ANSWER KEY

INTERNAL ASSESSMENT - 1, 14E304 - EMT.

PART A

A1: $\vec{A} = 2\vec{a}_x - 2\vec{a}_y$

$\vec{B} = -\vec{a}_x + 2\vec{a}_z$

$$\vec{A}\cdot\vec{B} = (2\vec{a}_x - 2\vec{a}_y)\cdot(-\vec{a}_x + 2\vec{a}_z) \quad (1)$$

$$= -2\,\vec{a}_x\cdot\vec{a}_x + 4\,\vec{a}_x\cdot\vec{a}_z + 2\,\vec{a}_x\cdot\vec{a}_y - 4\vec{a}_y\cdot\vec{a}_z$$

$$= -2+0+0+0 = \underline{\underline{-2}} \quad (1)$$

A2: $x = r\cos\varphi$ $r = \sqrt{x^2+y^2}$

$y = r\sin\varphi$ (or) $\varphi = \tan^{-1}\frac{y}{x}$

$z = z$ $z = z$

A3: $\vec{E} = \dfrac{Q_1}{4\pi\varepsilon_0 R_{1P}^2}\,\vec{a}_{1P}$ (or) $\vec{E} = \dfrac{Q_1}{4\pi\varepsilon_0 r^2}\,\vec{a}_r$

P → Position of any other charge around Q_1

r → radius of sphere r

A4: $A(3,-2,1) \Rightarrow \vec{A} = 3\vec{a}_x - 2\vec{a}_y + \vec{a}_z$

$B(-3,-3,5) \Rightarrow \vec{B} = -3\vec{a}_x - 3\vec{a}_y + 5\vec{a}_z$

$\vec{BA} = \vec{A} - \vec{B} = 6\vec{a}_x + \vec{a}_y - 4\vec{a}_z$

$|\vec{BA}| = \sqrt{6^2+1^2+4^2} = \sqrt{36+1+16} = \sqrt{53}$

$\therefore$ unit vector $\vec{a}_{BA} = \dfrac{\vec{BA}}{|\vec{BA}|} = \dfrac{6\vec{a}_x + \vec{a}_y - 4\vec{a}_z}{7.28}$

$$\boxed{\vec{a}_{BA} = 0.824\,\vec{a}_x + 0.137\,\vec{a}_y - 0.549\,\vec{a}_z}$$

A5: A Point charge means that the electric charge which is spreaded on a surface or space whose geometrical dimensions are very ~~very~~ small compared to the other dimensions, in which the effect of electric field is to be studied. Thus a point charge has a location but not the dimensions.

A6: $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\vec{a_r}$ ---①

$\vec{D} = \frac{Q}{4\pi r^2}\vec{a_r}$ ---②

② ÷ ① $\frac{\vec{D}}{\vec{E}} = \epsilon_0$ $\boxed{\therefore \vec{D} = \epsilon_0 \vec{E}}$

A7: Gauss's law is used to find $\vec{D}$ or $\vec{E}$ due to some symmetric charge distributions like

(i) Point charge

(ii) Line charge

(iii) surface charge

(iv) volume charge

(v) co-axial cable and so-on

A8: The Potential difference can be expressed as the difference between the absolute potentials of the two points (or) work done per unit charge.

$\boxed{V_{AB} = V_A - V_B \quad V}$

unit is (J/C)

$V = -\int \vec{E}.\vec{dL}$

A9: For Electric field $\vec{E}$ to be solenoidal

$\nabla.\vec{E} = 0$ where $\nabla = \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}$

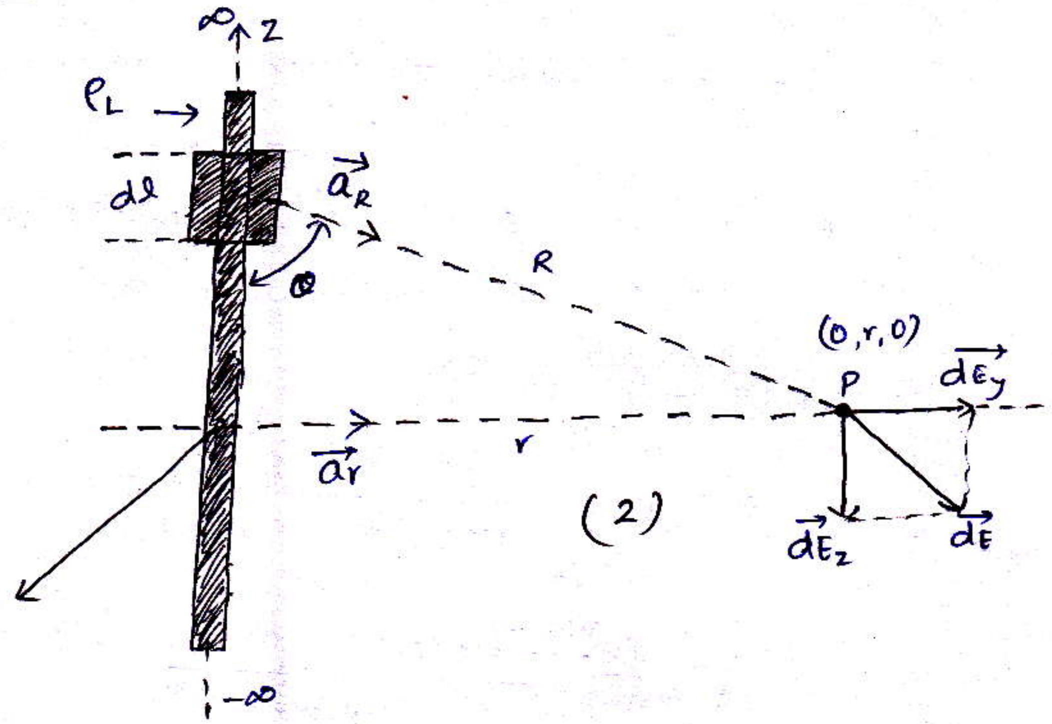
A10:

Two point charges of equal magnitude but opposite sign, seperated by a very small distance given rise to an electric dipole.

Dipole moment is denoted by $\vec{P}$. If the vector directed from $-Q$ to Q is $\vec{d}$, then the dipole moment is defined as

$\vec{P} = Q\vec{d}$ (C.m).

B1 (a) (i): ELECTRIC FIELD INTENSITY DUE TO INFINITE LINE CHARGE



As line charge is along z-axis, $dl = dz$

$\therefore dQ = \rho_L dL = \rho_L dz$ — ①

The co-ordinates of dQ are $(0,0,z)$ while co-ordinates of point P are $(0,r,0)$. Hence the distance vector $\vec{R}$ can be written as

$$\vec{R} = \vec{r_p} - \vec{r_{dl}} = [r\vec{a_y} - z\vec{a_z}]$$

$$\therefore |\vec{R}| = \sqrt{r^2+z^2} \quad \text{— ②}$$

$$\therefore \vec{a_R} = \frac{r\vec{a_y} - z\vec{a_z}}{\sqrt{r^2+z^2}} \quad \text{— ③}$$

$$\therefore \vec{dE} = \frac{dQ}{4\pi\varepsilon_0 R^2}\vec{a_R} \quad \text{— ④}$$

sub ① ② & ③ in ④, we get

$$\vec{dE} = \frac{\rho_L dz}{4\pi\varepsilon_0 R^2}\left[\frac{r\vec{a_y} - z\vec{a_z}}{\sqrt{r^2+z^2}}\right] = \frac{\rho_L dz}{4\pi\varepsilon_0(\sqrt{r^2+z^2})^2}\left[\frac{r\vec{a_y} - z\vec{a_z}}{\sqrt{r^2+z^2}}\right] \quad ⑤$$

Z component will be zero,

$$\therefore \vec{dE} = \frac{\rho_L dz}{4\pi\varepsilon_0(\sqrt{r^2+z^2})^2}\frac{r a_y}{\sqrt{r^2+z^2}} \qquad (4)$$

Now by integrating $\vec{dE}$ over the z-axis from $-\infty$ to ∞ we get total $\vec{E}$ at point P

$$\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_L}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \, r\vec{a_y} \, dz \quad \text{—} (6)$$

use substitution, $z = r\tan\theta$ ie $r = \dfrac{z}{\tan\theta}$

$dz = r\sec^2\theta \, d\theta$ $\Big\}$ — (7)

for $z = -\infty$ $\quad \theta = \tan^{-1}\left(\frac{z}{r}\right) = \tan^{-1}(-\infty) = -90^\circ$

$z = \infty$ $\quad \theta = \tan^{-1}(\infty) = 90^\circ$ $\Big\}$ - (8)

sub (7) & (8) in (6)

$$\vec{E} = \int_{\theta=-\pi/2}^{\theta=\pi/2} \frac{\rho_L}{4\pi\varepsilon_0 \left[r^2 + r^2\tan^2\theta\right]^{3/2}} \, r \times r\sec^2\theta \, d\theta \, \vec{a_y}$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{r^2 \sec^2\theta \, d\theta}{r^3 \left[1+\tan^2\theta\right]^{3/2}} \, \vec{a_y}$$

w.k.t $1 + \tan^2\theta = \sec^2\theta$

$$\therefore \vec{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{r^2 \sec^2\theta \, d\theta}{r^3 (\sec^2\theta)^{3/2}} \, \vec{a_y}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta} \, d\theta \, \vec{a_y}$$

w.k.t $\dfrac{1}{\sec\theta} = \cos\theta$ and $\int \cos\theta \, d\theta = \sin\theta$

$$\therefore \vec{E} = \frac{\rho_L}{4\pi\varepsilon_0 r} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} \vec{a_y}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right] \vec{a_y}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \left[1 - (-1)\right] \vec{a_y} = \frac{\rho_L}{\cancel{4}_{2}\pi\varepsilon_0 r} \cancel{2} \, \vec{a_y}$$

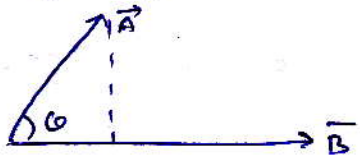
$$\boxed{\therefore \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \vec{a_y} \ \text{V/m}} \qquad (4)$$

(4)

B1) a) ii) **DOT PRODUCT AND CROSS PRODUCT** :

DOT PRODUCT: It is defined as the product of the magnitude of $\vec{A}$, the magnitude of $\vec{B}$ and the cosine of smaller angle between them.

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos \theta_{AB}.}$$ (1)



The result of such a dot product is scalar hence it is also called as scalar product.

(2)

Properties of Dot product:

1. If the two vectors are parallel to each other ie, $\theta = 0$, then $\cos \theta_{AB} = 1$. Thus $\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}|$ for $\parallel^{el}$ vectors.
2. If the two vector are prependicular each other ie, $\theta = 90^\circ$ then $\cos \theta = 0^\circ$. Thus $\vec{A} \cdot \vec{B} = 0$ for $\perp^r$ vectors.
3. If the dot product of vector with itself is performed, the result is square of the magnitude of that vector.
   $$\vec{A} \cdot \vec{A} = |\vec{A}|\,|\vec{A}| \cos 0 = |\vec{A}|^2$$
4. Any unit vector dotted with itself is unity $\vec{a_x} \cdot \vec{a_x} = \vec{a_y} \cdot \vec{a_y} = \vec{a_z} \cdot \vec{a_z} = 1$
5. The dot product obeys commutative & Distributive law
   (ie) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$, $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

CROSS PRODUCT: cross product is a vector quantity and has a direction $\perp r$ to the plane, containing the two vectors $\vec{A}$ and $\vec{B}$. Mathematically cross product is expressed as

(1)

$$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin \theta_{AB}\, \vec{a_N}$$

(1)

Properties of cross product:

1. The commutative law is not applicable to the cross product thus $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
2. Reversing the order of the vectors $\vec{A}$ and $\vec{B}$, a unit vector $\vec{a_N}$ reverses its direction hence we can write
   $$\vec{A} \times \vec{B} = -[\vec{B} \times \vec{A}]$$
3. The cross product is not associative, thus
   $$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$$
4. With respect to addition cross product is distributive, thus
   $$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$
5. If two vectors are $\parallel^{el}$ to each other (ie) they are in same direction then $\theta = 0^\circ$ & hence cross product of such two vectors is zero (5)

6. $\vec{A} \times \vec{A} = 0$ [cross product to itself]

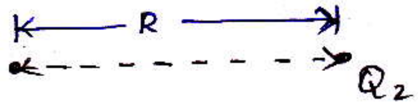
7. cross product of unit vectors

$\vec{a_x} \times \vec{a_y} = \vec{a_z}$; $\vec{a_y} \times \vec{a_z} = \vec{a_x}$; $\vec{a_z} \times \vec{a_x} = \vec{a_y}$

B1) b) i) **COULOMB'S LAW:**

The coulomb's law states that force b/w the two point charges Q_1 and Q_2,

1. Acts along the line joining the two point charges.
2. Is directly proportional to the product (Q_1, Q_2) of the two charges.
3. Is inversely proportional to the square of distance b/w them.



consider two point charges Q_1 & Q_2 separated by distance R.

Force F b/w two charges is expressed as

$$F \propto \frac{Q_1 Q_2}{R^2}$$

Force depends upon the medium also

$$\therefore \boxed{F = K \frac{Q_1 Q_2}{R^2}} \qquad K = \frac{1}{4\pi\varepsilon}$$

$\varepsilon \rightarrow$ permittivity of the medium in which charges are enclosed

In general $\varepsilon = \varepsilon_0 \varepsilon_r$ (4)

ε_0 = permittivity of free space or vaccum

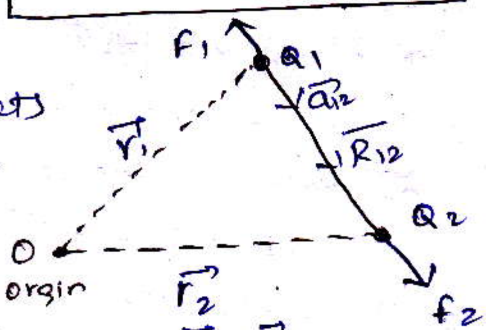
ε_r = Relative permittivity of the medium w.r.t free space

for free space $\varepsilon_r = 1$ $\quad \varepsilon = \varepsilon_0$

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2}}$$

Vector form of coulomb's law:

The force exerted by Q_1 on Q_2 acts along the direction $\vec{R_{12}}$ where $\vec{a_{12}}$ is unit vector along $\vec{R_{12}}$.



Force in vector form

$$\boxed{\vec{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} \vec{a_{12}}} \; ; \; \vec{a_{12}} = \frac{r_{12}}{|\vec{r_{12}}|} = \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|} \quad (4)$$

III^{ly} Force f_1

$$\boxed{\vec{F_1} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2} \vec{a_{21}}} \qquad \vec{a_{21}} = -\vec{a_{12}}$$

$$\boxed{\therefore \vec{F_1} = -\vec{F_2}} \quad (6)$$

B1 (b) (ii) $d = 10\,cm$ medium Vaccum.

$Q_1 = 4nC$

$Q_2 = 6NC$ (1)

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} = \frac{4\times10^{-9}\times6\times10^{-6}}{4\pi\times8.854\times10^{-12}\times(10\times10^{-2})^2} \quad (2)$$

$$= \frac{4\times6\times10^{-15}}{4\pi\times8.854\times10^{-14}\times10\times10^{-2}}$$

$$= \frac{24\times10^{-15}\times10^{14}\times10^{2}}{4\pi\times8.854\times10^{2}} = \frac{24\times10^{+1}}{4\pi\times8.854\times10^{2}}$$

$$\boxed{\vec{F} = 21.557\times10^{-3}\ N\text{-}m} \quad (2)$$

Medium : KEROSENE

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0\epsilon_r R^2} = \frac{4\times10^{-9}\times6\times10^{-6}}{4\pi\times8.854\times10^{-12}\times2\times(10\times10^{-2})^2}$$

$$\boxed{\vec{F} = 10.70785\times10^{-3}\ N.m} \quad (2)$$

B2 a(i)

GAUSS'S LAW:

The electric flux passing thro' any closed surface is equal to the total charge enclosed by that surface

mathematically Gauss's law can be expressed as

$$\psi = \oint_S \vec{D}\cdot\vec{ds} = Q = \text{charge enclosed}$$

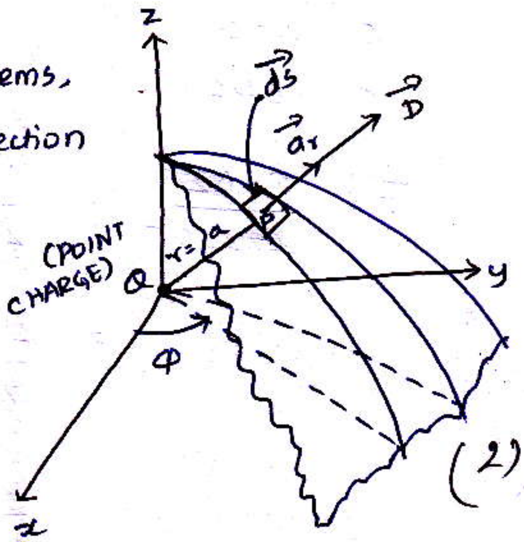
$\psi \rightarrow$ Electric flux

$\vec{D} \Rightarrow$ Electric flux density

$\vec{ds} \rightarrow$ differential surface area (3)

$Q \rightarrow$ Total charge

PROOF FOR GAUSS'S LAW:



(2)

In spherical co-ordinate systems, the ds normal to radial direction $\vec{a_r}$ is given by

$$\boxed{ds = r^2 \sin\theta \, d\theta \, d\phi} \quad \text{---} \ 1$$

W.K.T

$$\therefore \ \boxed{r = a} \quad \text{---} \ 2$$

sub 2 in 1 we get

$$ds = a^2 \sin\theta \, d\theta \, d\phi \quad \text{---} \ 3$$

$$\therefore \ \vec{ds} = ds \, \vec{a_n} \quad \text{---} \ 4$$

$$\& \ \vec{a_n} = \vec{a_r}$$

$$\boxed{\therefore \ ds = a^2 \sin\theta \, d\theta \, d\phi \ \vec{a_r}} \quad \text{---} \ 5 \qquad (2)$$

Now $\vec{D}$ due to point charge is given by,

$$\vec{D} = \frac{Q}{4\pi r^2} \vec{a_r} = \frac{Q}{4\pi a^2} \vec{a_r} \quad [\text{as } r = a]$$

$\therefore \ \vec{D} \cdot \vec{ds} = |\vec{D}| |\vec{ds}| \cos\theta'$ where θ' is angle b/w $\vec{D}$ & $\vec{ds}$

$$\left. \text{where } |\vec{D}| = \frac{Q}{4\pi a^2} \qquad |\vec{ds}| = a^2 \sin\theta \, d\theta \, d\phi \right\} \text{---} \ 6$$

The normal to $\vec{ds}$ is $\vec{a_r}$ while $\vec{D}$ also acts along $\vec{a_r}$ angle b/w $\vec{ds}$ and $\vec{D}$ is zero (ie) $\theta' = 0$.

$$\therefore \ \vec{D} \cdot \vec{ds} = |\vec{D}| |\vec{ds}| \cos 0 = |\vec{D}| |\vec{ds}|$$

$$= \frac{Q}{4\pi \cancel{a^2}} \times \cancel{a^2} \sin\theta \, d\theta \, d\phi$$

$$\boxed{\vec{D} \cdot \vec{ds} = \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi} \quad \text{---} \ 7 \qquad (2)$$

$$\therefore \ \psi = \oint_S \vec{D} \cdot \vec{ds}$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$$

(1) 8

$$\Rightarrow \ \boxed{\psi = Q}$$

B2) a) ii) Given $\vec{A} = 2xy\,\vec{a_x} + z\,\vec{a_y} + yz^2\,\vec{a_z}$

$$\nabla \cdot \vec{A} = \text{div}\,\vec{A} = \left[\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right] \cdot \left[2xy\,\vec{a_x} + z\,\vec{a_y} + yz^2\,\vec{a_z}\right]$$

$$= \frac{\partial}{\partial x}[2xy] + \frac{\partial}{\partial y}[z] + \frac{\partial}{\partial z}[yz^2]$$

$$\boxed{\nabla \cdot \vec{A} = 2y + 0 + 2zy} \quad (2)$$

$\nabla \cdot \vec{A}$ at $P(2,-1,3)$

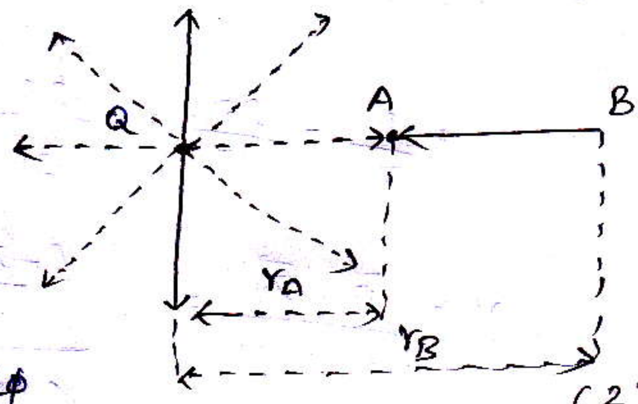
$$\nabla \cdot \vec{A} = 2(-1) + 0 + 2(3)(-1)$$

$$= -2 - 6 = -8$$

$$\boxed{\nabla \cdot \vec{A} = -8} \quad (3)$$

B2) b) i) Potential due to point charge:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a_r}$$



The differential length in spherical system is

$$\vec{dL} = dr\,\vec{a_r} + r\,d\theta\,\vec{a_\theta} + r\sin\theta\,d\phi\,\vec{a_\phi}$$

(2)

Hence potential difference V_{AB} btn points A & B is given by

$$V_{AB} = -\int_B^A \vec{E} \cdot \vec{dl} \qquad \text{Here } B = r_B,\ A = r_A$$

$$\therefore V_{AB} = -\int_{r_B}^{r_A} \vec{E} \cdot \vec{dL} = -\int_{r_B}^{r_A} \left(\frac{Q}{4\pi\varepsilon_0 r^2}\vec{a_r}\right) \left(dr\,\vec{a_r} + r\,d\theta\,\vec{a_\theta} + r\sin\theta\,d\phi\,\vec{a_\phi}\right)$$

$$= \int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{-Q}{4\pi\varepsilon_0}\int_{r_B}^{r_A}\frac{1}{r^2}\,dr$$

$$\boxed{V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right] V} \quad (5)$$

(9)

B2) b) ii)

Given $\vec{P} = 3\vec{a_x} - 5\vec{a_y} + 10\vec{a_z}$ nC

at $Q(1,-2,4)$

To find V at $P(2,3,4)$

$$V = \frac{\vec{P}\cdot\vec{a_r}}{4\pi\varepsilon_0 r^2} \quad (1)$$

$Q(1,-2,4)$ $P(2,3,4)$

$$\vec{r} = \vec{P} - \vec{Q} = (2-1)\vec{a_x} + (3+2)\vec{a_y} + (4-4)\vec{a_z}$$

$$\vec{r} = \vec{a_x} + 5\vec{a_y}$$

$$|\vec{r}| = \sqrt{1+25} = \sqrt{26} \qquad \vec{a_r} = \frac{\vec{a_x} + 5\vec{a_y}}{\sqrt{26}} \quad (2)$$

$$\vec{P}\cdot\vec{a_r} = (3\vec{a_x} - 5\vec{a_y} + 10\vec{a_z})\left(\frac{\vec{a_x}+5\vec{a_y}}{\sqrt{26}}\right)$$

$$= \frac{3}{\sqrt{26}} - \frac{25}{\sqrt{26}} = \frac{-22}{\sqrt{26}} \times 10^{-9} \text{ as } \vec{P} \text{ is in nC}$$

$$\therefore V = \frac{\vec{P}\cdot\vec{a_r}}{4\pi\varepsilon_0 r^2} = \frac{\left(\frac{-22}{\sqrt{26}}\right)\times 10^{-9}}{4\times 3.14\times 8.854\times 10^{-12}\times(\sqrt{26})^2}$$

$$\boxed{V = -1.489\ V} \quad (5)$$

—x—

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