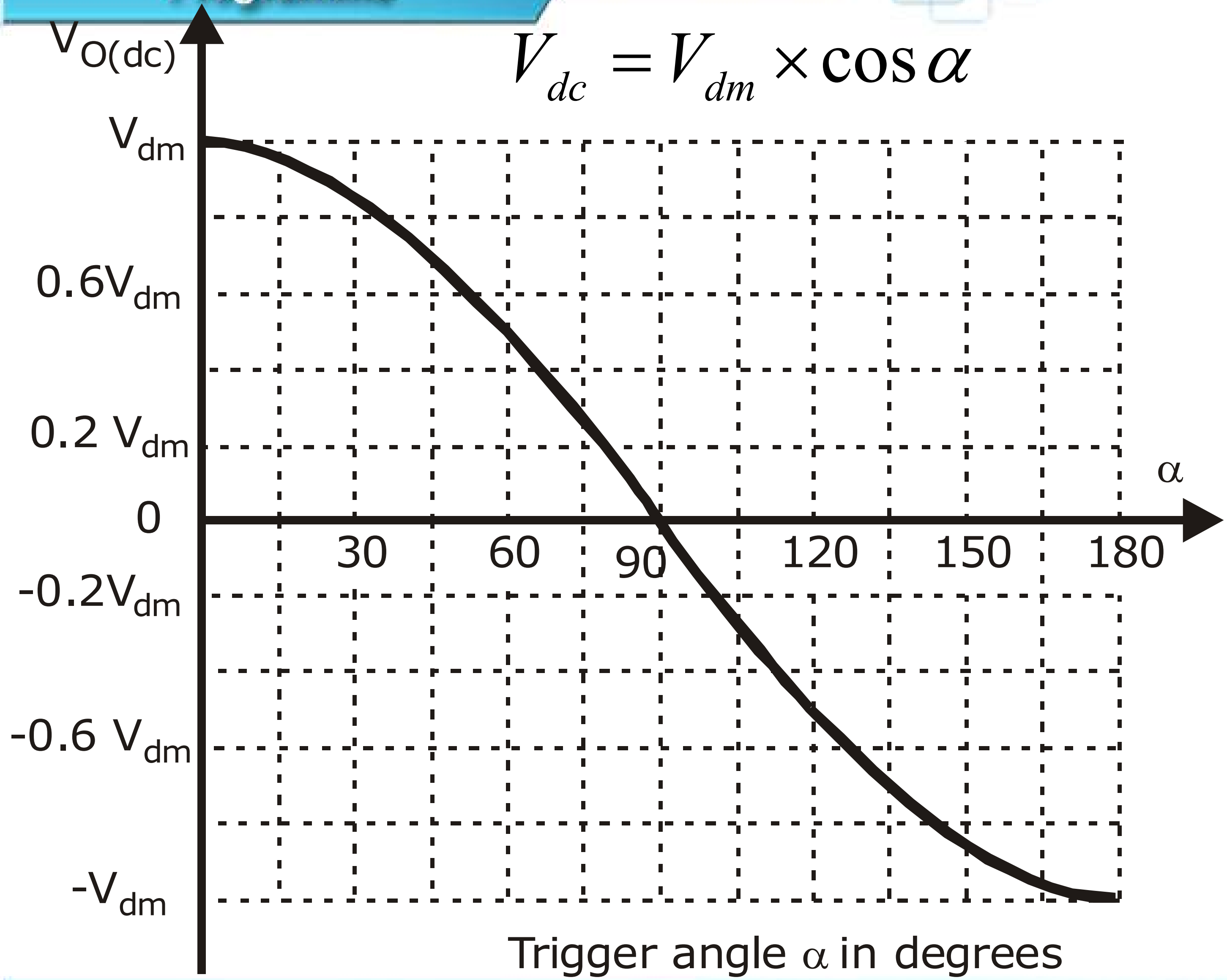
To plot the control characteristic of a Single Phase Full Converter for constant & continuous load current operation.

We use the equation for the average/ dc output voltage

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	

$$V_{dc} = V_{dm} \times \cos\alpha$$



- During the period from $\omega t = \alpha$ to π the input voltage v_S and the input current i_S are both positive and the power flows from the supply to the load.
- The converter is said to be operated in the rectification mode

Controlled Rectifier Operation

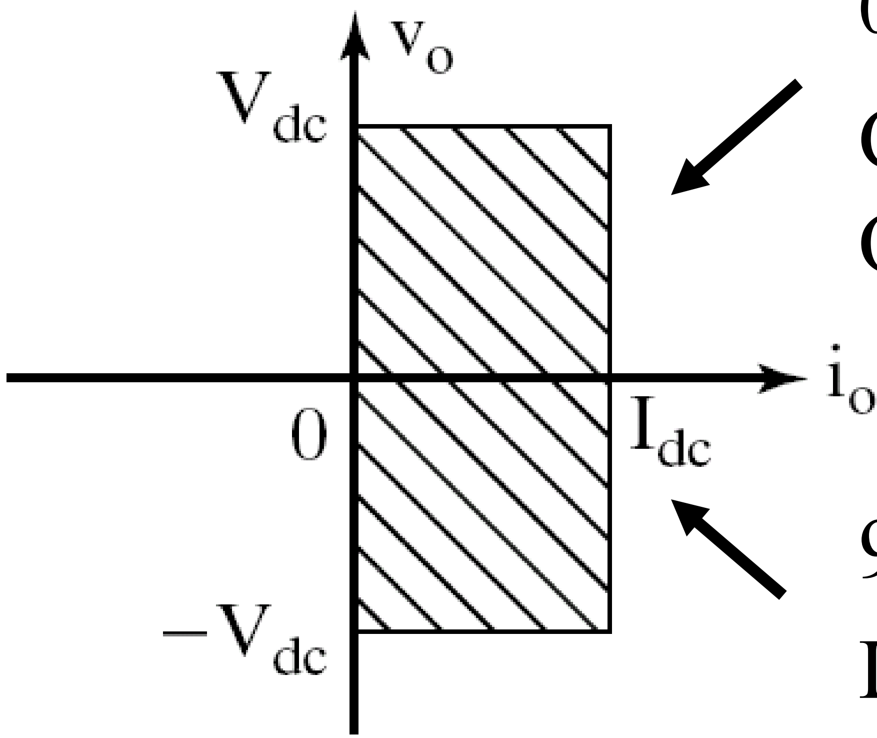
for $0 < \alpha < 90^0$

- During the period from $\omega t = \pi$ to $(\pi+\alpha)$, the input voltage v_S is negative and the input current i_S is positive and the output power becomes negative and there will be reverse power flow from the load circuit to the supply.
- The converter is said to be operated in the inversion mode.

Line Commutated Inverter Operation

for $90^0 < \alpha < 180^0$

Two Quadrant Operation of a Single Phase Full Converter



$0 < \alpha < 90^0$

Controlled Rectifier Operation

$90^0 < \alpha < 180^0$

Line Commutated Inverter Operation

To Derive An Expression For The RMS Value Of The Output Voltage

The rms value of the output voltage is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2 . d(\omega t)\right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} v_O^2 . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t.d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \sin^2 \omega t.d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \frac{(1-\cos 2\omega t)}{2}.d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} d(\omega t) - \int_{\alpha}^{\pi+\alpha} \cos 2\omega t.d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\omega t\right)\Big/_{\alpha}^{\pi+\alpha} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi+\alpha}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\pi+\alpha-\alpha\right) - \left(\frac{\sin 2\left(\pi+\alpha\right) - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\pi\right) - \left(\frac{\sin\left(2\pi+2\alpha\right) - \sin 2\alpha}{2}\right)\right]};$$

$$\sin\left(2\pi+2\alpha\right) = \sin 2\alpha$$

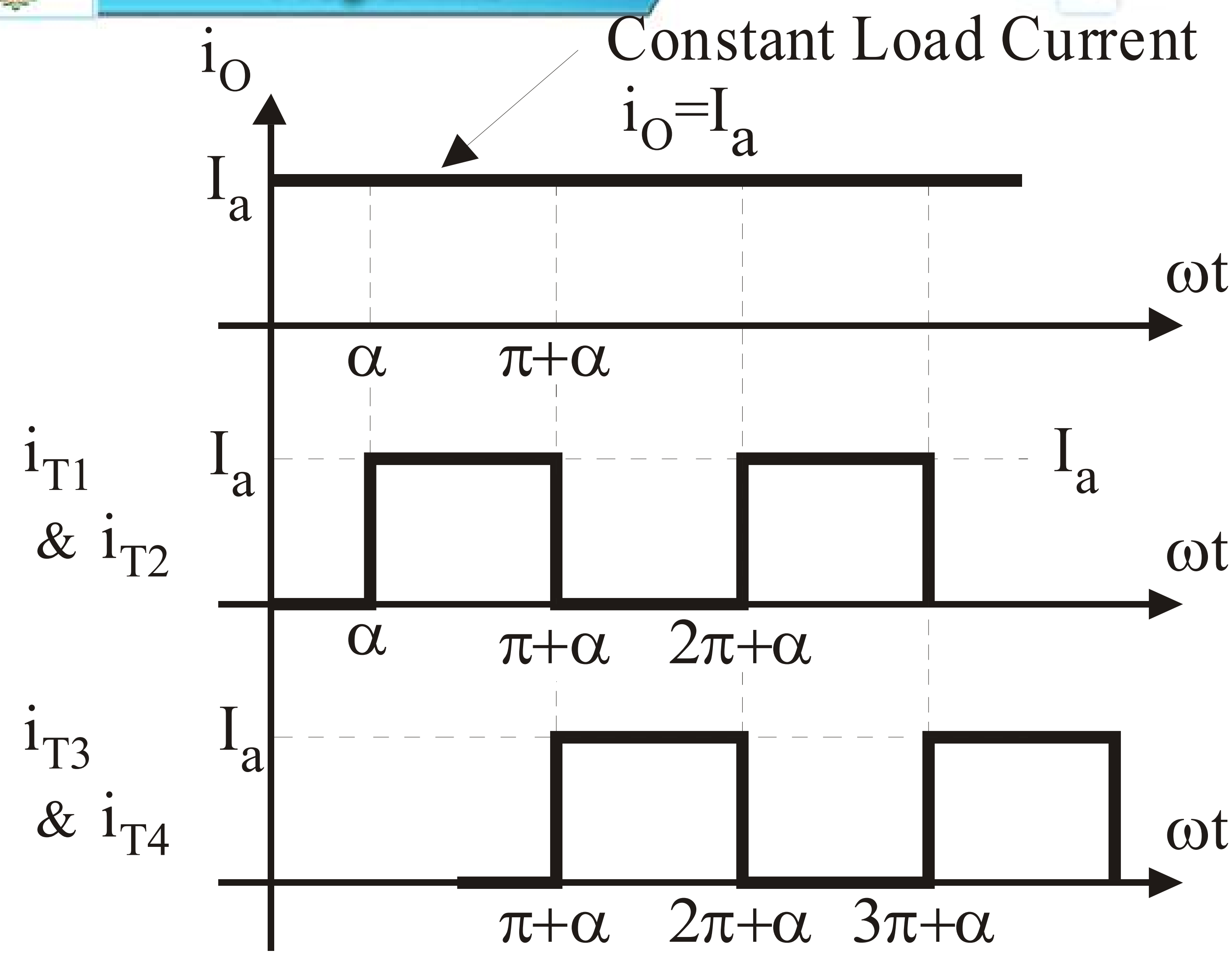
$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin 2\alpha - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}(\pi) - 0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S$$

Hence the rms output voltage is same as the rms input supply voltage

Thyristor Current Waveforms


Constant Load Current
i_O=I_a
i_O
I_a
ωt
α
π+α
i_T1
& i_T2
I_a
I_a
ωt
α
π+α
2π+α
i_T3
& i_T4
I_a
ωt
π+α
2π+α
3π+α

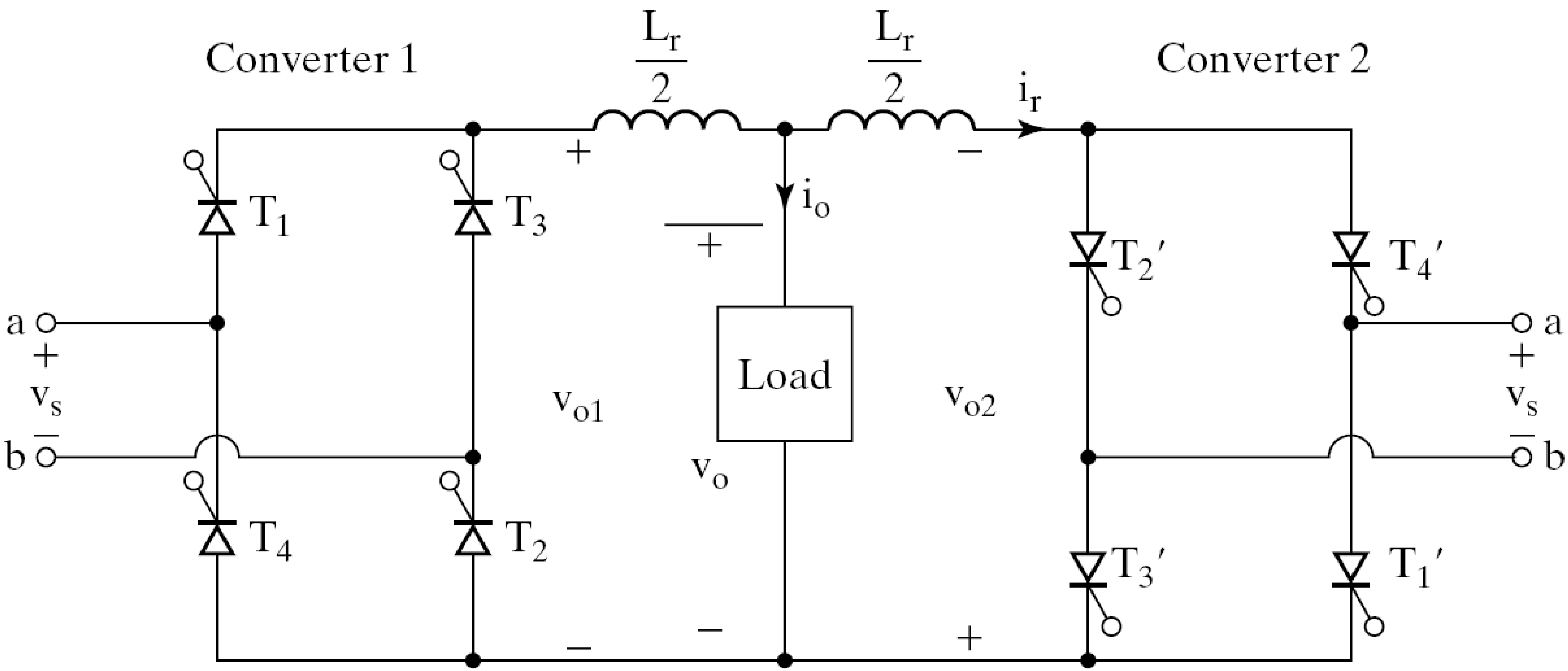
The rms thyristor current can be calculated as

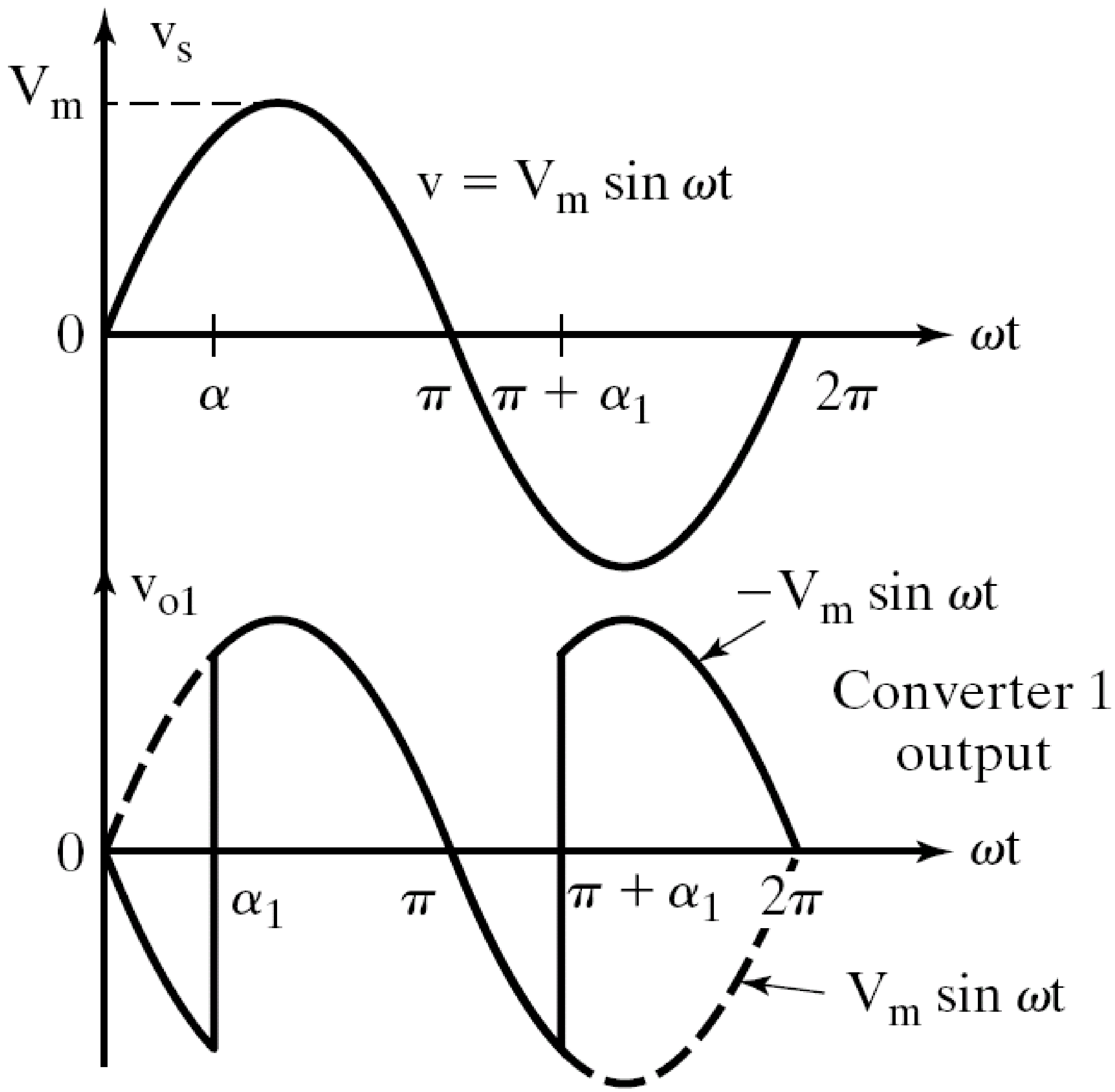
$$I_{T(RMS)} = \frac{I_{O(RMS)}}{\sqrt{2}}$$

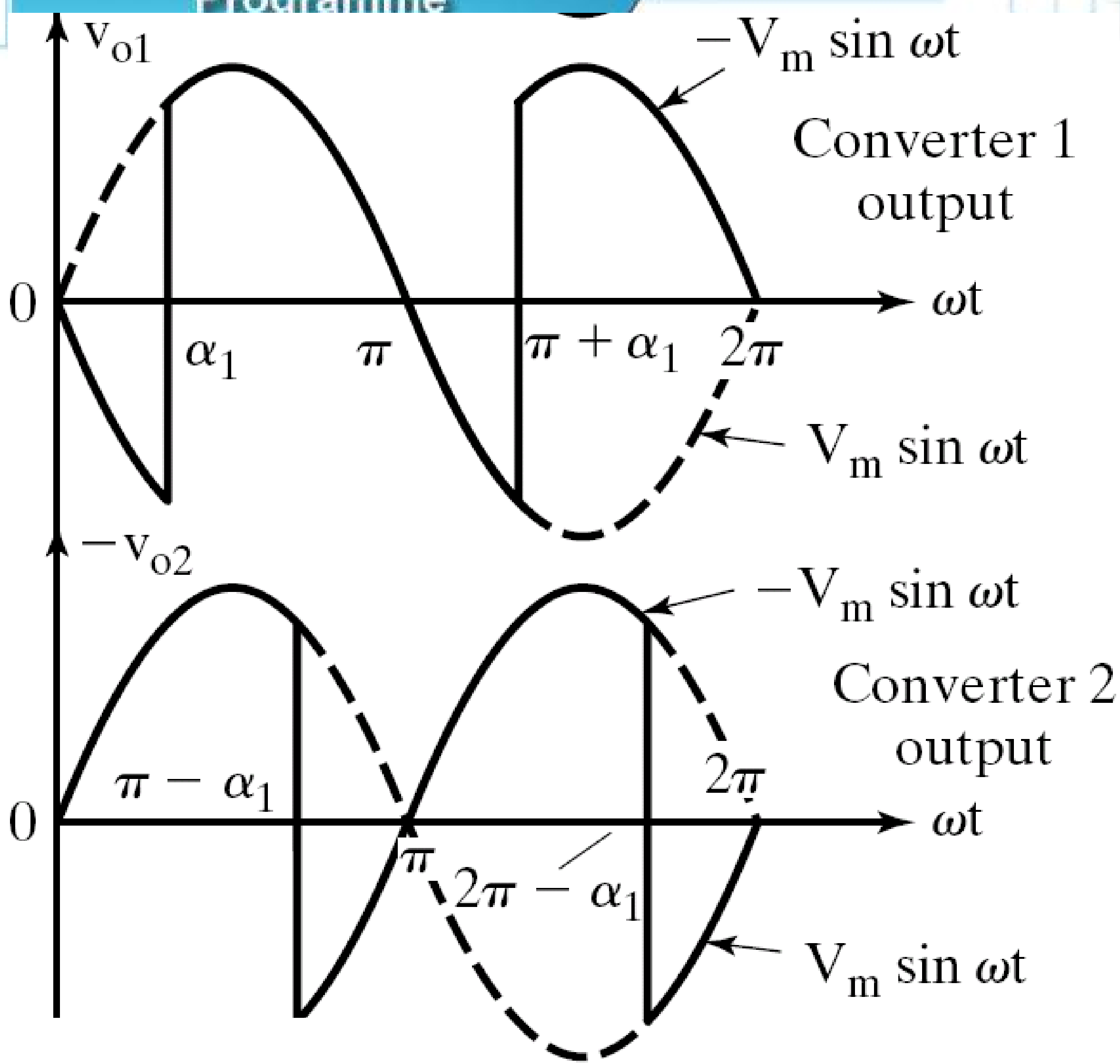
The average thyristor current can be calculated as

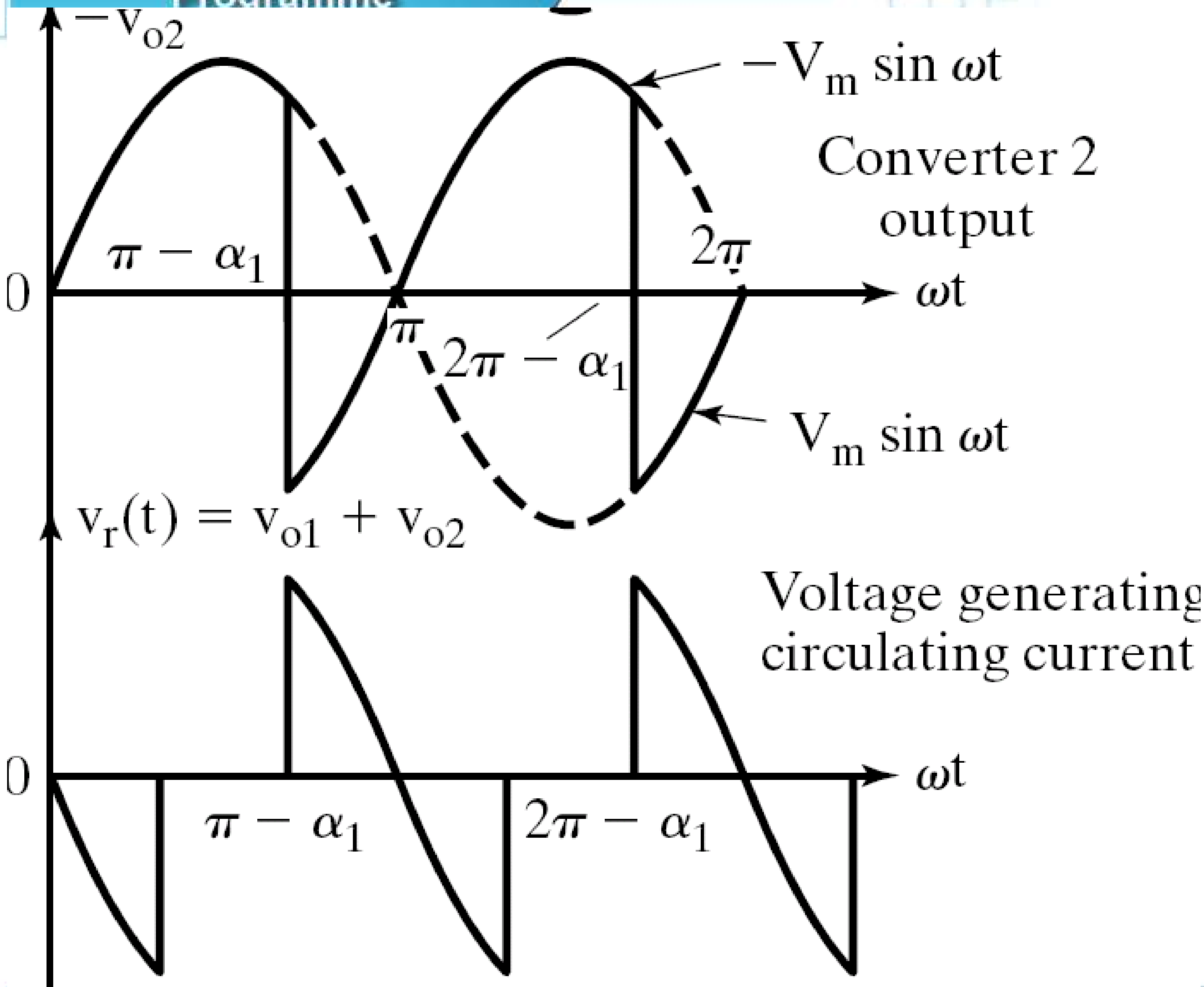
$$I_{T(Avg)} = \frac{I_{O(dc)}}{2}$$

Single Phase Dual Converter




v_s
V_m
$v = V_m \sin \omega t$
0
ωt
α
π
$\pi + \alpha_1$
2π
v_{o1}
$-V_m \sin \omega t$
Converter 1
output
0
ωt
α_1
π
$\pi + \alpha_1$
2π
$V_m \sin \omega t$


v_{o1}
$-V_m \sin \omega t$
Converter 1
output
0
ωt
α_1
π
$\pi + \alpha_1$
2π
$V_m \sin \omega t$
$-v_{o2}$
$-V_m \sin \omega t$
Converter 2
output
$\pi - \alpha_1$
2π
0
ωt
π
$2\pi - \alpha_1$
$V_m \sin \omega t$


$-v_{o2}$
$-V_m \sin \omega t$
Converter 2
output
0
$\pi - \alpha_1$
2π
ωt
π
$2\pi - \alpha_1$
$V_m \sin \omega t$
$v_r(t) = v_{o1} + v_{o2}$
Voltage generating
circulating current
0
ωt
$\pi - \alpha_1$
$2\pi - \alpha_1$

The average dc output voltage of converter 1 is

$$V_{dc1} = \frac{2V_m}{\pi}\cos\alpha_1$$

The average dc output voltage of converter 2 is

$$V_{dc2} = \frac{2V_m}{\pi}\cos\alpha_2$$

In the dual converter operation one converter is operated as a controlled rectifier with $\alpha < 90^0$ & the second converter is operated as a line commutated inverter in the inversion mode with $\alpha > 90^0$

$$\therefore \qquad V_{dc1} = -V_{dc2}$$

$$\frac{2V_m}{\pi}\cos\alpha_1 = \frac{-2V_m}{\pi}\cos\alpha_2 = \frac{2V_m}{\pi}\left(-\cos\alpha_2\right)$$

$$\therefore \qquad \cos\alpha_1 = -\cos\alpha_2$$

or

$$\cos\alpha_2 = -\cos\alpha_1 = \cos\left(\pi - \alpha_1\right)$$

$$\therefore \qquad \alpha_2 = \left(\pi - \alpha_1\right) \text{ or}$$

$$\left(\alpha_1 + \alpha_2\right) = \pi \text{ radians}$$

Which gives

$$\alpha_2 = \left(\pi - \alpha_1\right)$$

To Obtain an Expression for the Instantaneous Circulating Current

- v_{O1} = Instantaneous o/p voltage of converter 1.
- v_{O2} = Instantaneous o/p voltage of converter 2.
- The circulating current i_r can be determined by integrating the instantaneous voltage difference (which is the voltage drop across the circulating current reactor L_r), starting from $\omega t = (2\pi - \alpha_1)$.
- As the two average output voltages during the interval $\omega t = (\pi + \alpha_1)$ to $(2\pi - \alpha_1)$ are equal and opposite their contribution to the instantaneous circulating current i_r is zero.

$$i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} v_r . d(\omega t)\right] \; ; \qquad v_r = (v_{O1} - v_{O2})$$

As the o/p voltage v_{O2} is negative

$$v_r = (v_{O1} + v_{O2})$$

$$\therefore \qquad i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} (v_{O1} + v_{O2}).d(\omega t)\right];$$

$$v_{O1} = -V_m \sin\omega t \text{ for } (2\pi - \alpha_1) \text{ to } \omega t$$

$$i_r = \frac{V_m}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} -\sin\omega t.d(\omega t) - \int_{(2\pi-\alpha_1)}^{\omega t} \sin\omega t.d(\omega t)\right]$$

$$i_r = \frac{2V_m}{\omega L_r}(\cos\omega t - \cos\alpha_1)$$

The instantaneous value of the circulating current depends on the delay angle.

For trigger angle (delay angle) $\alpha_1 = 0$,
the magnitude of circulating current becomes min.
when $\omega t = n\pi,\ n = 0, 2, 4, ...$ & magnitude becomes
max. when $\omega t = n\pi,\ n = 1, 3, 5, ...$
If the peak load current is I_p, one of the
converters that controls the power flow
may carry a peak current of

$$\left(I_p + \frac{4V_m}{\omega L_r} \right),$$

where

$$I_p = I_{L(\max)} = \frac{V_m}{R_L},$$

&

$$i_{r(\max)} = \frac{4V_m}{\omega L_r} = \text{ max. circulating current}$$

The Dual Converter Can Be Operated In Two Different Modes Of Operation

- Non-circulating current (circulating current free) mode of operation.
- Circulating current mode of operation.

Non-Circulating Current Mode of Operation

- In this mode only one converter is operated at a time.
- When converter 1 is ON, $0 < \alpha_1 < 90^0$
- V_{dc} is positive and I_{dc} is positive.
- When converter 2 is ON, $0 < \alpha_2 < 90^0$
- V_{dc} is negative and I_{dc} is negative.

Circulating Current Mode Of Operation

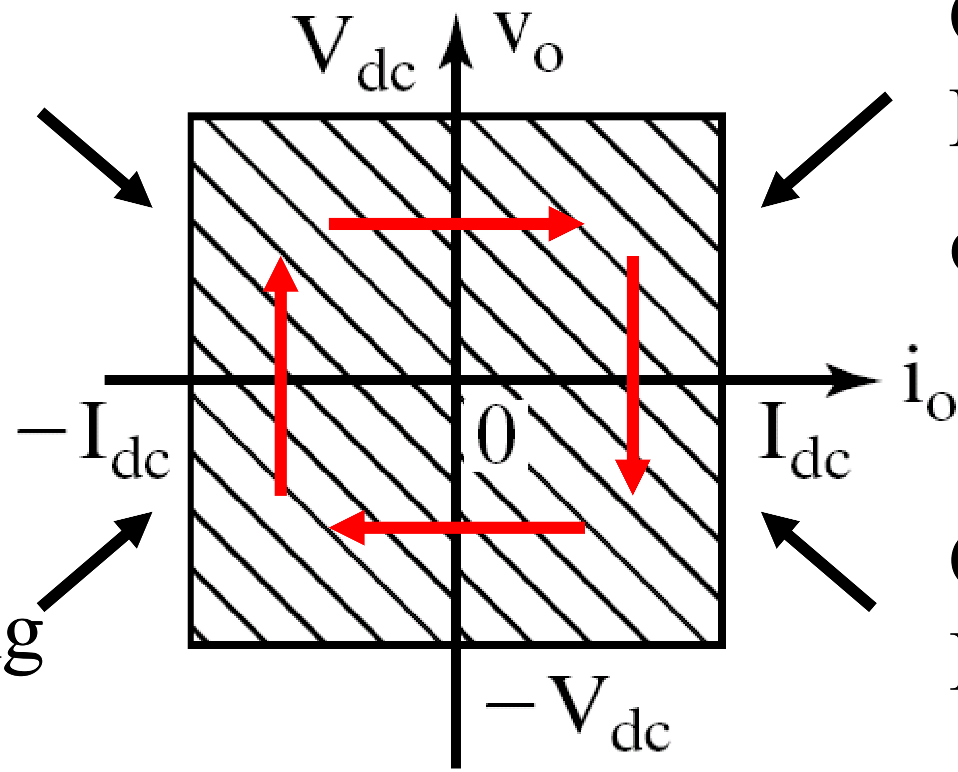
- In this mode, both the converters are switched ON and operated at the same time.
- The trigger angles α_1 and α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^0$; $\alpha_2 = (180^0 - \alpha_1)$.

- When $0 < \alpha_1 < 90^0$, converter 1 operates as a controlled rectifier and converter 2 operates as an inverter with $90^0 < \alpha_2 < 180^0$.
- In this case V_{dc} and I_{dc}, both are positive.
- When $90^0 < \alpha 1 < 180^0$, converter 1 operates as an Inverter and converter 2 operated as a controlled rectifier by adjusting its trigger angle α_2 such that $0 < \alpha_2 < 90^0$.
- In this case V_{dc} and I_{dc}, both are negative.

Four Quadrant Operation

Conv. 2
Inverting
$\alpha_2 > 90^0$

Conv. 1
Rectifying
$\alpha_1 < 90^0$



Conv. 2
Rectifying
$\alpha_2 < 90^0$

Conv. 1
Inverting
$\alpha_1 > 90^0$

Advantages of Circulating Current Mode Of Operation

- The circulating current maintains continuous conduction of both the converters over the complete control range, independent of the load.
- One converter always operates as a rectifier and the other converter operates as an inverter, the power flow in either direction at any time is possible.

- As both the converters are in continuous conduction we obtain faster dynamic response. i.e., the time response for changing from one quadrant operation to another is faster.

Disadvantages of Circulating Current Mode Of Operation

- There is always a circulating current flowing between the converters.
- When the load current falls to zero, there will be a circulating current flowing between the converters so we need to connect circulating current reactors in order to limit the peak circulating current to safe level.
- The converter thyristors should be rated to carry a peak current much greater than the peak load current.