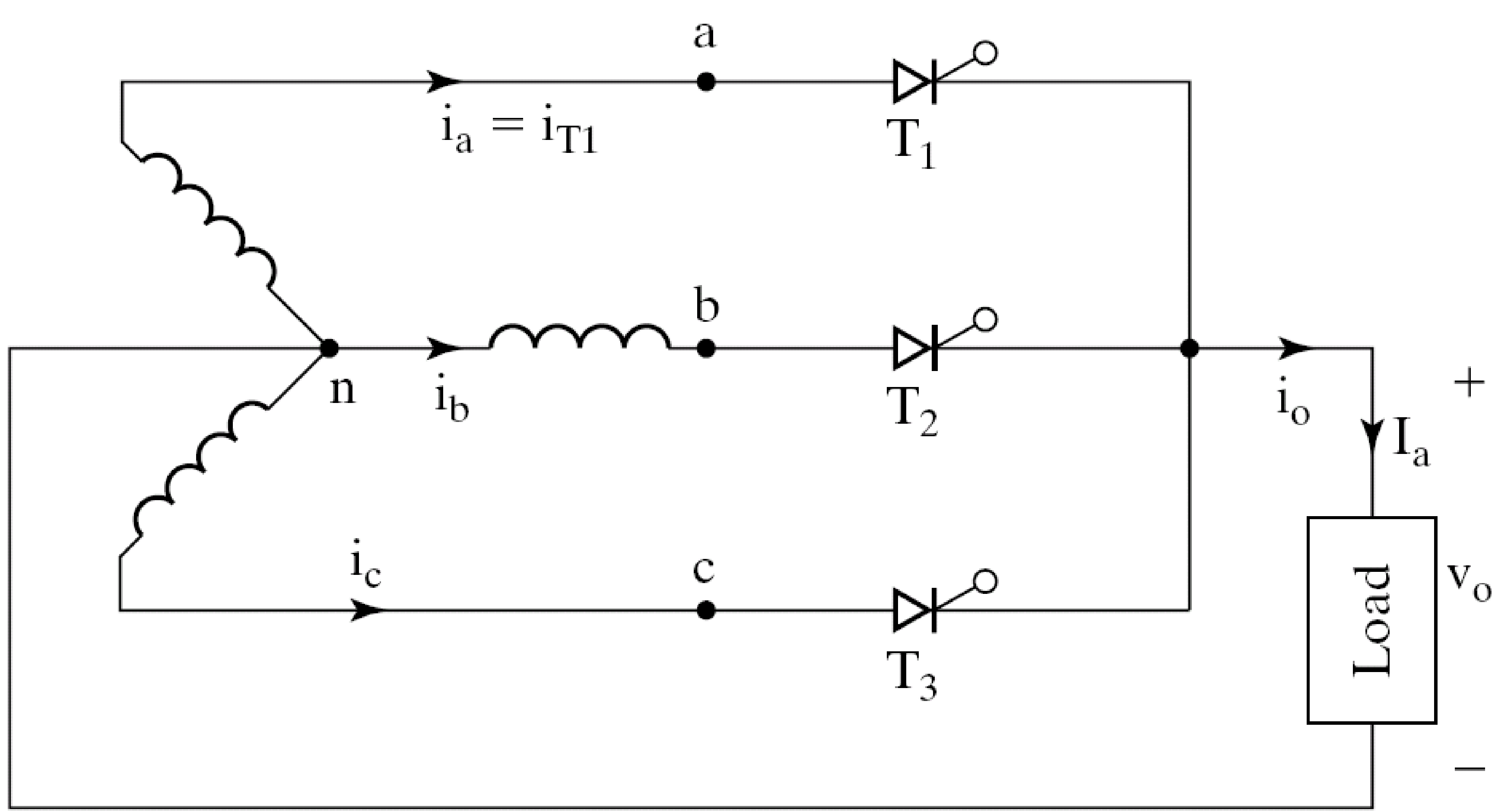
Three Phase Controlled Rectifiers

3 Phase Controlled Rectifiers

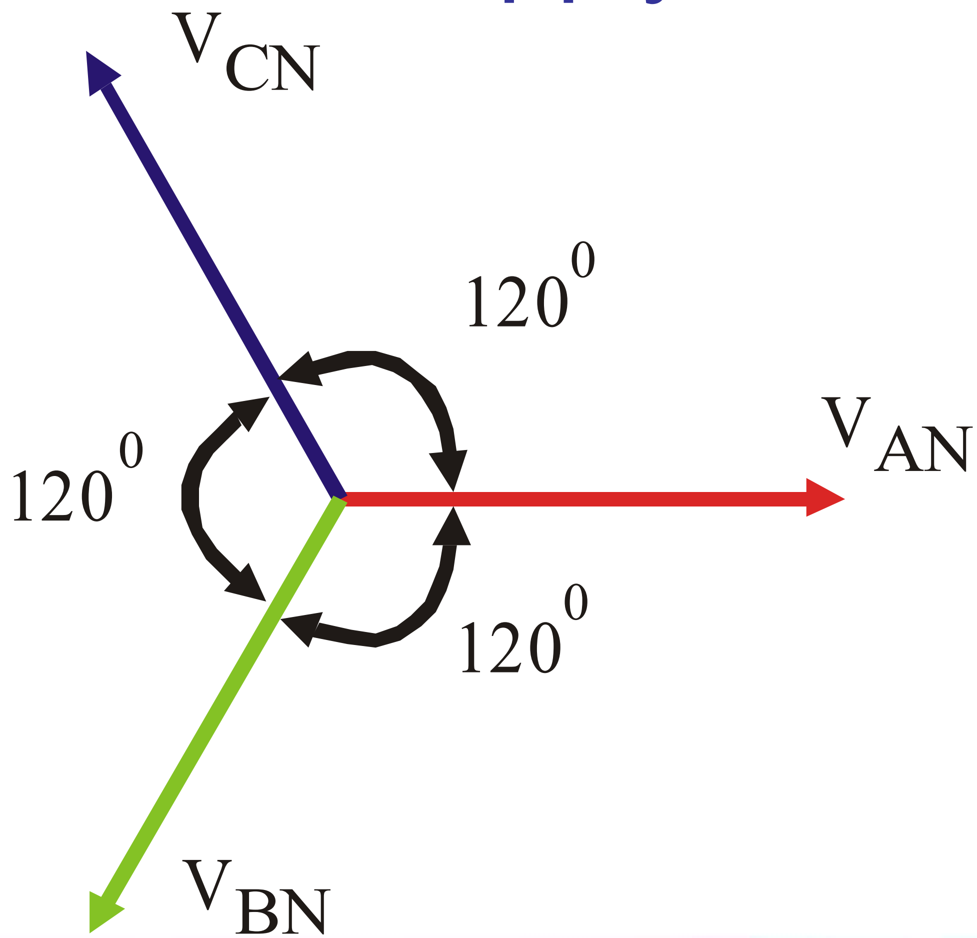
- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage.
- Higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current.

- Extensively used in high power variable speed industrial dc drives.
- Three single phase half-wave converters can be connected together to form a three phase half-wave converter.

3-Phase
Half Wave Converter
(3-Pulse Converter)
with
RL Load
Continuous & Constant
Load Current Operation


a
$i_a = i_{T1}$
T_1
b
n
i_b
T_2
i_o
+
I_a
i_c
c
T_3
Load
v_o
–

Vector Diagram of 3 Phase Supply Voltages



$$v_{RN} = v_{AN}$$

$$v_{YN} = v_{BN}$$

$$v_{BN} = v_{CN}$$

3 Phase Supply Voltage Equations

We deifine three line to neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t;$$

$$V_m = \text{ Max. Phase Voltage}$$

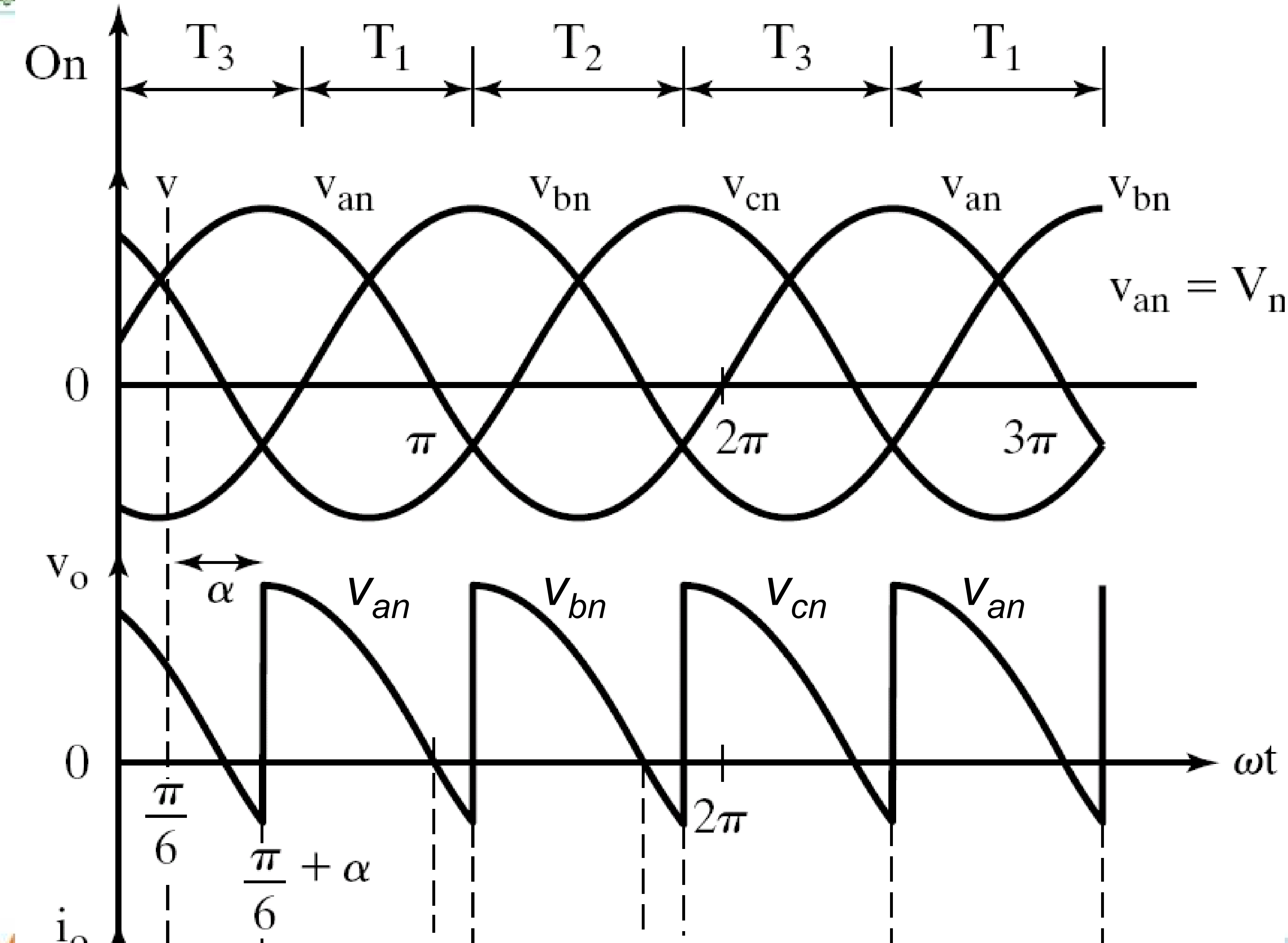
$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3} \right)$$

$$= V_m \sin\left(\omega t - 120^0 \right)$$

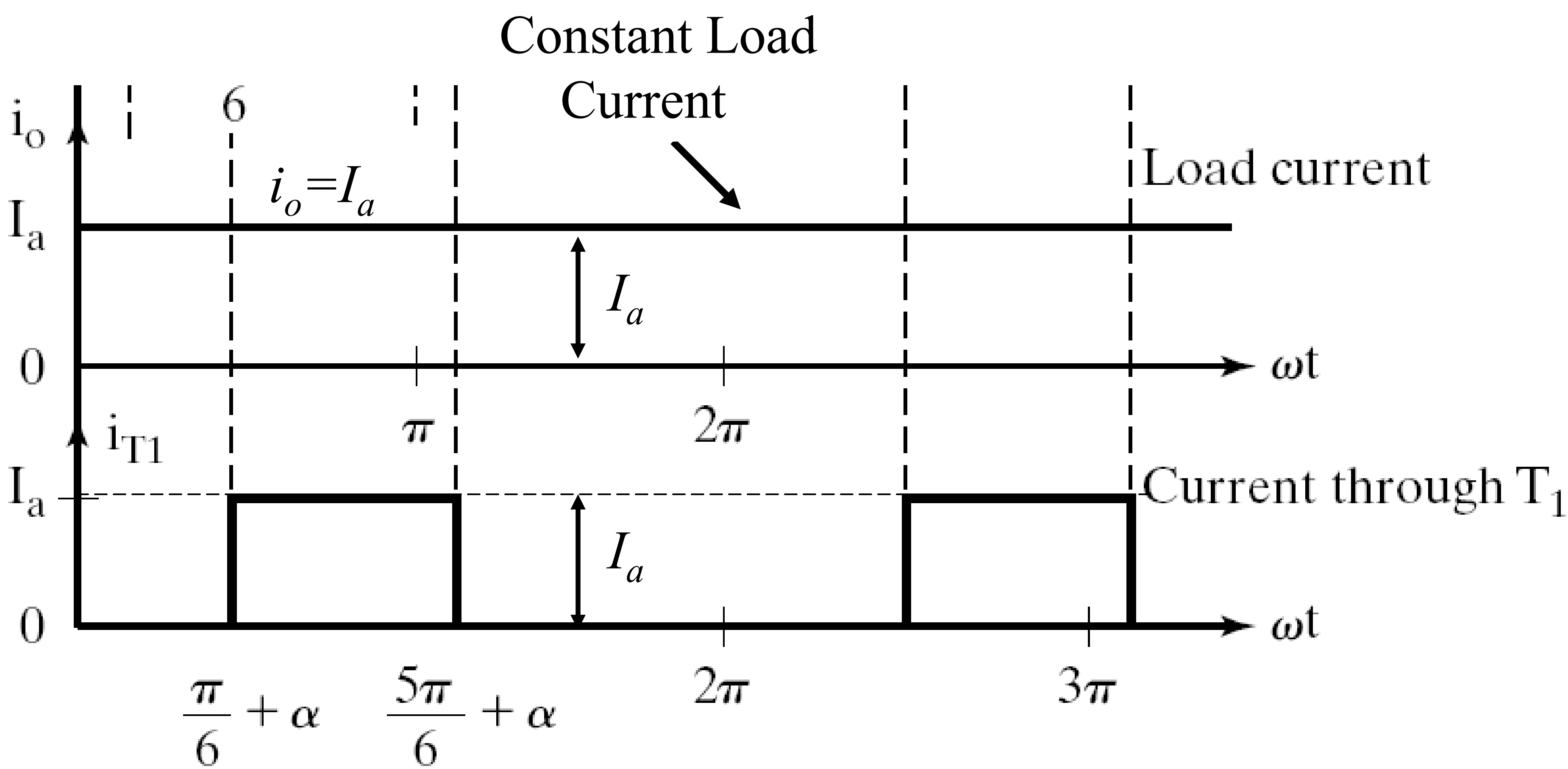
$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3} \right)$$

$$= V_m \sin\left(\omega t + 120^0 \right)$$

$$= V_m \sin\left(\omega t - 240^0 \right)$$


On
T3
T1
T2
T3
T1
v
van
vbn
vcn
van
vbn
van = Vn
0
π
2π
3π
vo
α
Van
Vbn
Vcn
Van
0
ωt
π/6
2π
π/6 + α
io

Each thyristor conducts for $2\pi/3$ (120^0)



To Derive an Expression for the Average Output Voltage of a 3-Phase Half Wave Converter with RL Load for Continuous Load Current

$$T_1 \ is \ triggered \ at \ \omega t = \left(\frac{\pi}{6} + \alpha\right) = \left(30^0 + \alpha\right)$$

$$T_2 \ is \ triggered \ at \ \omega t = \left(\frac{5\pi}{6} + \alpha\right) = \left(150^0 + \alpha\right)$$

$$T_3 \ is \ triggered \ at \ \omega t = \left(\frac{7\pi}{6} + \alpha\right) = \left(270^0 + \alpha\right)$$

$$\text{Each thytistor conducts for } 120^0 \text{ or } \frac{2\pi}{3} \text{ radians}$$

If the reference phase voltage is $v_{RN} = v_{an} = V_m \sin \omega t$, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin \omega t . d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin \omega t . d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\left(-\cos \omega t\right)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

Note from the trigonometric relationship

$$\cos(A+B) = (\cos A.\cos B - \sin A.\sin B)$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}\right)\cos(\alpha) + \sin\left(\frac{5\pi}{6}\right)\sin(\alpha) + \cos\left(\frac{\pi}{6}\right).\cos(\alpha) - \sin\left(\frac{\pi}{6}\right)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos(150^0)\cos(\alpha) + \sin(150^0)\sin(\alpha) + \cos(30^0).\cos(\alpha) - \sin(30^0)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\begin{aligned}&-\cos\left(180^0-30^0\right)\cos(\alpha)+\sin\left(180^0-30^0\right)\sin(\alpha)\\ &+\cos\left(30^0\right).\cos(\alpha)-\sin\left(30^0\right)\sin(\alpha)\end{aligned}\right]$$

Note: $\cos\left(180^0-30^0\right)=-\cos\left(30^0\right)$

$\sin\left(180^0-30^0\right)=\sin\left(30^0\right)$

$$\therefore V_{dc} = \frac{3V_m}{2\pi}\left[\begin{aligned}&+\cos\left(30^0\right)\cos(\alpha)+\sin\left(30^0\right)\sin(\alpha)\\ &+\cos\left(30^0\right).\cos(\alpha)-\sin\left(30^0\right)\sin(\alpha)\end{aligned}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\cos\left(30^0\right)\cos\left(\alpha\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\times\frac{\sqrt{3}}{2}\cos\left(\alpha\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\sqrt{3}\cos\left(\alpha\right)\right] = \frac{3\sqrt{3}V_m}{2\pi}\cos\left(\alpha\right)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi}\cos\left(\alpha\right)$$

Where $V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}\ V_m}{2\pi}$$

Where V_m is the peak phase voltage.

And the normalized average output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

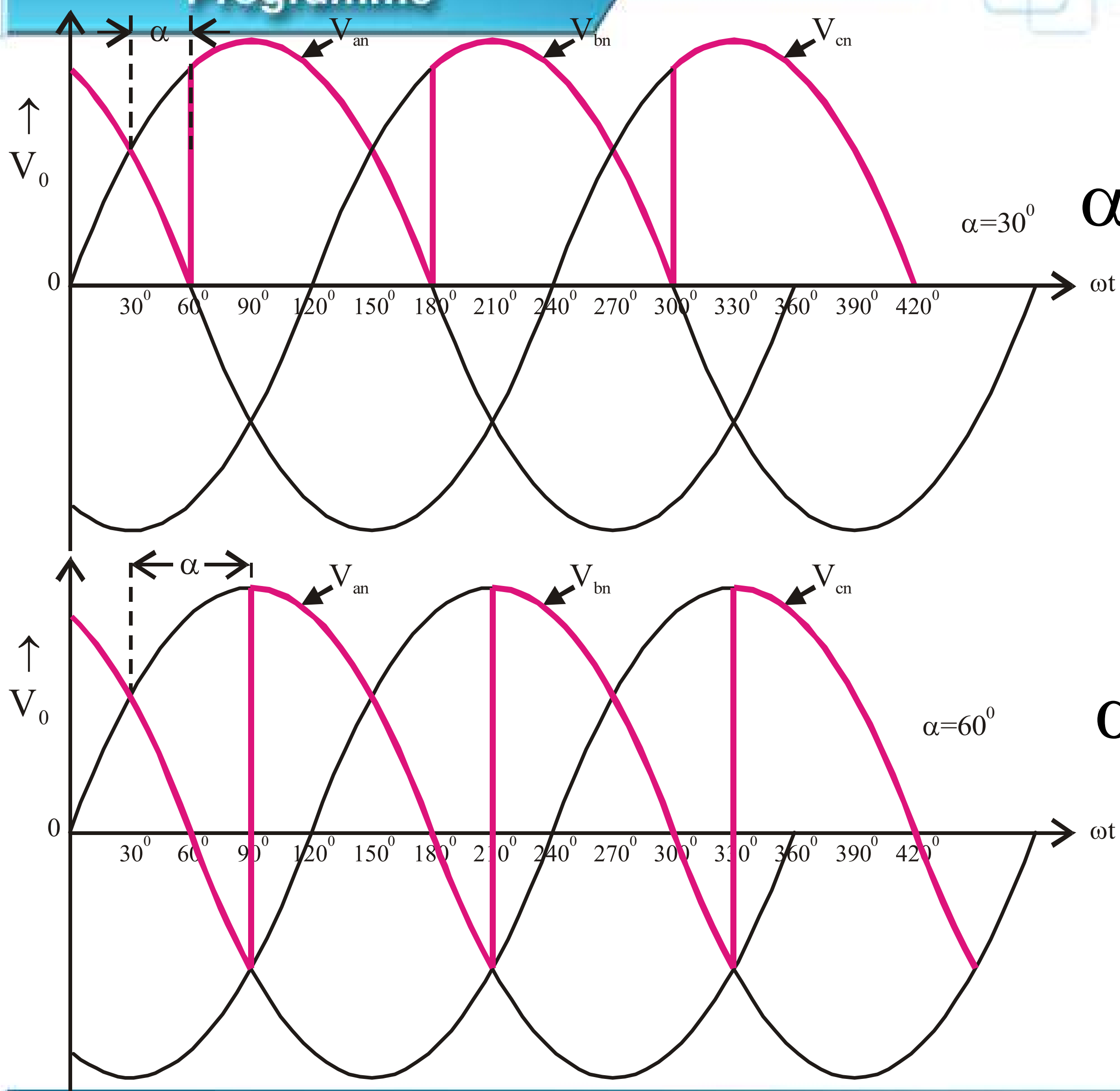
The rms value of output voltage is found by using the equation

$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

and we obtain

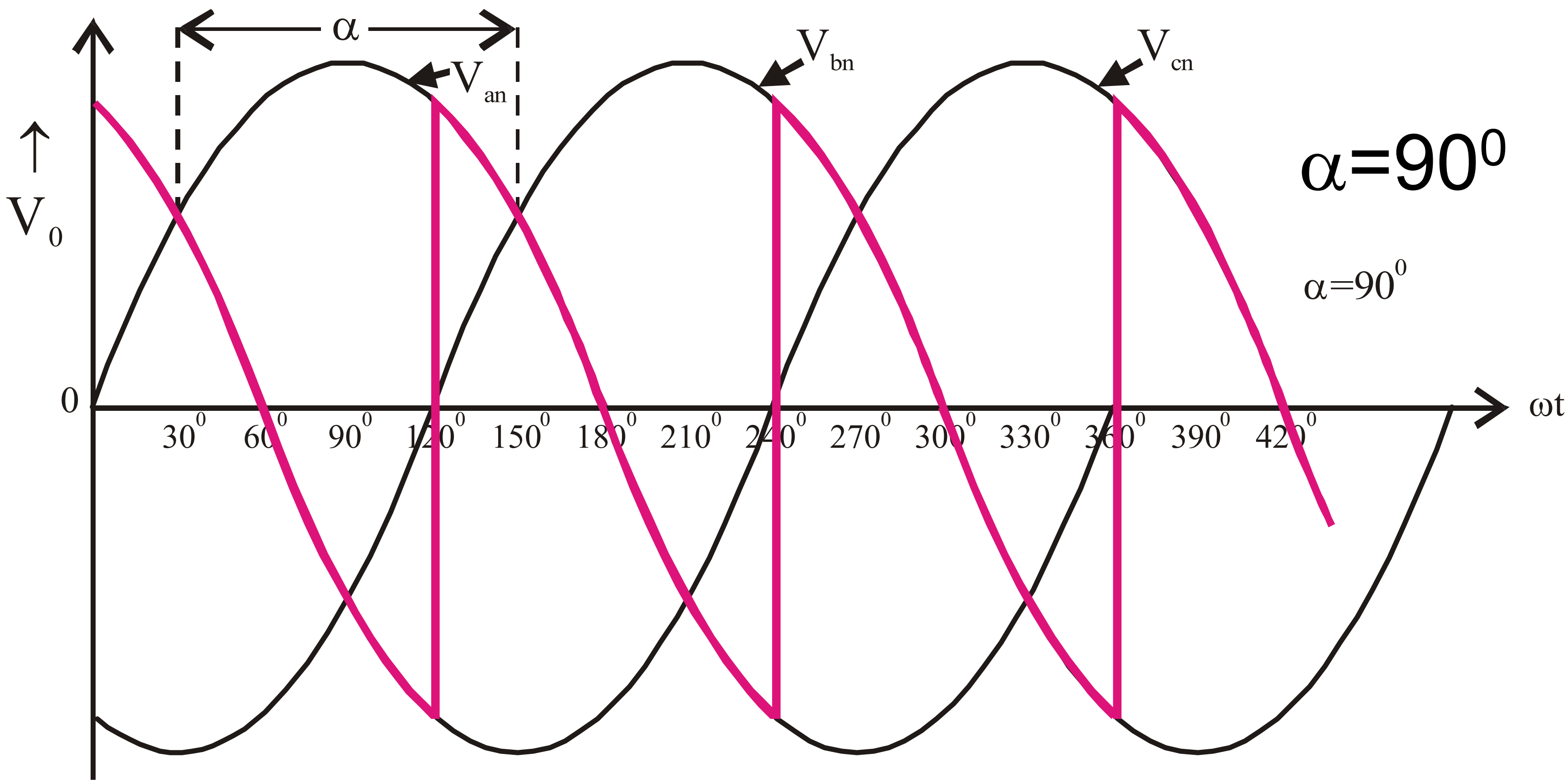
$$V_{O(RMS)} = \sqrt{3} V_m \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$$

3 Phase Half Wave Controlled Rectifier Output Voltage Waveforms For RL Load at Different Trigger Angles

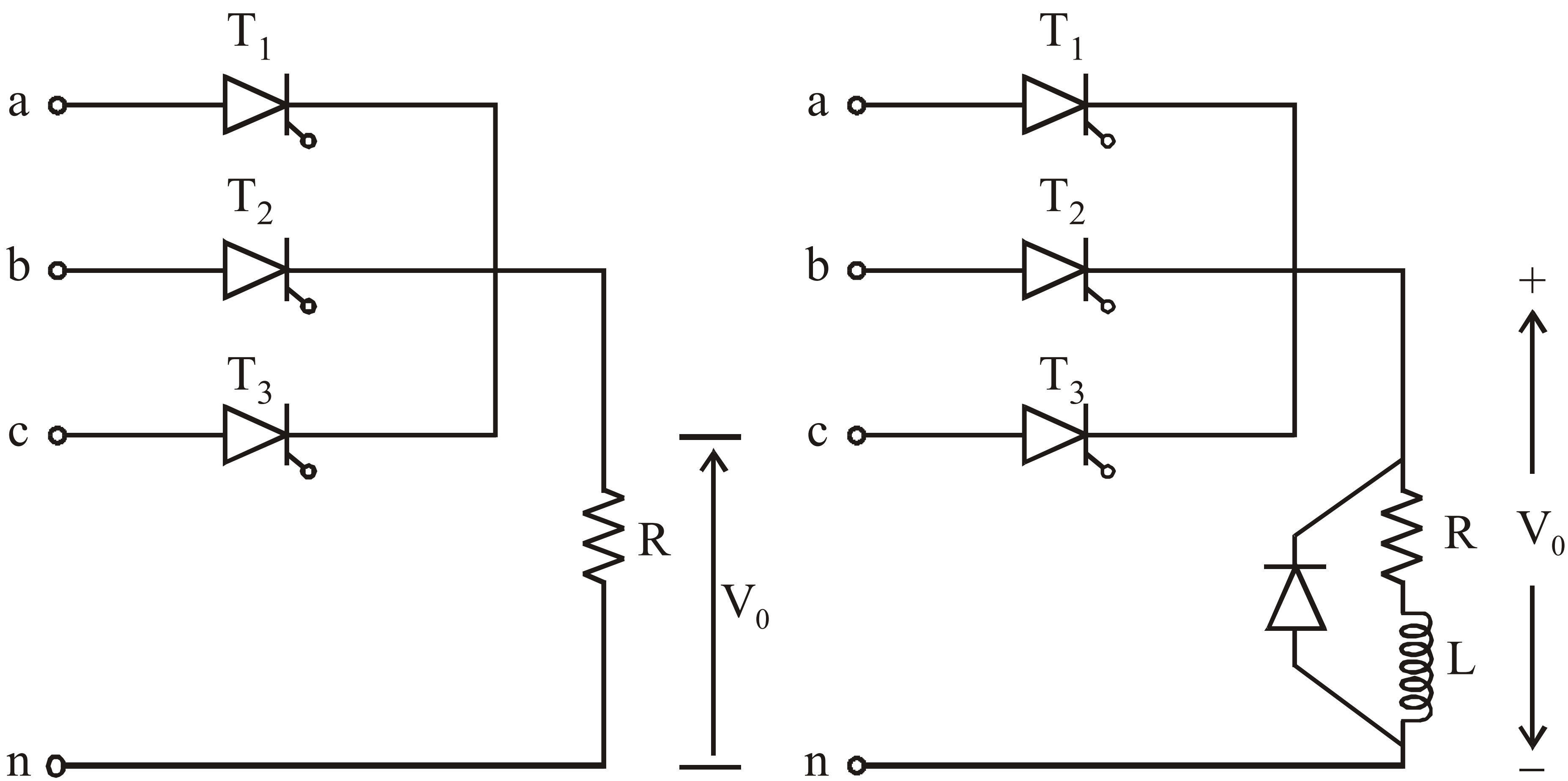


$\alpha = 30^0$

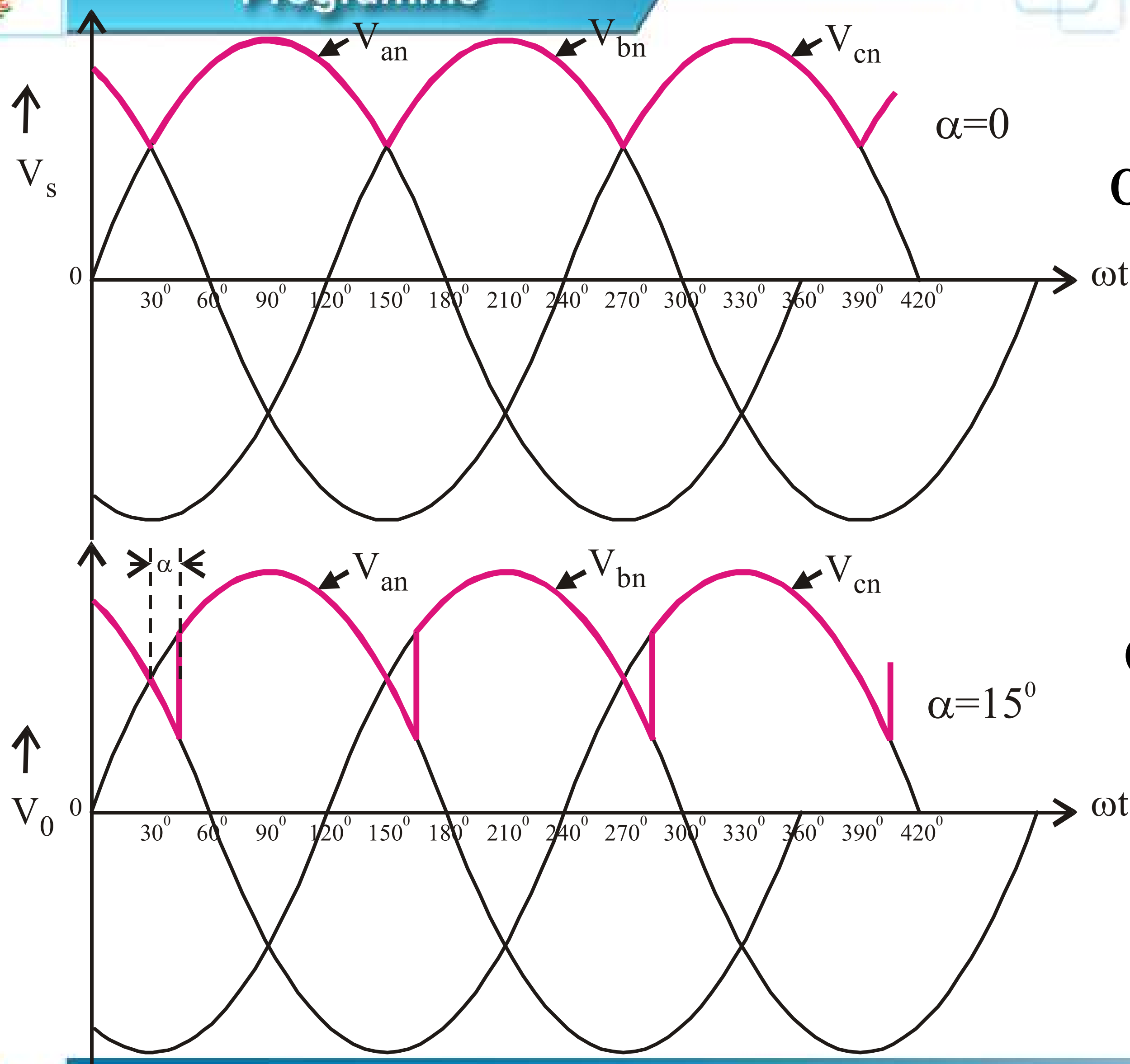
$\alpha = 60^0$


α
V_{an}
V_{bn}
V_{cn}
V_0
0
ωt
30⁰ 60⁰ 90⁰ 120⁰ 150⁰ 180⁰ 210⁰ 240⁰ 270⁰ 300⁰ 330⁰ 360⁰ 390⁰ 420⁰
α=90⁰
α=90⁰

3 Phase Half Wave Controlled Rectifier With R Load and RL Load with FWD

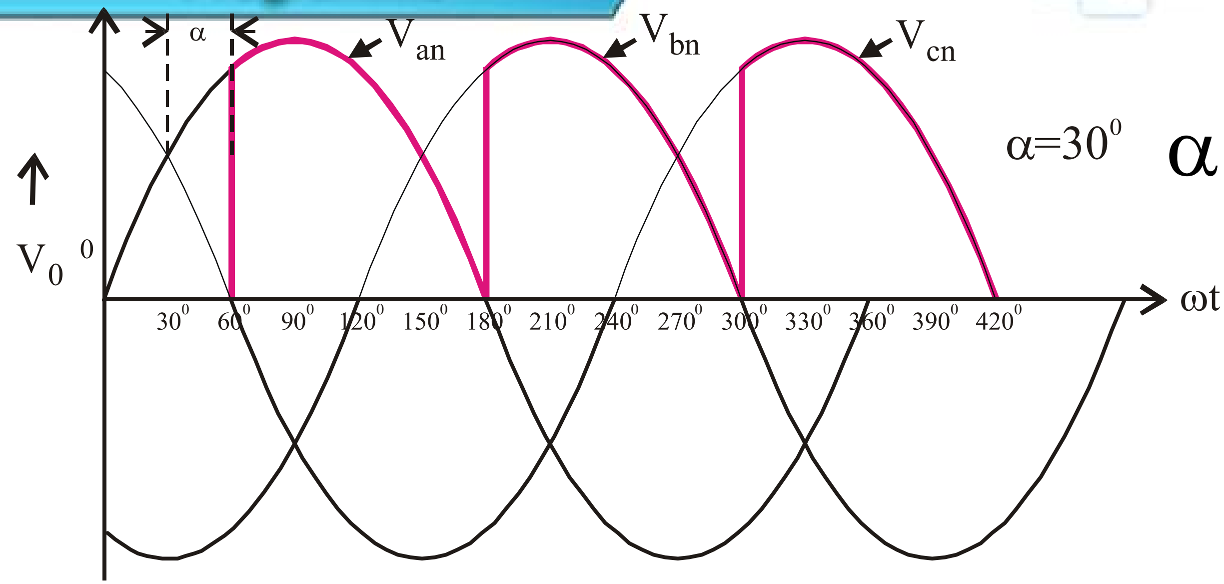

T1
T2
T3
a
b
c
n
R
V0
T1
T2
T3
a
b
c
n
+
R
V0
L
−

3 Phase Half Wave Controlled Rectifier Output Voltage Waveforms For R Load or RL Load with FWD at Different Trigger Angles

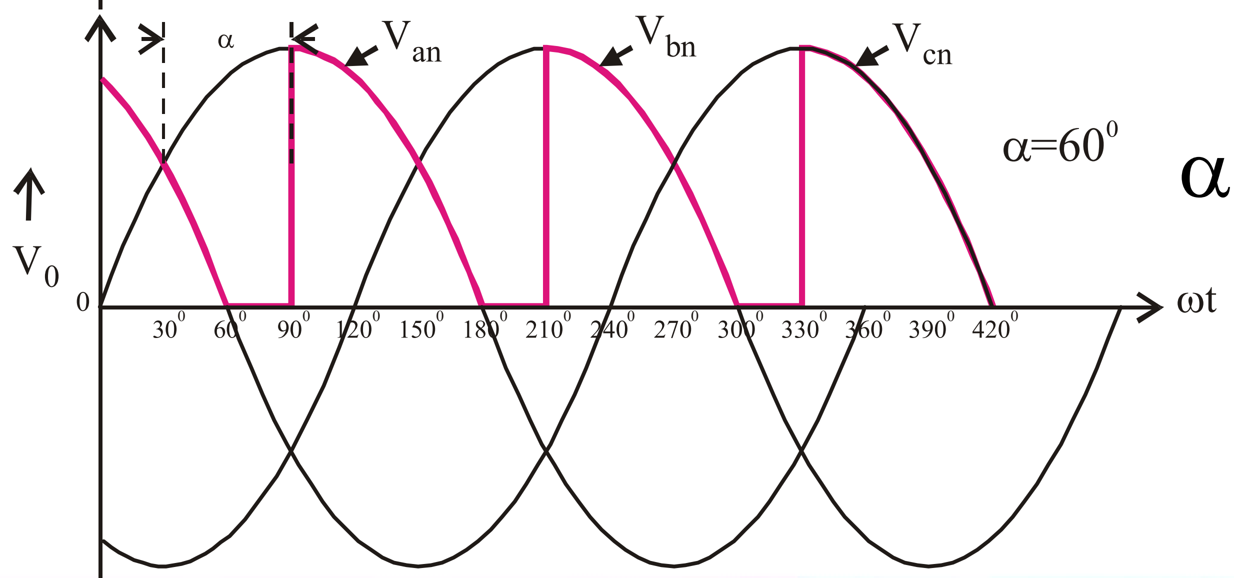


$\alpha = 0^0$

$\alpha = 15^0$



$\alpha = 30^0$



$\alpha = 60^0$

To Derive An Expression For The Average Or Dc Output Voltage Of A 3 Phase Half Wave Converter With Resistive Load Or RL Load With FWD

$$T_1 \ is \ triggered \ at \ \omega t = \left(\frac{\pi}{6}+\alpha\right) = \left(30^0+\alpha\right)$$

$$T_1 \ conducts \ from \ \left(30^0+\alpha\right) \ to \ 180^0 \ ;$$

$$v_O = v_{an} = V_m \sin \omega t$$

$$T_2 \ is \ triggered \ at \ \omega t = \left(\frac{5\pi}{6}+\alpha\right) = \left(150^0+\alpha\right)$$

$$T_2 \ conducts \ from \ \left(150^0+\alpha\right) \ to \ 300^0 \ ;$$

$$v_O = v_{bn} = V_m \sin\left(\omega t - 120^0\right)$$

$$T_3 \ \text{is triggered at} \ \omega t = \left(\frac{7\pi}{6} + \alpha\right) = \left(270^0 + \alpha\right)$$

$$T_3 \ \text{conducts from} \ \left(270^0 + \alpha\right) \ \text{to} \ 420^0 \quad ;$$

$$v_O = v_{cn} = V_m \sin\left(\omega t - 240^0\right)$$

$$= V_m \sin\left(\omega t + 120^0\right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} v_O . d(\omega t)\right]$$

$$v_O = v_{an} = V_m \sin \omega t; \text{ for } \omega t = \left(\alpha + 30^0\right) \text{ to } \left(180^0\right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} V_m \sin \omega t . d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\alpha+30^0}^{180^0} \sin \omega t . d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha+30^0}^{180^0}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos 180^0 + \cos\left(\alpha + 30^0\right)\right]$$

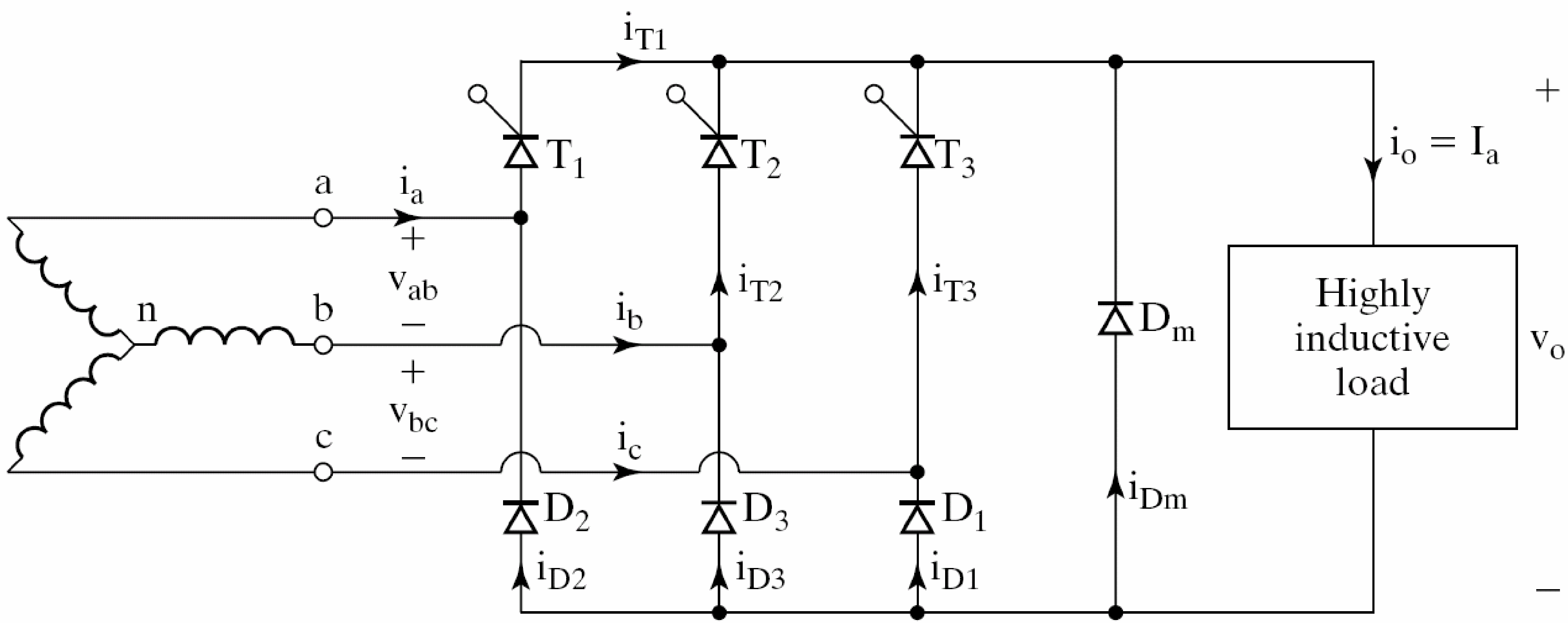
$\because \quad \cos 180^0 = -1,$ we get

$$V_{dc} = \frac{3V_m}{2\pi}\left[1 + \cos\left(\alpha + 30^0\right)\right]$$

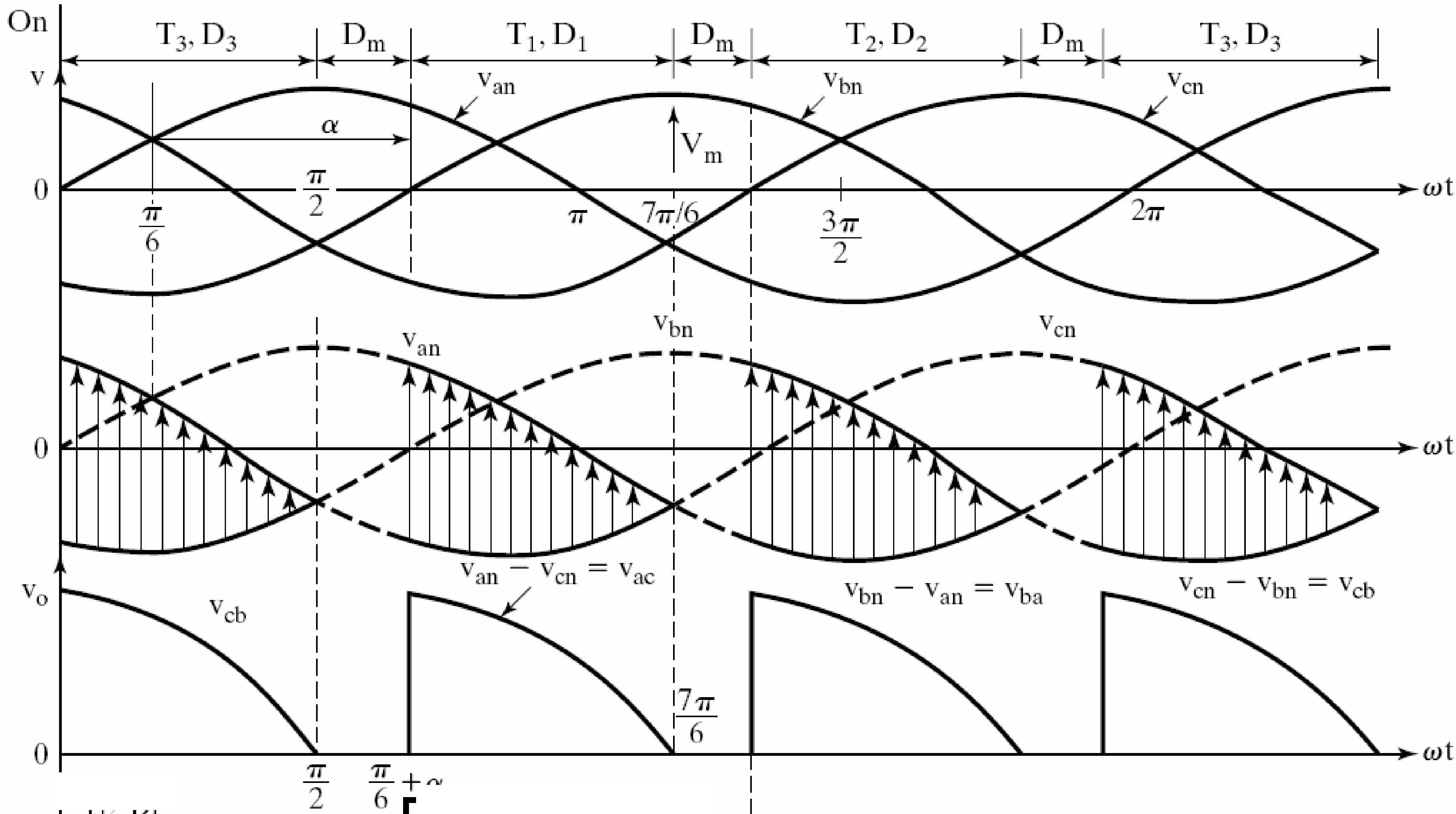
Three Phase Semiconverters

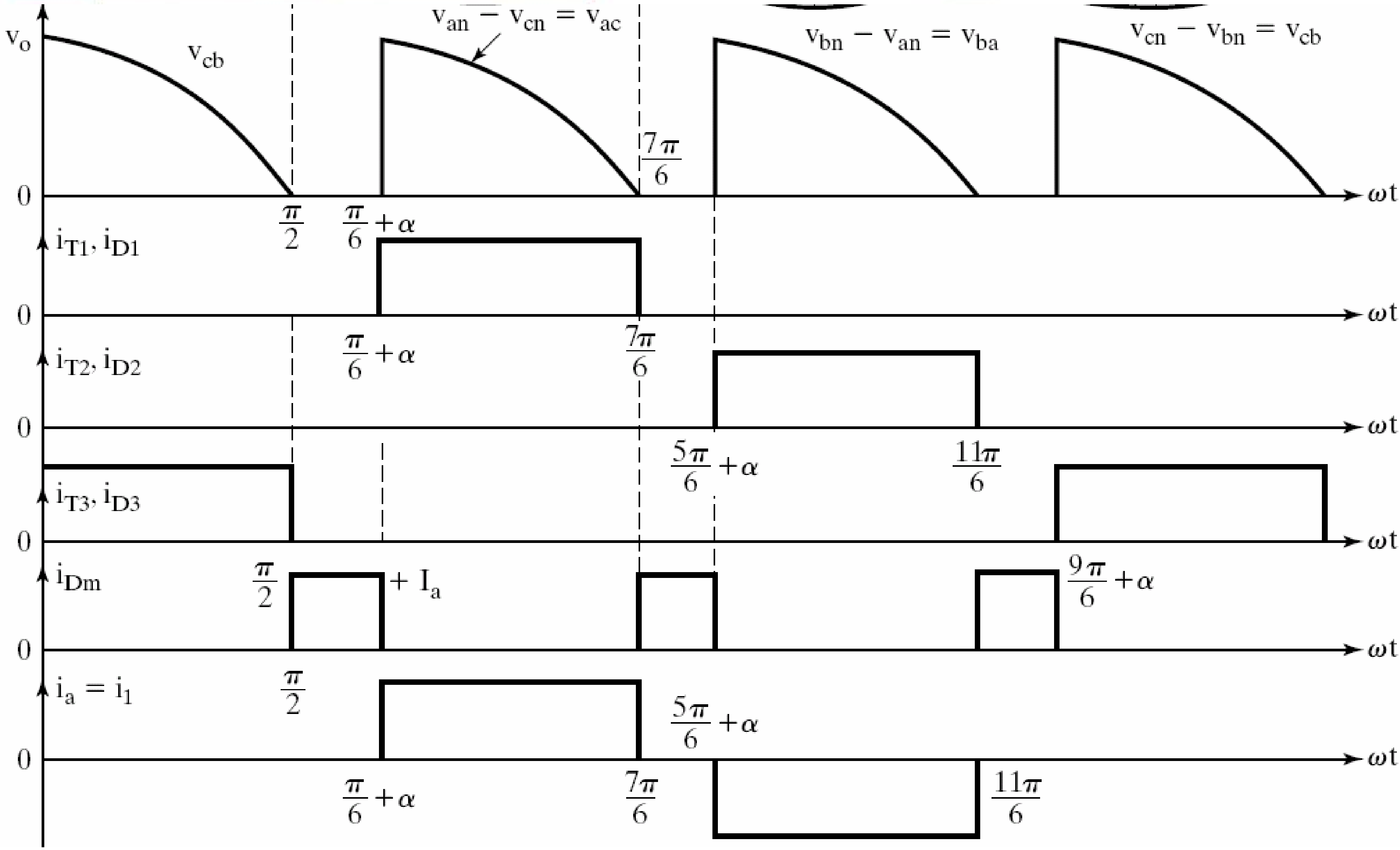
- 3 Phase semiconverters are used in Industrial dc drive applications upto 120kW power output.
- Single quadrant operation is possible.
- Power factor decreases as the delay angle increases.
- Power factor is better than that of 3 phase half wave converter.

3 Phase
Half Controlled Bridge Converter
(Semi Converter)
with Highly Inductive Load &
Continuous Ripple free Load Current


iT1
T1
T2
T3
a
ia
+
vab
−
n
b
iT2
iT3
ib
Dm
Highly
inductive
load
io = Ia
+
vo
−
+
vbc
−
c
ic
D2
D3
D1
iD2
iD3
iD1
iDm

Wave forms of 3 Phase Semiconverter for $\alpha > 60^0$


On
T_3, D_3
D_m
T_1, D_1
D_m
T_2, D_2
D_m
T_3, D_3
v
v_{an}
v_{bn}
v_{cn}
α
V_m
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{2}$
π
$7\pi/6$
$\frac{3\pi}{2}$
2π
v_{an}
v_{bn}
v_{cn}
0
ωt
v_o
v_{cb}
$v_{an} - v_{cn} = v_{ac}$
$v_{bn} - v_{an} = v_{ba}$
$v_{cn} - v_{bn} = v_{cb}$
$\frac{7\pi}{6}$
0
ωt
$\frac{\pi}{2}$
$\frac{\pi}{6} + \alpha$


v_o
v_{cb}
$v_{an} - v_{cn} = v_{ac}$
$v_{bn} - v_{an} = v_{ba}$
$v_{cn} - v_{bn} = v_{cb}$
$\frac{7\pi}{6}$
0
$\frac{\pi}{2}$
$\frac{\pi}{6}+\alpha$
ωt
i_{T1}, i_{D1}
i_{T2}, i_{D2}
$\frac{7\pi}{6}$
i_{T3}, i_{D3}
$\frac{5\pi}{6}+\alpha$
$\frac{11\pi}{6}$
i_{Dm}
$+ I_a$
$\frac{9\pi}{6}+\alpha$
$i_a = i_1$

3 phase semiconverter output ripple frequency of output voltage is $3f_S$

The delay angle α can be varied from 0 to π

During the period

$$30^0 \le \omega t < 210^0$$

$$\frac{\pi}{6} \le \omega t < \frac{7\pi}{6}, \text{ thyristor } T_1 \text{ is forward biased}$$

If thyristor T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha \right)$,

T_1 & D_1 conduct together and the line to line voltage

v_{ac} appears across the load.

At $\omega t = \frac{7\pi}{6}$, v_{ac} becomes negative & FWD D_m conducts.

The load current continues to flow through FWD D_m;

T_1 and D_1 are turned off.

If FWD D_m is not used the T_1 would continue to conduct until the thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right)$, and Free wheeling action would be accomplished through T_1 & D_2.

If the delay angle $\alpha \leq \frac{\pi}{3}$, each thyristor conducts for $\frac{2\pi}{3}$ and the FWD D_m does not conduct.

We deifine three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \qquad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$

V_m is the peak phase voltage of a wye-connected source

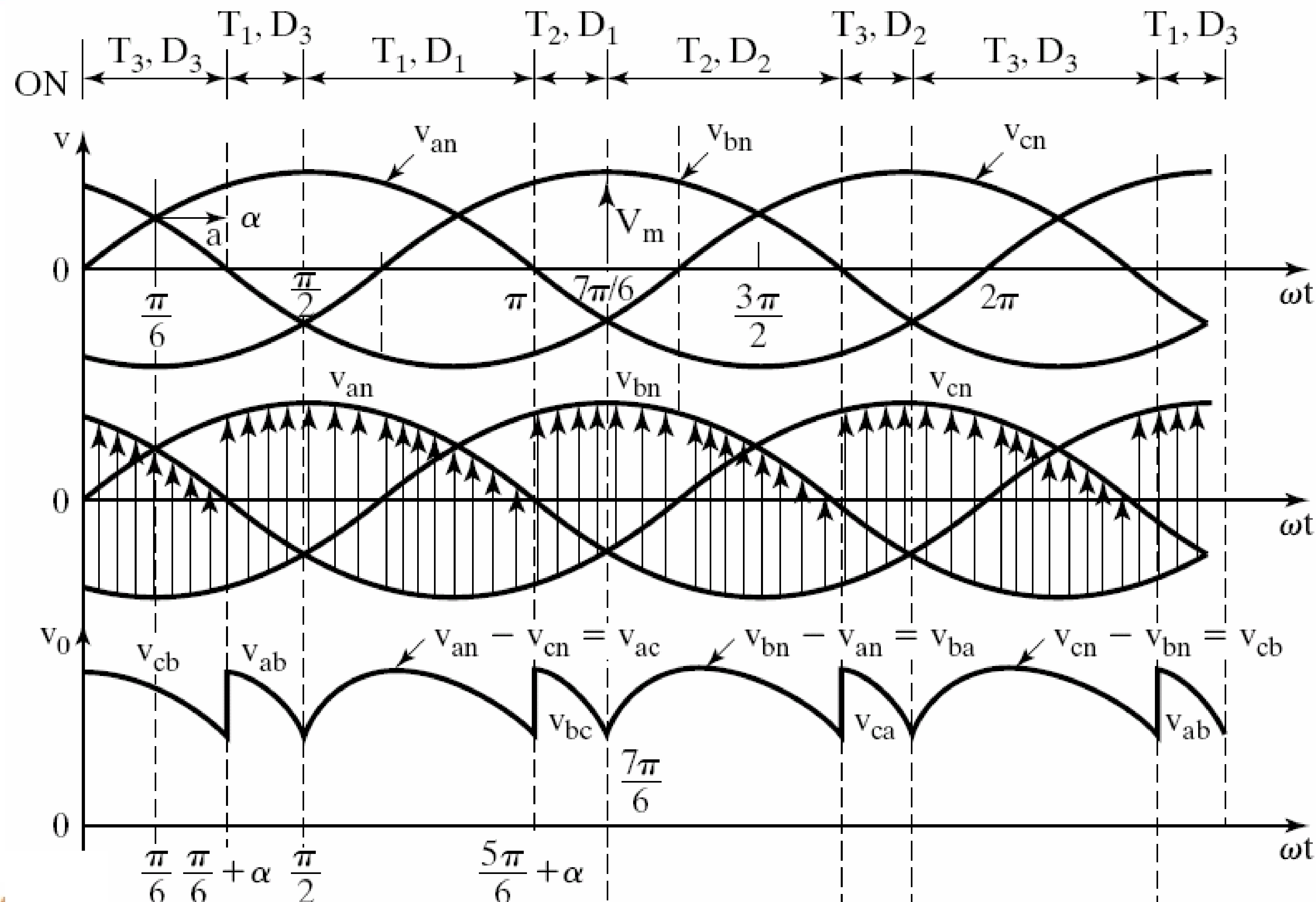
$$v_{RB} = v_{ac} = \left(v_{an} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

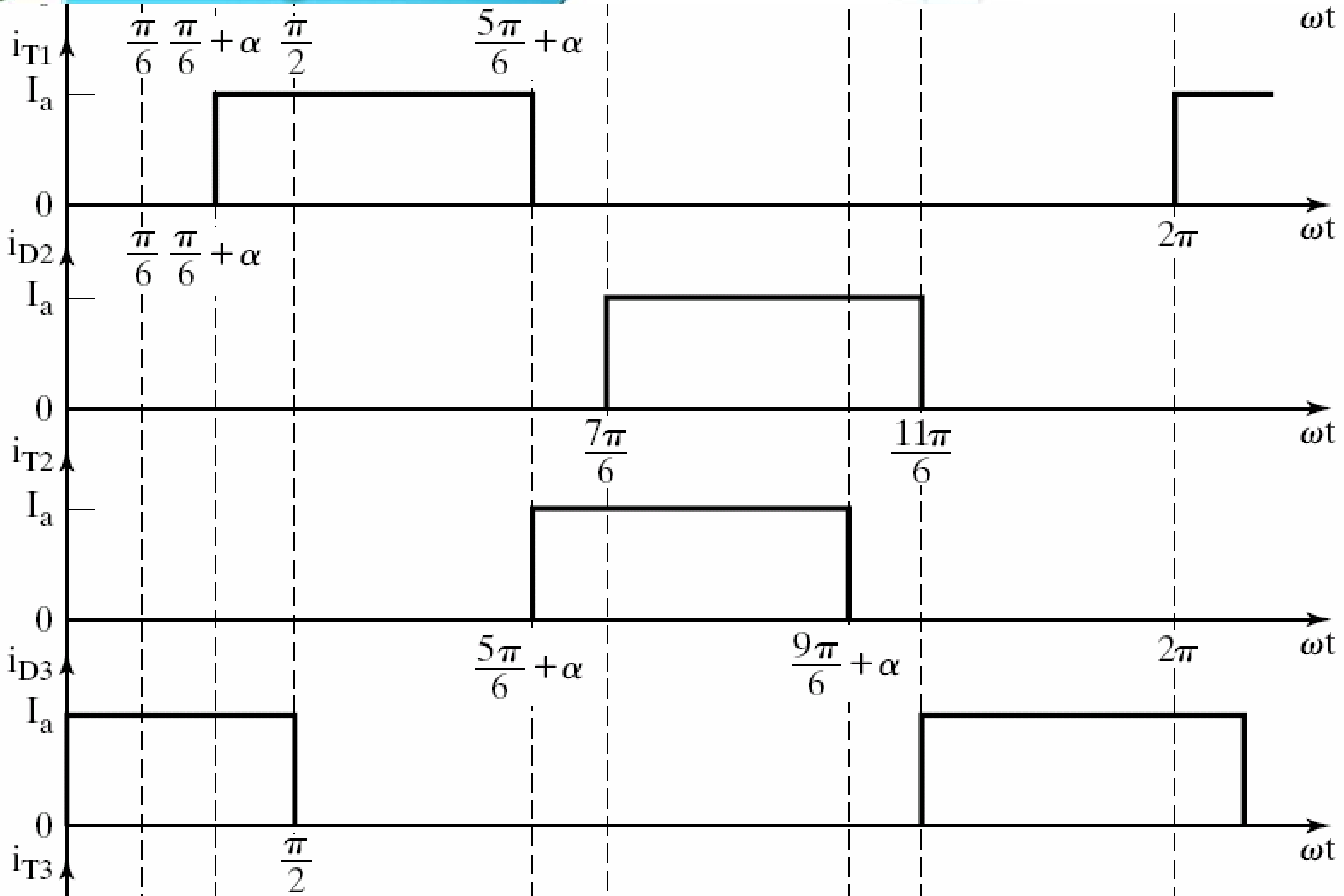
$$v_{YR} = v_{ba} = \left(v_{bn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{5\pi}{6}\right)$$

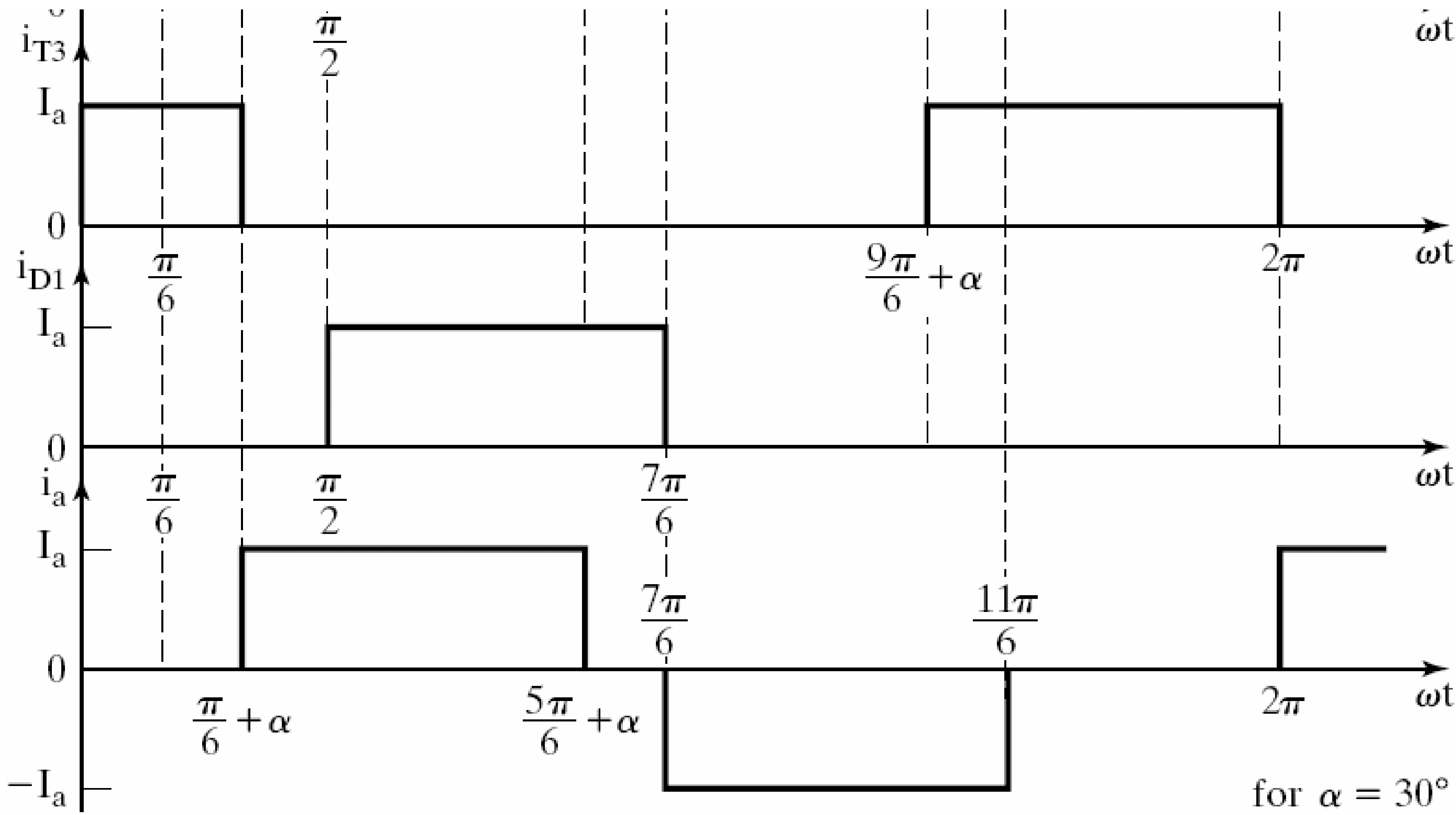
$$v_{BY} = v_{cb} = \left(v_{cn} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

Wave forms of 3 Phase Semiconverter for $\alpha \leq 60^0$


ON
T_3, D_3
T_1, D_3
T_1, D_1
T_2, D_1
T_2, D_2
T_3, D_2
T_3, D_3
T_1, D_3
v
v_{an}
v_{bn}
v_{cn}
α
a
V_m
0
$\frac{\pi}{6}$
$\frac{\pi}{2}$
π
$7\pi/6$
$\frac{3\pi}{2}$
2π
ωt
v_{an}
v_{bn}
v_{cn}
0
ωt
v_0
v_{cb}
v_{ab}
$v_{an} - v_{cn} = v_{ac}$
$v_{bn} - v_{an} = v_{ba}$
$v_{cn} - v_{bn} = v_{cb}$
v_{bc}
v_{ca}
v_{ab}
$\frac{7\pi}{6}$
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha$
$\frac{\pi}{2}$
$\frac{5\pi}{6} + \alpha$


i_{T1}
I_a
0
$\frac{\pi}{6}$ $\frac{\pi}{6}+\alpha$ $\frac{\pi}{2}$ $\frac{5\pi}{6}+\alpha$
ωt
i_{D2}
I_a
0
$\frac{\pi}{6}$ $\frac{\pi}{6}+\alpha$
2π
ωt
i_{T2}
I_a
0
$\frac{7\pi}{6}$ $\frac{11\pi}{6}$
ωt
i_{D3}
I_a
0
$\frac{5\pi}{6}+\alpha$ $\frac{9\pi}{6}+\alpha$ 2π
ωt
i_{T3}
$\frac{\pi}{2}$
ωt


i_{T3}
I_a
0
$\frac{\pi}{2}$
ωt
i_{D1}
$\frac{\pi}{6}$
$\frac{9\pi}{6}+\alpha$
2π
I_a
0
i_a
$\frac{\pi}{6}$
$\frac{\pi}{2}$
$\frac{7\pi}{6}$
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
$\frac{\pi}{6}+\alpha$
$\frac{5\pi}{6}+\alpha$
$-I_a$
for $\alpha = 30°$

To derive an Expression for the Average Output Voltage of 3 Phase Semiconverter for $\alpha > \pi / 3$ and Discontinuous Output Voltage

For $\alpha \geq \frac{\pi}{3}$ and discontinuous output voltage:

the Average output voltage is found from

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{7\pi/6} v_{ac}.d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3}\ V_m \sin\left(\omega t - \frac{\pi}{6}\right) d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

$$V_{dc} = \frac{3V_{mL}}{2\pi}(1+\cos\alpha)$$

$V_{mL} = \sqrt{3}V_m =$ Max. value of line-to-line supply voltage

The maximum average output voltage that occurs at a delay angle of $\alpha = 0$ is

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi}$$

The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1+\cos\alpha\right)$$

The rms output voltage is found from

$$V_{O(rms)} = \left[\frac{3}{2\pi}\int_{\pi/6+\alpha}^{7\pi/6} v_{ac}^2 .d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} 3V_m^2 \sin^2\left(\omega t - \frac{\pi}{6}\right) d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3} V_m \left[\frac{3}{4\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

Average or DC Output Voltage of a 3-Phase Semiconverter for $\alpha \leq \pi / 3$, and Continuous Output Voltage

For $\alpha \leq \frac{\pi}{3}$, and continuous output voltage

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{\pi/2} v_{ab}.d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}.d(\omega t)\right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1+\cos\alpha\right)$$

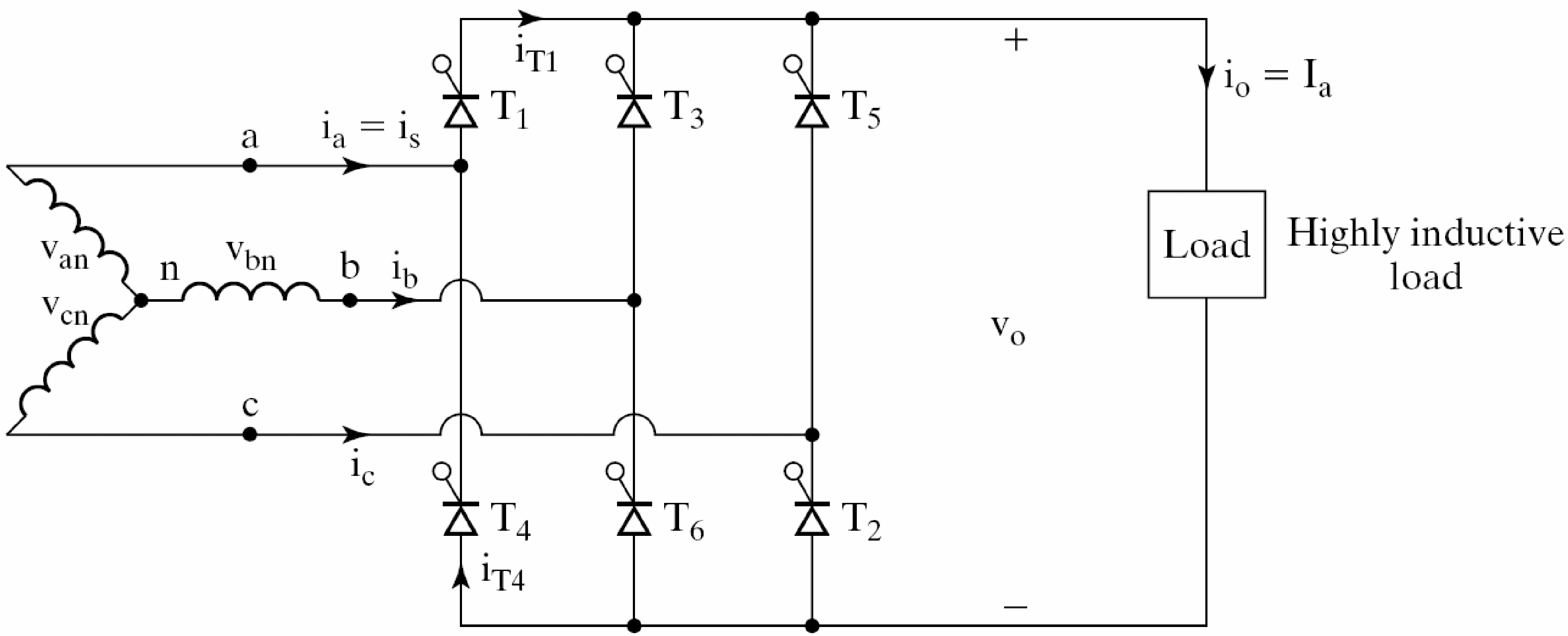
RMS value of o/p voltage is calculated by using the equation

$$V_{O(rms)} = \left[\frac{3}{2\pi}\int_{\pi/6+\alpha}^{\pi/2} v_{ab}^2 . d\left(\omega t\right) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}^2 . d\left(\omega t\right)\right]^{\frac{1}{2}}$$

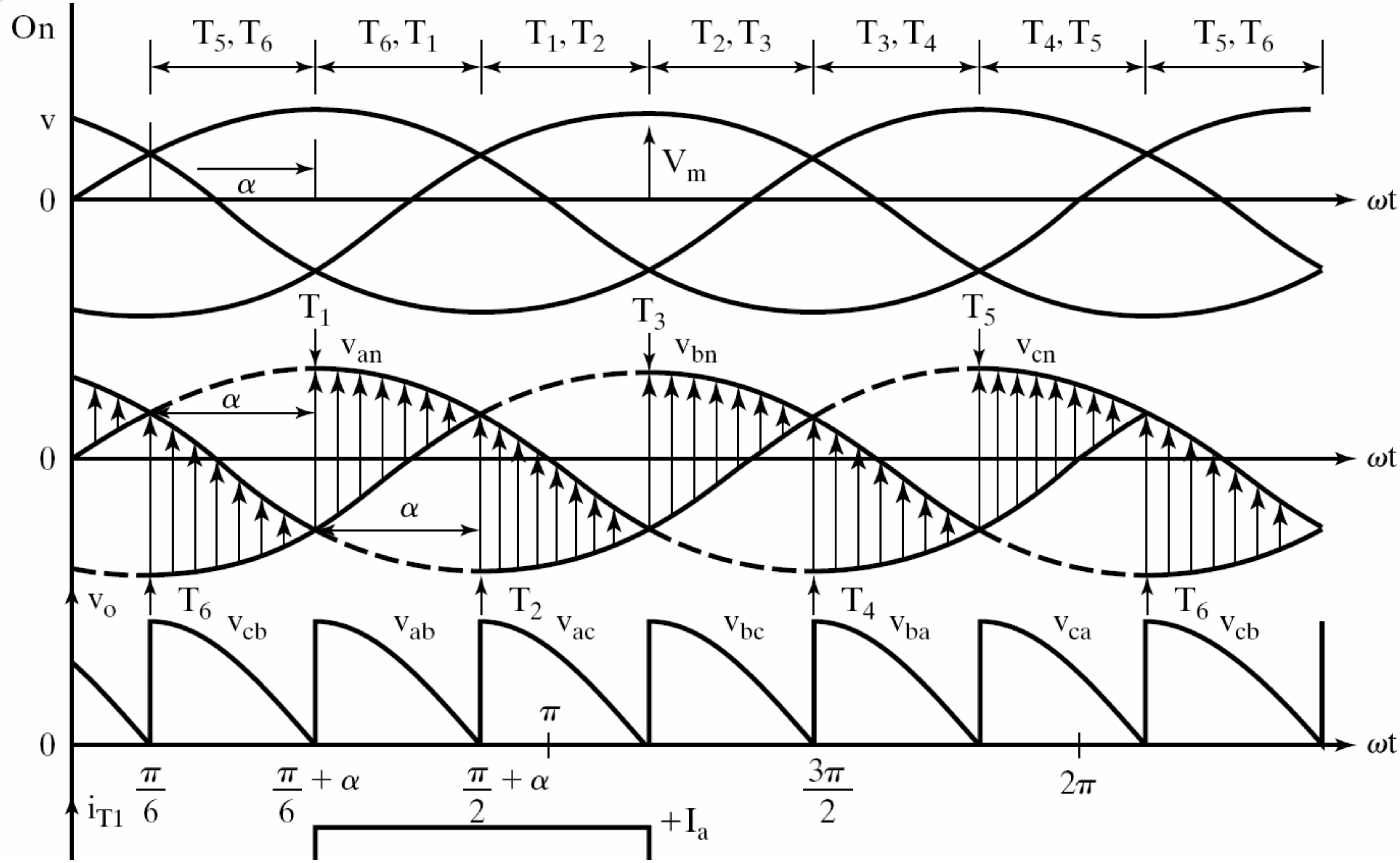
$$V_{O(rms)} = \sqrt{3}V_m\left[\frac{3}{4\pi}\left(\frac{2\pi}{3}+\sqrt{3}\cos^2\alpha\right)\right]^{\frac{1}{2}}$$

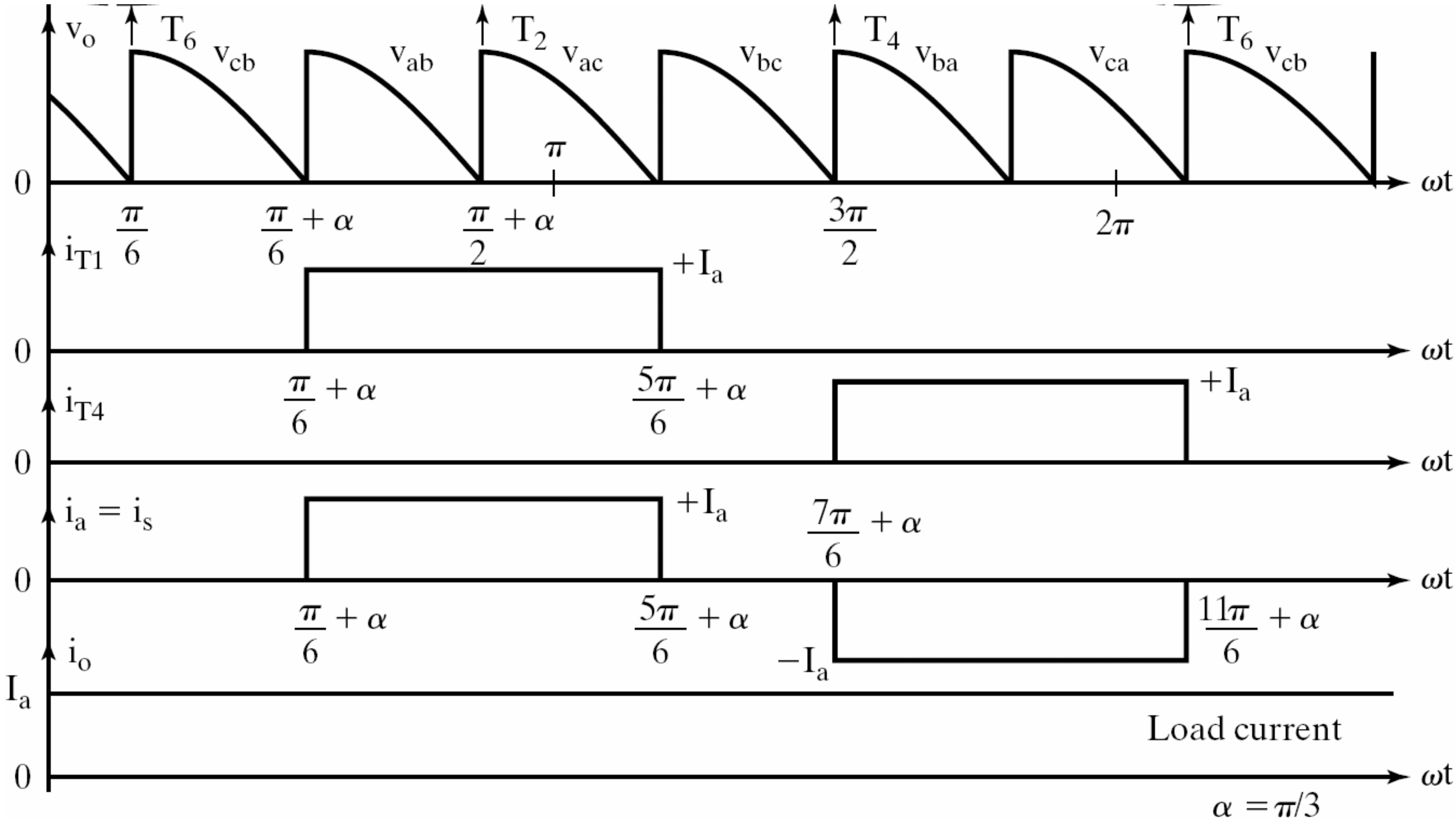
Three Phase Full Converter

- 3 Phase Fully Controlled Full Wave Bridge Converter.
- Known as a 6-pulse converter.
- Used in industrial applications up to 120kW output power.
- Two quadrant operation is possible.



i_{T1}
T_1
T_3
T_5
+
$i_o = I_a$
a
$i_a = i_s$
v_{an}
n
v_{bn}
b
i_b
v_{cn}
Load
Highly inductive load
v_o
c
i_c
T_4
T_6
T_2
i_{T4}
–


On
T5, T6
T6, T1
T1, T2
T2, T3
T3, T4
T4, T5
T5, T6
v
0
α
Vm
ωt
T1
T3
T5
van
vbn
vcn
α
0
ωt
α
vo
T6
vcb
vab
T2
vac
vbc
T4
vba
vca
T6
vcb
π
0
ωt
π/6
π/6 + α
π/2 + α
3π/2
2π
iT1
+Ia



v_o
T_6 v_{cb}
v_{ab}
T_2 v_{ac}
v_{bc}
T_4 v_{ba}
v_{ca}
T_6 v_{cb}
π
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha$
$\frac{\pi}{2} + \alpha$
$\frac{3\pi}{2}$
2π
i_{T1}
$+I_a$
0
ωt
i_{T4}
$\frac{\pi}{6} + \alpha$
$\frac{5\pi}{6} + \alpha$
$+I_a$
0
ωt
$i_a = i_s$
$+I_a$
$\frac{7\pi}{6} + \alpha$
0
ωt
$\frac{\pi}{6} + \alpha$
$\frac{5\pi}{6} + \alpha$
$\frac{11\pi}{6} + \alpha$
$-I_a$
i_o
I_a
Load current
0
ωt
$\alpha = \pi/3$

- The thyristors are triggered at an interval of $\pi / 3$.
- The frequency of output ripple voltage is $6f_S$.
- T_1 is triggered at $\omega t = (\pi/6 + \alpha)$, T_6 is already conducting when T_1 is turned ON.
- During the interval $(\pi/6 + \alpha)$ to $(\pi/2 + \alpha)$, T_1 and T_6 conduct together & the output load voltage is equal to $v_{ab} = (v_{an} - v_{bn})$

- T_2 is triggered at $\omega t = (\pi/2 + \alpha)$, T_6 turns off naturally as it is reverse biased as soon as T_2 is triggered.
- During the interval $(\pi/2 + \alpha)$ to $(5\pi/6 + \alpha)$, T_1 and T_2 conduct together & the output load voltage $v_O = v_{ac} = (v_{an} - v_{cn})$
- Thyristors are numbered in the order in which they are triggered.
- The thyristor triggering sequence is 12, 23, 34, 45, 56, 61, 12, 23, 34, ...

We deifine three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \quad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3} \right) = V_m \sin\left(\omega t - 120^0 \right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3} \right) = V_m \sin\left(\omega t + 120^0 \right)$$

$$= V_m \sin\left(\omega t - 240^0 \right)$$

V_m is the peak phase voltage of a wye-connected source

The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

To Derive An Expression For The Average Output Voltage Of 3-phase Full Converter With Highly Inductive Load Assuming Continuous And Constant Load Current

The output load voltage consists of 6 voltage pulses over a period of 2π radians, Hence the average output voltage is calculated as

$$V_{O(dc)} = V_{dc} = \frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O . d\omega t \quad ;$$

$$v_O = v_{ab} = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right).d\omega t$$

$$V_{dc} = \frac{3\sqrt{3} V_m}{\pi} \cos\alpha = \frac{3 V_{mL}}{\pi} \cos\alpha$$

Where $V_{mL} = \sqrt{3} V_m$ = Max. line-to-line supply voltage

The maximum average dc output voltage is obtained for a delay angle $\alpha = 0$,

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3} V_m}{\pi} = \frac{3 V_{mL}}{\pi}$$

The normalized average dc output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

The rms value of the output voltage is found from

$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O^2 . d(\omega t) \right]^{\frac{1}{2}}$$

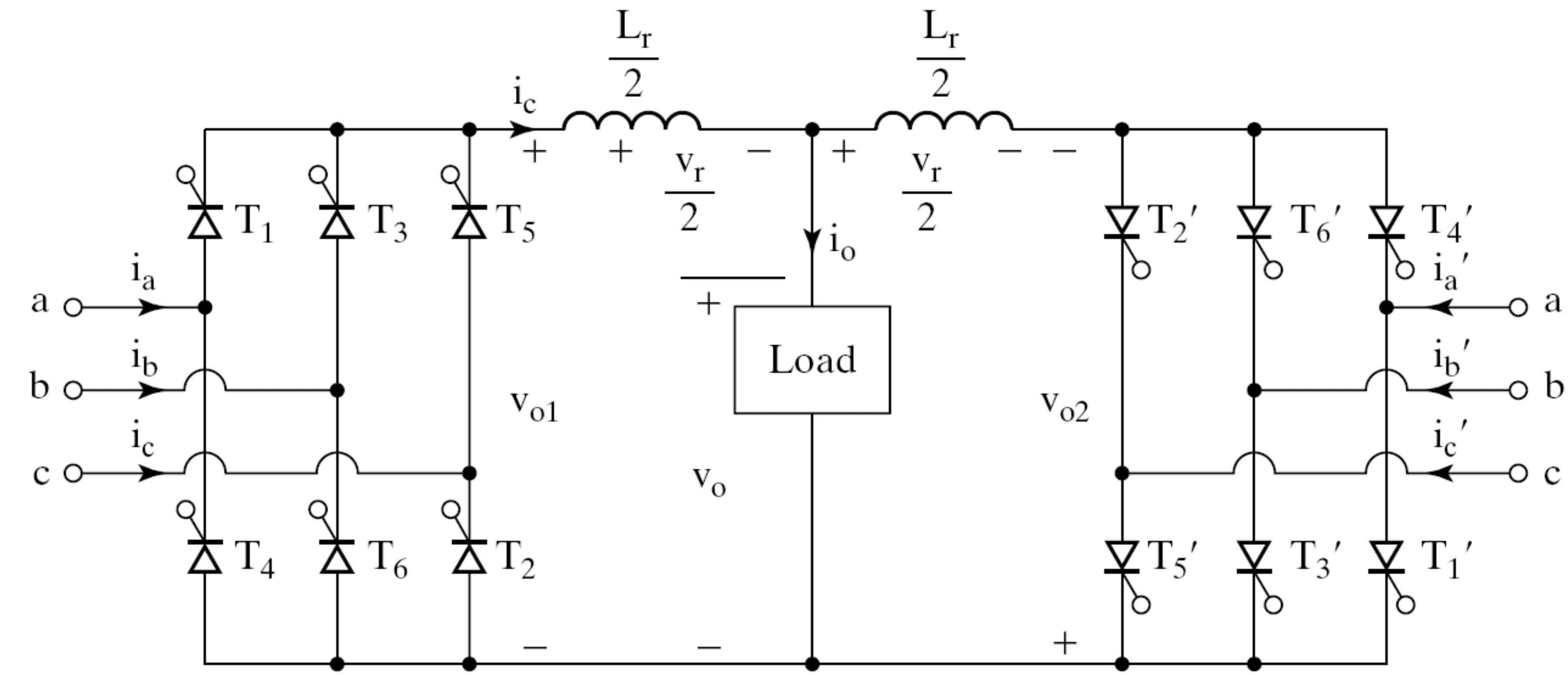
$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_{ab}^2 .d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} 3V_m^2 \sin^2\left(\omega t + \frac{\pi}{6} \right) .d(\omega t) \right]^{\frac{1}{2}}$$

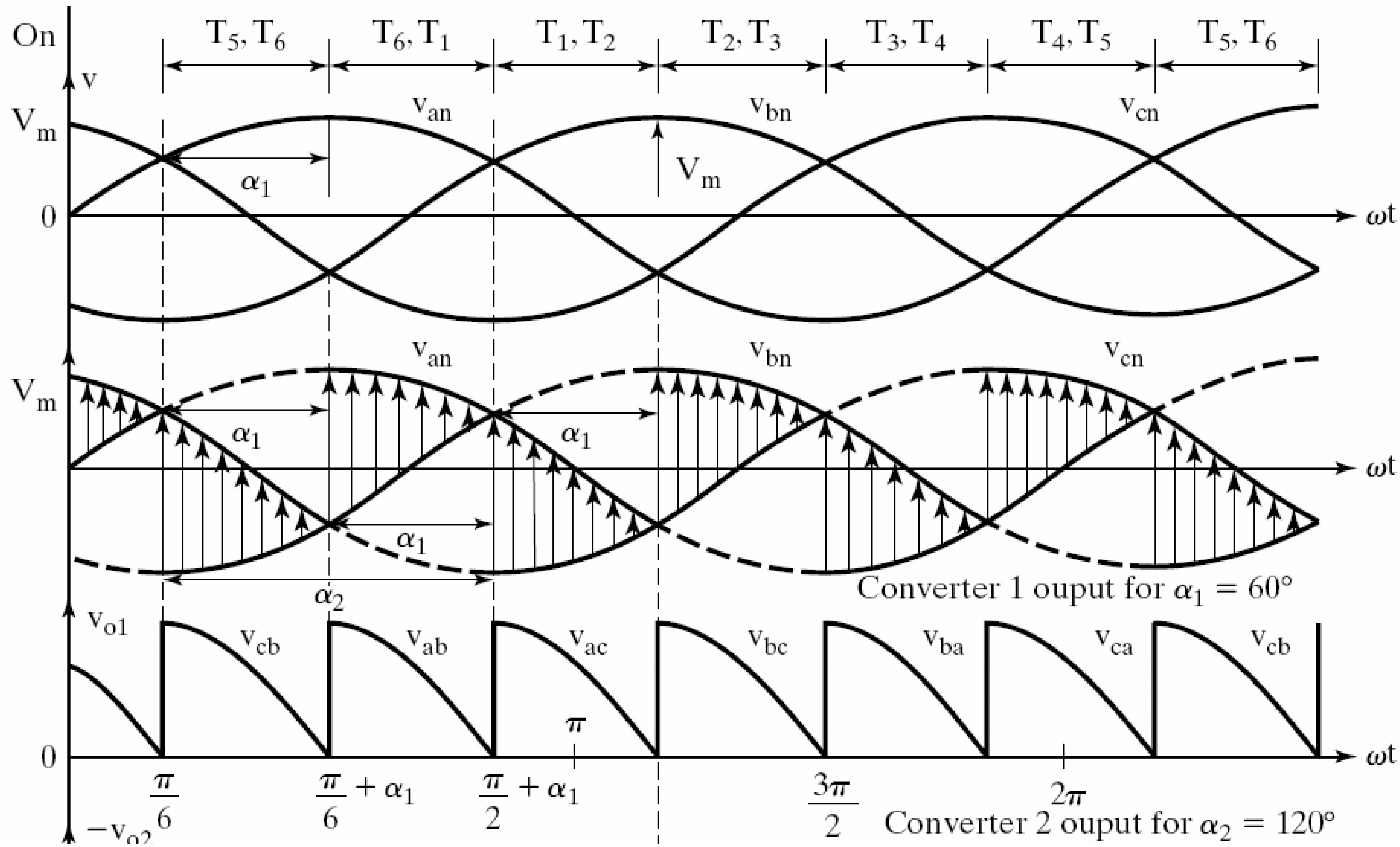
$$V_{O(rms)} = \sqrt{3} V_m \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha \right)^{\frac{1}{2}}$$

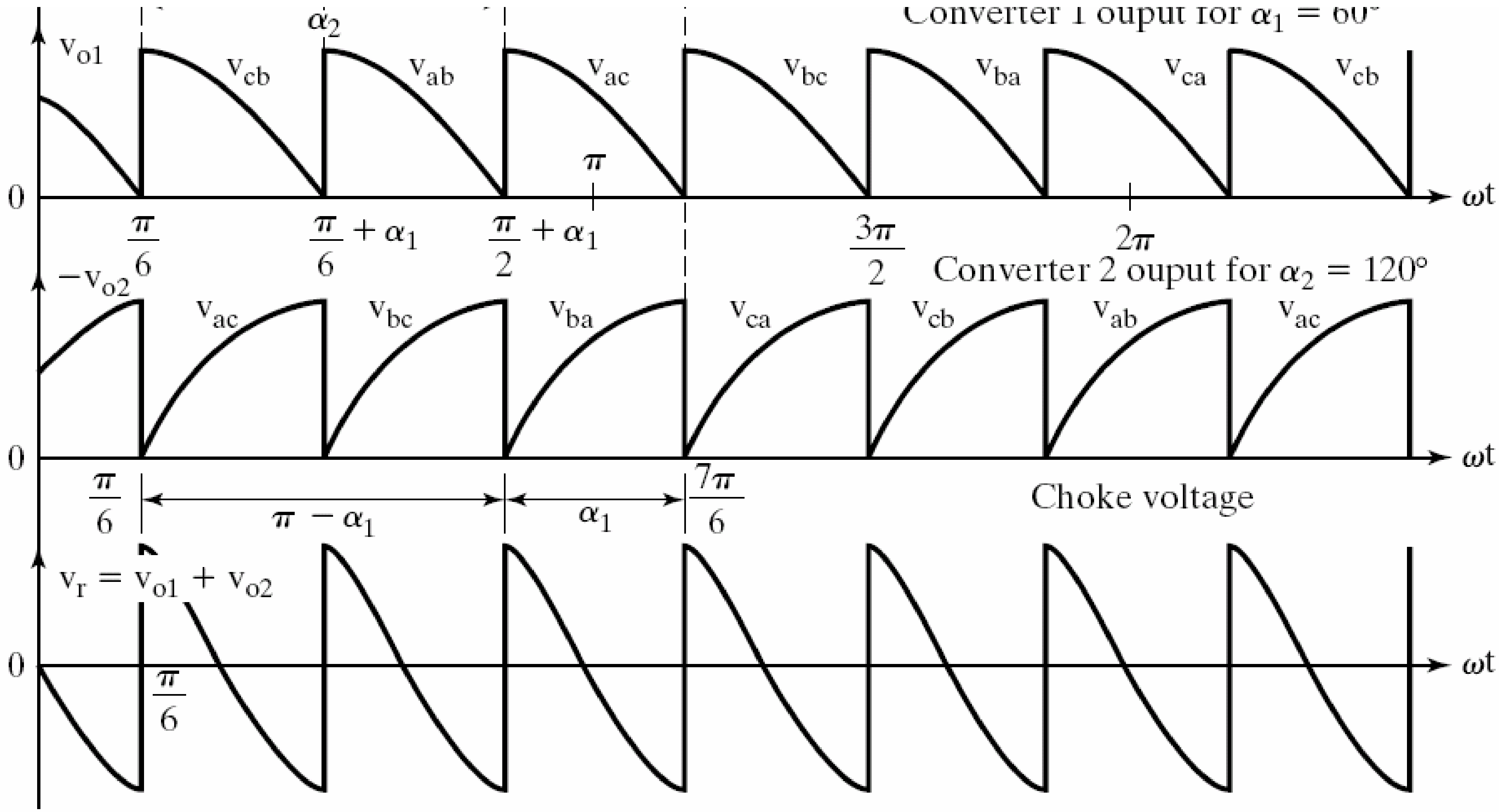
Three Phase Dual Converters

- For four quadrant operation in many industrial variable speed dc drives , 3 phase dual converters are used.
- Used for applications up to 2 mega watt output power level.
- Dual converter consists of two 3 phase full converters which are connected in parallel & in opposite directions across a common load.



$\frac{L_r}{2}$
$\frac{L_r}{2}$
i_c
+ + $\frac{v_r}{2}$ −
+ $\frac{v_r}{2}$ − −
T_1 T_3 T_5
T_2' T_6' T_4'
i_o
i_a i_b i_c
i_a' i_b' i_c'
a b c
a b c
+
Load
v_{o1}
v_{o2}
v_o
T_4 T_6 T_2
T_5' T_3' T_1'
− − +


On
T5, T6
T6, T1
T1, T2
T2, T3
T3, T4
T4, T5
T5, T6
v
Vm
van
vbn
vcn
α1
Vm
0
ωt
Vm
van
vbn
vcn
α1
α1
α1
α2
ωt
Converter 1 ouput for α1 = 60°
vo1
vcb
vab
vac
vbc
vba
vca
vcb
π
0
ωt
π/6
π/6 + α1
π/2 + α1
3π/2
2π
−vo2
Converter 2 ouput for α2 = 120°


Converter 1 ouput for $\alpha_1 = 60°$
v_{o1}
α_2
v_{cb}
v_{ab}
v_{ac}
v_{bc}
v_{ba}
v_{ca}
v_{cb}
0
π
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha_1$
$\frac{\pi}{2} + \alpha_1$
$\frac{3\pi}{2}$
2π
Converter 2 ouput for $\alpha_2 = 120°$
$-v_{o2}$
v_{ac}
v_{bc}
v_{ba}
v_{ca}
v_{cb}
v_{ab}
v_{ac}
0
ωt
$\frac{\pi}{6}$
$\pi - \alpha_1$
α_1
$\frac{7\pi}{6}$
Choke voltage
$v_r = v_{o1} + v_{o2}$
0
$\frac{\pi}{6}$
ωt

Outputs of Converters 1 & 2

- During the interval $(\pi/6 + \alpha_1)$ to $(\pi/2 + \alpha_1)$, the line to line voltage v_{ab} appears across the output of converter 1 and v_{bc} appears across the output of converter 2

We deifine three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ;$$

$$V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$

The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

To obtain an Expression for the Circulating Current

- If v_{O1} and v_{O2} are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval $(\pi/6 + \alpha_1) \leq \omega t \leq (\pi/2 + \alpha_1)$ is given by

$$v_r = v_{O1} + v_{O2} = v_{ab} - v_{bc}$$

$$v_r = \sqrt{3}V_m\left[\sin\left(\omega t + \frac{\pi}{6}\right) - \sin\left(\omega t - \frac{\pi}{2}\right)\right]$$

$$v_r = 3V_m\cos\left(\omega t - \frac{\pi}{6}\right)$$

The circulating current can be calculated by using the equation

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6}+\alpha_1}^{\omega t} v_r \, . d(\omega t)$$

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6}+\alpha_1}^{\omega t} 3V_m \cos\left(\omega t - \frac{\pi}{6}\right) . d(\omega t)$$

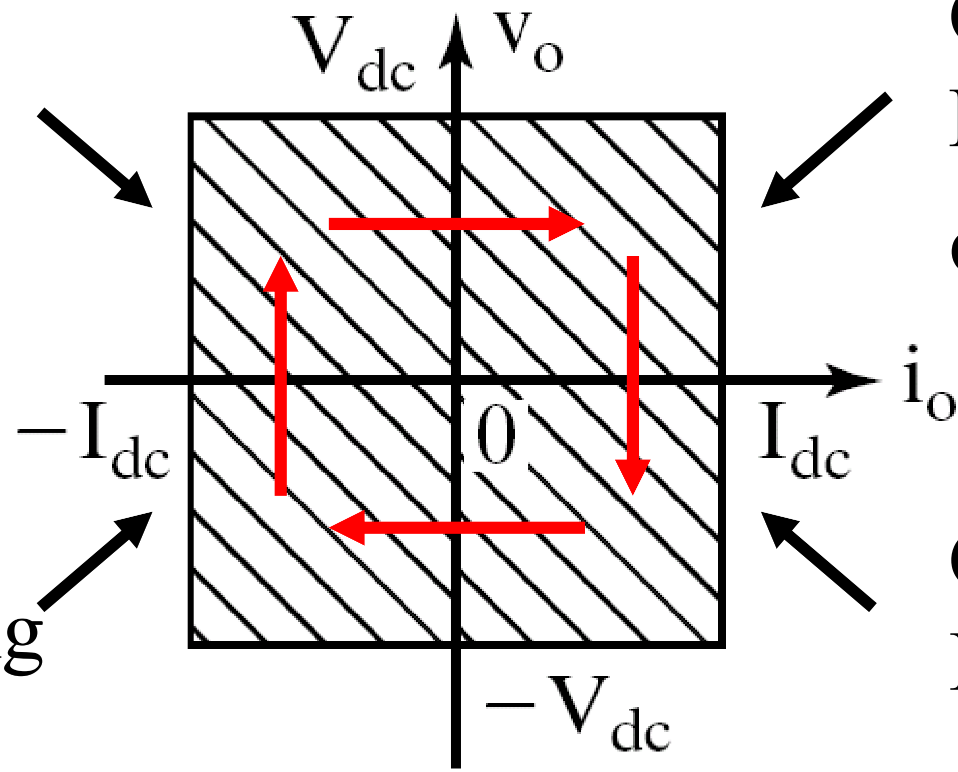
$$i_r(t) = \frac{3V_m}{\omega L_r}\left[\sin\left(\omega t - \frac{\pi}{6}\right) - \sin\alpha_1\right]$$

$$i_{r(\max)} = \frac{3V_m}{\omega L_r}$$

Four Quadrant Operation

Conv. 2
Inverting
$\alpha_2 > 90^0$

Conv. 1
Rectifying
$\alpha_1 < 90^0$



Conv. 2
Rectifying
$\alpha_2 < 90^0$

Conv. 1
Inverting
$\alpha_1 > 90^0$

- There are two different modes of operation.
  - Circulating current free (non circulating) mode of operation
  - Circulating current mode of operation

Non Circulating Current Mode Of Operation

- In this mode of operation only one converter is switched on at a time
- When the converter 1 is switched on,

  For $\alpha_1 < 90^0$ the converter 1 operates in the Rectification mode

  V_{dc} is positive, I_{dc} is positive and hence the average load power P_{dc} is positive.
- Power flows from ac source to the load

- When the converter 1 is on,

  For $\alpha_1 > 90^0$ the converter 1 operates in the Inversion mode

  V_{dc} is negative, I_{dc} is positive and the average load power P_{dc} is negative.
- Power flows from load circuit to ac source.

- When the converter 2 is switched on,

  For $\alpha_2 < 90^0$ the converter 2 operates in the Rectification mode

  V_{dc} is negative, I_{dc} is negative and the average load power P_{dc} is positive.
- The output load voltage & load current reverse when converter 2 is on.
- Power flows from ac source to the load

- When the converter 2 is switched on,

  For $\alpha_2 > 90^0$ the converter 2 operates in the Inversion mode

  V_{dc} is positive, I_{dc} is negative and the average load power P_{dc} is negative.
- Power flows from load to the ac source.
- Energy is supplied from the load circuit to the ac supply.

Circulating Current Mode Of Operation

- Both the converters are switched on at the same time.
- One converter operates in the rectification mode while the other operates in the inversion mode.
- Trigger angles α_1 & α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^0$

- When $\alpha_1 < 90^0$, converter 1 operates as a controlled rectifier. α_2 is made greater than 90^0 and converter 2 operates as an Inverter.
- V_{dc} is positive & I_{dc} is positive and P_{dc} is positive.

- When $\alpha_2 < 90^0$, converter 2 operates as a controlled rectifier. α_1 is made greater than 90^0 and converter 1 operates as an Inverter.
- V_{dc} is negative & I_{dc} is negative and P_{dc} is positive.