UNIT-V

ELECTROMAGNETIC WAVES.

→ where both fields are independent

In previous chapters we delt the concepts of electrostatic and magnetostatic fields, which donot change w.r.to time. Hence these fields are called as static fields or time invarient fields.

In this chapter – time varying or dynamic fields.

Maxwell's eq – describe relationship btn time varying electric & Magnetic fields

FARADAY'S LAW & LENZ'S LAW:

Time varying electric field & magnetic field are interdependent

In 1820, Prof. Hans christian Oersted demonstrated that a compass needle deflected due to an electric current. After 10 years, Michael faraday, a British scientist proved that a magnetic field could produce a current.

According to faraday's experiment, a static magnetic field cannot produce any current flow, but with a time varying field, an electro motive force (emf) is induced, which may drive a current in a closed path or circuit.

This emf is nothing but a voltage that induces from changing magnetic fields or motion of the conductors in a magnetic field.

STATEMENT:

The electromagnetic force (e.m.f) induced in a closed path (or circuit) is proportional to rate of change of magnetic flux enclosed by the closed path [or linked with the circuit]

Faraday's law can be stated as

$$e = -N\frac{d\phi}{dt} \text{ volts}$$

$N \rightarrow$ No. of turns in the circuit

$e \rightarrow$ Induced e.m.f

The minus sign indicates that the direction of the induced e.m.f is such that to produce a current which will produce a magnetic field which will oppose the original field.

DISPLACEMENT CURRENT DENSITY & DISPLACEMENT CURRENT

For static electromagnetic fields, according to amperes circuital law, we can write

$$\nabla \times \vec{H} = \vec{J} \quad \text{--- (1)}$$

Taking divergence on both sides we get

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} \quad \text{--- (2)}$$

But according to vector identity, divergence of the curl of any vector field is zero "

$$\therefore \nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} = 0 \quad \text{--- (3)}$$

But the equation of continuity is given by,

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \quad \text{[Page No 3 of chapter 3] --- (4)}$$

From eqn (4) it is clear that when $\frac{\partial \rho_v}{\partial t} = 0$, then only eqn (3) becomes true. Thus equation (3) and (4) are not compatible for time varying fields.

Thus equation (1) must be modified by adding one unknow term say $\vec{N}$

$\therefore$ The equation (1) becomes

$$\nabla \times \vec{H} = \vec{J} + \vec{N} \quad \text{--- (5)}$$

Again after taking divergence on both the sides,

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} + \nabla \cdot \vec{N} = 0 \quad \text{--- (6)}$$

As $\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$, to get correct conditions we must write,

$$\nabla \cdot \vec{N} = \frac{\partial \rho_v}{\partial t} \quad \text{--- (7)}$$

But according to Gauss's law

$$\rho_v = \nabla \cdot \vec{D}$$

Lenz law

The direction of induced emf is such that it opposes the cause producing it.

$e = -N\frac{d\phi}{dt}$ volts

$e = \oint \bar{E} \cdot \bar{dL}$

Induced emf is equal to the of voltage in the circuit E change of change Magnetic flux passing through a specified area

$\phi = \int_S B \cdot \bar{dS}$ B = Magnetic flux density

$\therefore e = -N\frac{d\phi}{dt} = -\frac{d\phi}{dt}$ $\boxed{N = 1}$

$e = -\frac{d}{dt}\int_S \bar{B} \cdot \bar{ds}$

$e = \oint \bar{E} \cdot \bar{dl} = -\frac{d}{dt}\int_S \bar{B} \cdot \bar{ds}$

$\oint E \cdot dl = -\int \frac{\partial \bar{B}}{\partial t} \bar{ds}$

Partial derivative as time varying

The emf induced in stationary closed path due to time varying $\bar{B}$ is called statically induced emf or transformer emf. If time varying closed path. dynamically induced emf or motional emf

Thus replacing ρ_v by $\nabla \cdot \vec{D}$ in 7 we get

$$\nabla \cdot \vec{N} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D}) = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

comparing two sides of the equation,

$$\vec{N} = \frac{\partial \vec{D}}{\partial t}$$

Now we can write Ampere's circuital law in point form

as,

$$\boxed{\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}} \quad --- (8)$$

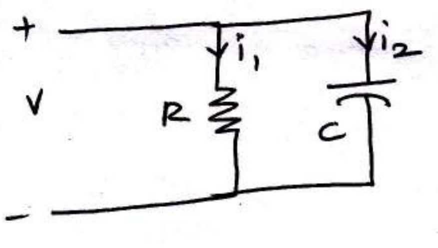
conduction current density
c indicates that the current is due to the moving charges.

represents current density (A/m²). As this quantity is obtained from time varying electric flux density. This is also called displacement density. This is also called displacement current density ($\vec{J}_D$) (or) displacement current density ($\vec{J}_D$)

$$\boxed{\therefore \quad \nabla \times \vec{H} = \vec{J}_c + \vec{J}_D}$$

PHYSICAL SIGNIFICANCE OF DISPLACEMENT CURRENT:

consider a ||el circuit with Resistor 'R' and capacitor 'C'



consider that this ||el combination is driven by the Time varying i.e sinusoidal voltage V.

It is obvious that the nature of current thro' the resistor R i.e i_1 is different than that flowing thro' capacitor C i.e i_2.

The current flowing thro' R is due to due to the actual motion of charges. Thus the current thro' resistor can be written as,

$$\boxed{i_1 = \frac{V}{R}} \quad --- (1)$$

This is called conduction current as the current is flowing because of actual motion of charges. Let it be denoted by i_c.

Let A be the cross-sectional area of resistor,

then the conduction current density is given by,

$$\vec{J_c} = \frac{i_c}{A} = \sigma \vec{E} \quad \text{---} (2) \qquad \frac{i_c}{A} \quad i_c = \frac{V}{R} \quad = \frac{V}{RA} = \sigma \frac{V}{A} = \sigma \vec{E} \qquad \sigma = \frac{1}{R} \quad E = \frac{V}{A}$$

Now assume that the initial charge on a capacitor is zero. Then for time varying voltage applied across $\parallel^{el}$ plate capacitor, the current thro' the capacitor is given by,

$$i_2 = C\frac{dV}{dt}$$

$$i_2 = \frac{\varepsilon A}{d}\frac{dV}{dt} \quad \text{---} (3) \qquad \left[\because C = \frac{\varepsilon A}{d} \text{ for } \parallel^{el} \text{ plate capacitors}\right].$$

displacement current denoted by i_D

The Electric field produced by the voltage applied b/w the two plates is given by,

$$\vec{E} = \frac{V}{d}$$

$$V = (d)(\vec{E}) \quad \text{--} (4)$$

sub (4) in (3) we get

$$i_D = i_2 = \frac{\varepsilon A}{d}\frac{d}{dt}(d\vec{E})$$

$$i_D = \frac{\varepsilon A}{d} d \frac{d\vec{E}}{dt} \quad \rightarrow \text{As } d \text{ (distance) not varying with time}$$

$$\therefore i_D = \varepsilon A \frac{d\vec{E}}{dt}$$

$\therefore$ current density for i_D is

$$\vec{J_D} = \frac{i_D}{A}$$

$$\vec{J_D} = \frac{\varepsilon A}{A}\frac{d\vec{E}}{dt} \rightarrow \left(\frac{\varepsilon \cancel{A}}{\cancel{A}}\frac{d\vec{E}}{dt}\right)$$

$$\vec{J_D} = \varepsilon\frac{d\vec{E}}{dt} = \frac{d}{dt}(\varepsilon\vec{E}) \Rightarrow \varepsilon\vec{E} = \vec{D}$$

$$\boxed{\vec{J_D} = \frac{\partial \vec{D}}{\partial t}}$$

$\vec{J_c}$ - conduction current density.

$\vec{J_D} \rightarrow$ Displacement current density

$$\boxed{J = \vec{J_c} + \vec{J_D}}$$

MAXWELL'S EQUATION:

We have seen that a static electric field $\vec{E}$ can exist without a magnetic field $\vec{H}$ demonstrated by a capacitor with a static charge Q.

IIIly a conductor with a constant current I has a magnetic field $\vec{H}$ in the absence of an electric field $\vec{E}$.

But in the case of Time varying fields, $\vec{E}$ & $\vec{H}$ doesnot exists without each other.

Maxwell's Equations are nothing but a set of four expressions derived from Ampere's circuital law, Faraday's law, Gauss's law for electric field and Gauss's law for magnetic field.

MAXWELL's EQUATION FOR STATIC FIELDS:

MAXWELL's EQUATION DERIVED FROM FARADAY's LAW:

A] MAXWELL's EQUATION DERIVED FROM FARADAY's LAW

According to the basic concept from electrostatic field, the workdone over a closed path (or) closed contour (i.e. starting point same as terminating point) is always zero.

Mathematically it is represented as,

$$\oint \vec{E} \cdot \vec{dL} = 0.$$

The above equation is called integral form of Maxwell's equation derived from Faraday's law of static field.

Now using stoke's theorem converting the close line integral into the surface integral we get,

$$\oint \vec{E} \cdot \vec{dL} = \int_S (\nabla \times \vec{E}) \cdot \vec{ds} = 0$$

$$\therefore \int_S (\nabla \times \vec{E}) \cdot \vec{ds} = 0$$

But $\vec{ds}$ cannot be zero (i.e $\vec{ds} \neq 0$) That means,

$\boxed{\nabla \times \vec{E} = 0}$ → Point form or differential form of Maxwell's eqn derived from Faraday's law of static fields.

MAXWELL'S EQUATION DERIVED FROM AMPERE'S CIRCUITAL LAW.

According to basic concept of magnetostatics an Ampere's circuital law states that the line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path. Mathematically it is given as,

$$\oint \vec{H} \cdot \vec{dL} = I$$

Now the current enclosed is equal to the product of current density normal to the closed path and area of closed path. Hence we get,

$$I = \int_S \vec{J} \cdot \vec{ds} \quad \text{where } \vec{J} = \text{current density.}$$

Hence equating above equations we get,

$$\oint \vec{H} \cdot \vec{dL} = \int_S \vec{J} \cdot \vec{ds}$$

This above expression is called Integral form of maxwell's equation from Ampere circuital law for static field.

Now by applying stoke's theorem, L.H.S of above equation can be converted into surface integral

$$\therefore \oint \vec{H} \cdot \vec{dL} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \vec{J} \cdot \vec{ds}$$

Hence we get

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

differential form of Maxwell's eqn derived from Ampere circuital law for static field.

Maxwell's Equation derived from Gauss's law for Electrostatic fields.

According to Gauss's law of electrostatic fields, the electric flux passing thro' any closed surface is equal to the Total charge enclosed by that surface. Mathematically we can write,

$$\psi = \oint \vec{D} \cdot \vec{ds} = Q_{enclosed} \quad \text{--- (1)}$$

The most common form to represent Gauss's law is with volume charge density ρ_v. Hence we mathematically can write,

$$\oint \vec{D} \cdot \vec{ds} = \int_v \rho_v \, dv \quad \text{--- (2)}$$

The above equation is called integral form of maxwell's equation derived from Gauss's law for static electric field.

To establish relationship b/w $\vec{D}$ and ρ_v, converting closed surface integral into volume integral using divergence Theorem as,

$$\oint \vec{D} \cdot \vec{ds} = \int_v (\nabla \cdot \vec{D}) \cdot dv \quad \text{--- (3)}$$

comparing (2) and (3) we get

$\int_v (\nabla \cdot \vec{D}) \cdot dv = \int_v \rho_v \, dv$

$\rightarrow \nabla \cdot \vec{D} = \rho_v \rightarrow$ Point form or differential form of Maxwell's Equation derived from Gauss's law for static electric field.

MAXWELL'S EQUATION DERIVED FROM GAUSS'S LAW FOR MAGNETOSTATIC FIELD:

According to Gauss's law for magnetostatic field, the magnetic flux cannot reside in a closed surface due to non existance of single magnetic Pole.

Mathematically we can write,

$$\oint_s \vec{B} \cdot \vec{ds} = 0 .$$

The above equation is called Integral form of Maxwell's Equation derived from Gauss's law for static magnetic field.

Now using divergence Theorem, we can write

$$\oint_S \vec{B}.\vec{ds} = \int_V (\nabla.\vec{B})\, dv = 0$$

(ie) $\int_V (\nabla.\vec{B})\, dV = 0$

Now dV cannot be zero that means

$\nabla.\vec{B} = 0$

$\rightarrow$ Point form or differential form of maxwell's equation derived from Gauss's law for static magnetic field.

MAXWELL'S EQUATION FOR TIME VARYING FIELD:

MAXWELL'S EQUATION DERIVED FROM FARADAY'S LAW:

Now consider Faraday's law which relates e.m.f induced in a circuit to a circuit to the time rate of change of decrease of total magnetic flux linking the ckt. Thus we can write.

$$\oint \vec{E}.\vec{dL} = -\int_S \frac{\partial \vec{B}}{\partial t}.\vec{ds} \quad --- (1)$$

$\rightarrow$ Maxwell's Eqn derived from Faraday's law expressed in Integral form.

The Total electromotive force (e.m.f) induced in a closed path is equal to the negative surface integral of the rate of change of flux density w.r.to time over an entire surface bounded by the same closed path.

Using stokes theorem, we get

$$\int_S (\nabla\times\vec{E}).\vec{ds} = -\int_S \frac{\partial \vec{B}}{\partial t}.\vec{ds}$$

$$\therefore \nabla\times\vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

$\rightarrow$ Point form or differential form derived from faraday's law.

MAXWELL'S EQUATION DERIVED FROM AMPERE'S CIRCUITAL LAW:

According to Ampere's circuital law, the line integral of magnetic field intensity $\vec{H}$ around a closed path is equal to current enclosed by the path.

$$\oint \vec{H} \cdot \vec{dL} = I_{enclosed}.$$

$$I_{enclosed} = \int_S \vec{J} \cdot \vec{ds}$$

$$\therefore \oint \vec{H} \cdot \vec{dL} = \int_S \vec{J} \cdot \vec{ds}$$

Above expression can be made further general by adding displacement current density to conduction current density as follows,

$$\oint \vec{H} \cdot \vec{dL} = \int_S \left[\vec{J_c} + \frac{\partial \vec{D}}{\partial t} \right] \cdot \vec{ds} \rightarrow \text{Integral form}$$

Applying stokes theorem.

$$\oint \vec{H} \cdot \vec{dL} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \left[\vec{J_c} + \frac{\partial \vec{D}}{\partial t} \right] \cdot \vec{ds}$$

$$\therefore (\nabla \times \vec{H}) = \vec{J_c} + \frac{\partial \vec{D}}{\partial \vec{t}} \rightarrow \text{Point form.}$$

Statement:

The Total MMF around any closed path is equal to the surface integral of the conduction and displacement current densities over the entire surface bounded by the same closed path.

Total flux leaving out of a closed surface is equal to the total charge enclosed by a finite volume.

The surface integral of magnetic flux density over a closed surface is always equal to zero.

Differential form	Integral form	Significance
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \vec{dL} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$	Faraday's law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint \vec{H} \cdot \vec{dL} = I + \int_S \frac{\partial \vec{D}}{\partial t} \cdot ds$.	Ampere circuital law
$\nabla \cdot \vec{D} = \rho_V$	$\oint \vec{D} \cdot \vec{ds} = \int_S \rho_V \, dV$	Gauss's law
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot \vec{ds} = 0$	No isolated magnetic charges.

MAXWELL'S EQUATION FOR FREE SPACE:

Free space is a non-conducting medium in which volume charge density $\rho_V = 0$ and conductivity $\sigma = 0$

Point form	Integral form
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \vec{dL} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$
$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ $[\because \vec{J} = \sigma \vec{E}]$.	$\oint \vec{H} \cdot \vec{dL} = \int_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$
$\nabla \cdot \vec{D} = 0$	$\oint \vec{D} \cdot \vec{ds} = 0$
$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$.

MAXWELL'S EQUATION FOR GOOD conductors:

for good conductor $\vec{J} >> \frac{\partial \vec{D}}{\partial t}$ & $\rho_V = 0$.

$\therefore \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \vec{dL} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$
$\nabla \times \vec{H} = \vec{J}$	$\oint \vec{H} \cdot \vec{dL} = \int_S \vec{J} \cdot \vec{ds}$
$\nabla \cdot \vec{D} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$
$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$.

Comparison b/w Electric circuit and Magnetic circuit.

Electric circuit	Magnetic circuit
1. The Path traced by the current is called electric circuit	1. The path traced by the magnetic flux is called magnetic circuit
2. In electric ckt, emf is the driving force. It is measured in volts.	2. In magnetic circuit mmf is the driving force, it is measured in Ampere-turns.
3. Resistance R opposes the flow of current. $R = \frac{emf}{current} = \frac{l}{\sigma s} \Omega$	3. Reluctance $\mathbb{R}$ is opposed by the magnetic path. $\mathcal{R} = \frac{l}{\mu s} = \frac{mmf}{flux}$ A.t/wb.
4. conductivity σ	4. Permeability μ.
5. Field Intensity E	5. Field intensity H.
6. current density $J = \frac{\sigma E}{s}$ A/m^2	6. Flux density $B = \frac{\phi}{s} = \mu H$ wb/m^2
7. Reciprocal of resistance is conductance (G)	7. Reciprocal of reluctance is Permeance (R).
8. ohms law $e = IR$	8. ohms law $e_m = \phi R$.
9. Kirchoff's law: $\Sigma I = 0$, $\Sigma EMF = 0$	9. Kirchoff's law $\Sigma\phi = 0$, $\Sigma MMF = \Sigma \phi S = \Sigma H.l$.

ELECTROMAGNETIC WAVES:

The waves are the means of transporting energy (or) information from source to destination. The waves consisting of electric and magnetic fields are called electromagnetic waves.

The electromagnetic waves are said to be in existance is all the four Maxwell's equations are satisfied at the source point [where they are generated], at any point in the medium (thro' which they travel) and at the destination or load point (where they are received).

Basically the waves radiated from the source are with spherical wavefront, but at large distances from source the spherical waves become practically plane waves.

In general, the wave is a function of time & space.

eg: radio waves, light rays, radar beams, television signals etc.

GENERAL WAVE EQUATION:

To obtain general wave equations, let us assume that the electric and magnetic fields exist in a linear, homogeneous & isotropic medium with the parameters μ, ε and σ.

Also assume that the medium is source free which clearly gives the idea about the charge free medium.

Assume that the medium obeys the ohm's law i.e $\vec{J} = \sigma \vec{E}$. Then the Maxwell's equation is given by,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \implies \vec{B} = \mu \vec{H} \Rightarrow \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad --- (1)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \begin{array}{l} \vec{D} = \varepsilon \vec{E} \\ \vec{J} = \sigma \vec{E} \end{array} \Rightarrow \nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad -- (2)$$

$$\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \mu \vec{H} \Rightarrow \nabla \cdot \vec{H} = 0 \quad --- (3)$$

$$\nabla \cdot \vec{D} = 0 \Rightarrow \vec{D} = \varepsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0 \quad --- (4)$$

$[\because \rho_v = 0$ for free space$]$.

To eliminate $\vec{H}$ from (1), taking curl on both sides of equation (1), we get

$$\nabla \times (\nabla \times \vec{E}) = \cancel{-\mu \frac{\partial}{\partial t}(\nabla \times \vec{H})} - \mu \left(\nabla \times \frac{\partial \vec{H}}{\partial t}\right) \quad -- (5)$$

$\nabla \rightarrow$ indicates differentiation w.r.to space

while $\frac{\partial}{\partial t} \rightarrow$ indicates differentiation w.r.to time.

Both are independent of each other, the operations can be interchanged.

so we get

$$\nabla \times (\nabla \times \vec{E}) = -\mu \frac{\partial}{\partial t}(\nabla \times \vec{H}) \text{ --- } (6)$$

sub (2) in (6) we get

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}\left[\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right]$$

$$\nabla \times \nabla \times \vec{E} = -\mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \text{ ---- } (7)$$

Now according to vector identity,

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \text{ --- } (8)$$

sub (4) in (8) we get

$$\nabla \times \nabla \times \vec{E} = -\nabla^2 \vec{E} \text{ --- } (9)$$

sub (9) in (7) we get.

$$\boxed{+\nabla^2 \vec{E} = +\mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}} \text{ --- } (10)$$

This is the wave equation for the electric field $\vec{E}$. Now multiplying both the sides of (10) by ε, we get

$$\nabla^2(\varepsilon \vec{E}) = \mu\sigma \frac{\partial \varepsilon \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \varepsilon \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{D} = \mu\sigma \frac{\partial \vec{D}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{D}}{\partial t^2} \quad [\because \vec{D} = \varepsilon \vec{E}] \text{ --- } (11)$$

This is the wave equation for $\vec{D}$ in uniform medium.

To obtain wave equation of $\vec{H}$, take curl on both sides of (2), we get

$$\nabla \times (\nabla \times \vec{H}) = \nabla \times \sigma \vec{E} + \varepsilon \nabla \times \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{H} = \sigma(\nabla \times \vec{E}) + \varepsilon \frac{\partial}{\partial t}(\nabla \times \vec{E}) \text{ --- } (12)$$

sub (1) in (12) we get

$$\nabla \times \nabla \times \vec{H} = \sigma\left(-\mu \frac{\partial \vec{H}}{\partial t}\right) + \varepsilon \frac{\partial}{\partial t}\left[-\mu \frac{\partial \vec{H}}{\partial t}\right]$$

$$= -\mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{---} (13)$$

using vectro identiry

$$\nabla \times \nabla \times \vec{H} = \nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} \quad \text{---} (14)$$

sub (3) in (14) we get

$$\nabla \times \nabla \times \vec{H} = -\nabla^2 \vec{H} \quad \text{---} (15)$$

Eauivating (15) and (13) we get

$$-\nabla^2 \vec{H} = -\mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}} \quad \text{---} (16)$$

This is the wave eauation for magnetic field $\vec{H}$. Now multipiying both sides by μ we get

(16) $\times \mu$

$$\nabla^2(\mu \vec{H}) = \mu\sigma \frac{\partial \mu\vec{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mu\vec{H}}{\partial t^2}$$

$\because \mu\vec{H} = \vec{B}$

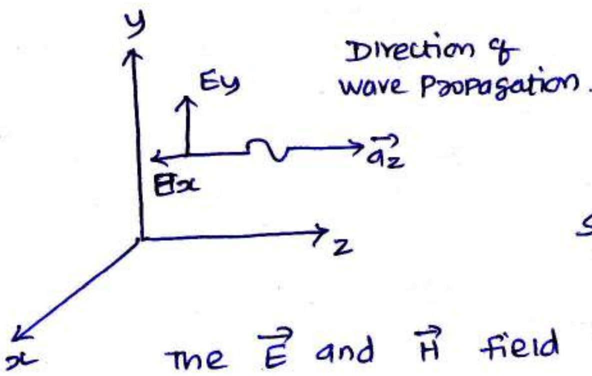
$$\boxed{\nabla^2 \vec{B} = \mu\sigma \frac{\partial \vec{B}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}} \quad \text{-----} (18)$$

This is the wave eauation of $\vec{B}$ in the unifoom medium.

UNIFORM PLANE WAVES IN FREE SPACE:

Consider an electromagnetic wave propagating thro' the free space. For free space $\sigma = 0$, consider that the electric field in the wave in x-direction only while the magnetic field in the y-direction only.

Both the fields i.e, electric and magnetic field do not vary with x and y but vary only with z. The fields also vary with time as wave propagates in the free space.



Basically plane waves means, the electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$ lie on the same plane.

The uniform plane wave means the $\vec{E}$ and $\vec{H}$ field vector are in same plane. moreover the amplitude and phase of field vectors $\vec{E}$ & $\vec{H}$ is constant over the planes $\parallel^{el}$ to each other.

Electric field vector is in $\vec{a_x}$ direction, while magnetic field vector is in $\vec{a_y}$ direction. That means $\vec{E}$ & $\vec{H}$ lie in x-y plane. So in any of the planes in the wave, the vectors $\vec{E}$ and $\vec{H}$ are independent of x and y. Thus we can conclude that $\vec{E}$ and $\vec{H}$ are functions of z and t only. Moreover as $\vec{E}$ and $\vec{H}$ are mutually $\perp^r$ to each other, the electromagnetic waves are also called as transverse electromagnetic waves.

Let us consider wave equations for $\vec{E}$ & $\vec{H}$ fields given by,

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{--- (1)}$$

$$\nabla^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{--- (2)}$$

But for free space $\sigma = 0$, $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$

sub these values in ① & ② we get

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{---} ③$$

$$\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{----} ④$$

consider eqn ③

$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{---} ⑤$$

But the wave travels in the z-direction, hence $\vec{E}$ is independent of x and y. Hence first two differential terms in above eqn are zero. Hence we can write

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{----} ⑥$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 \vec{E}}{\partial z^2} \quad \text{---} ⑦$$

Now according to the results in physics,

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c \quad \text{i.e.} \quad v^2 = \frac{1}{\mu_0 \varepsilon_0} = c^2$$

where $c = 3 \times 10^8 \, m/s$ = velocity of light

sub above relations in ⑦ we get

$$\boxed{\frac{\partial^2 \vec{E}}{\partial t^2} = v^2 \frac{\partial^2 \vec{E}}{\partial z^2}} \quad \text{---} ⑧$$

Above equation is other form of wave equation.

$|||^r$ to this we can write.

$$\boxed{\frac{\partial^2 \vec{H}}{\partial t^2} = v^2 \frac{\partial^2 \vec{H}}{\partial z^2}} \quad \text{----} ⑨$$

Reconsidering eqn ⑥

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

For the wave propagating in z-direction, $\vec{E}$ may have E_x & E_y component definately not E_z. According to assumption, $\vec{E}$ is in $\vec{a_x}$ direction, so let us consider that only E_x is present. Then we can rewrite above equation as,

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad \text{----} (10)$$

Let $\quad E_x = E_m e^{j\omega t}$

where $\quad E_m \rightarrow$ Amplitude of the electric field

$\omega \rightarrow$ Angular freq.

Partially differentiating E_x twice w.r.to t, we get

$$\frac{\partial^2 E_x}{\partial t^2} = E_m (j\omega)(j\omega) e^{j\omega t} = -\omega^2 E_m e^{j\omega t}$$

But $E_m e^{j\omega t} = E_x$

$$\therefore \frac{\partial^2 E_x}{\partial t^2} = -\omega^2 E_x$$

sub above eqn in (10) we get

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 [-\omega^2 E_x] = -\omega^2 \mu_0 \varepsilon_0 E_x \quad \text{----} (11)$$

Let $\frac{\partial}{\partial z} = D \quad$ i.e $\frac{\partial^2}{\partial z^2} = D^2 \quad \therefore$ (11) becomes.

$$D^2 E_x + \omega^2 \mu_0 \varepsilon_0 E_x = 0$$

Thus Auxillary Equation becomes,

$$(D^2 + \omega^2 \mu_0 \varepsilon_0) E_x = 0.$$

Hence equating bracket terms to zero, we get

$$D^2 + \omega^2 \mu_0 \varepsilon_0 = 0$$

(or) $\quad D^2 = -\omega^2 \mu_0 \varepsilon_0$

(or) $\quad D = \pm j\omega\sqrt{\mu_0 \varepsilon_0} = \pm j\beta$

where $\beta = \omega\sqrt{\mu_0\varepsilon_0}$ which is called Phase shift constant measured in rad/m.

Hence the solution of eqn (11) can be written as,

$$E_x = K_1 e^{-j\omega\sqrt{\mu_0 \varepsilon_0}\, z} + K_2 e^{j\omega\sqrt{\mu_0 \varepsilon_0}\, z}.$$

$$E_x = K_1 e^{-j\beta z} + K_2 e^{+j\beta z} \quad \text{----- (12)}$$

Let K_1 and K_2 be the constants w.r.to z but are functions of t. Let us assume K_1 and K_2 as.

$$K_1 = E_m^{+} e^{+j\omega t} \quad \&$$

$$K_2 = E_m^{-} e^{+j\omega t}.$$

sub K_1 and K_2 in (12) we get

$$E_x = E_m^{+} e^{j\omega t} e^{-j\beta z} + E_m^{-} e^{+j\omega t} e^{+j\beta z}$$

$$= E_m^{+} e^{j(\omega t - \beta z)} + E_m^{-} e^{j(\omega t + \beta z)}. \quad \text{--- (13)}$$

To find the electric field in the time domain, taking real part of (13), we get

$$E_x = Re\left[E_m^{+} e^{j(\omega t - \beta z)} + E_m^{-} e^{j(\omega t + \beta z)}\right]$$

$$E_x = \underbrace{E_m^{+}}_{\text{Travelling in } +z \text{ direction}} \cos(\omega t - \beta z) + \underbrace{E_m^{-}}_{\text{Travelling in } -ve\ z\text{-direction}} \cos(\omega t + \beta z) \ V/m.$$

Above equation is the sinusoidal function consisting of two components. one in forward direction and other in backward direction

POYNTING VECTOR & POYNTING THEOREM:

By means of Electromagnetic (EM) waves, an energy can be transported from transmitter to receiver. The energy stored in an electric field and magnetic field is transmitted at a certain rate of energy flow which can be calculated with the help of Poynting theorem.

As we know $\vec{E}$ & $\vec{H}$ are basic fields, $\vec{E}$ is electric field expressed in V/m ; while $\vec{H}$ is magnetic field measured in A/m. so if we take dot product of the two fields, dimensionally we get a unit V·A/m^2 (or) $\frac{\text{Watt}}{m^2}$. so this product of $\vec{E}$ & $\vec{H}$ gives a new quantity which is expressed as watt/m^2. This quantity is called Power density.

As $\vec{E}$ & $\vec{H}$ are vectors, to get power density we may carry out either dot product or cross product. The result of dot product is always a scalar quantity. But as Power flows in certain directions, it is a vector quantity. To illustrate this, consider that the field is transmitted in the form of an electromagnetic waves from an antenna. Both the fields are sinusoidal in nature.

The Power radiated from antenna has a particular direction. Hence to calculate a power density, we must carry out a cross product of $\vec{E}$ and $\vec{H}$.

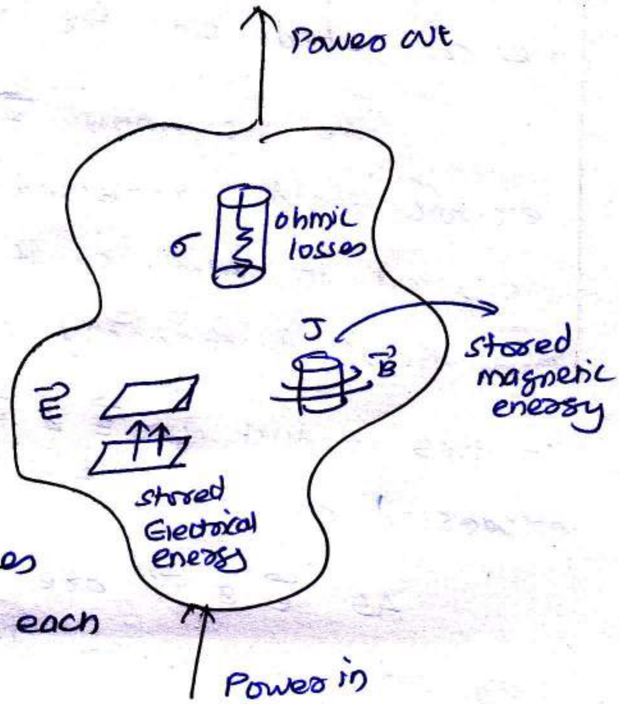
The Power density is given by

$$\vec{P} = \vec{E} \times \vec{H}.$$

Where $\vec{P}$ is called Poynting vector.

Poynting theorem is based on the law of conservation of energy in electromagnetism. Poynting theorem can be stated as,

The net power flowing out of a given volume V is equal to the time rate of decrease in the energy stored within volume 'V' minus the ohmic power dissipated.



suppose

$$\vec{E} = E_x \vec{a_x}$$

$$\vec{H} = H_y \vec{a_y} \text{ then}$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= E_x \vec{a_x} \times H_y \vec{a_y}$$

$$\boxed{\vec{P} = E_x H_y \vec{a_z}}$$

The above equation indicates $\vec{E}$, $\vec{H}$ and $\vec{P}$ are mutually $\perp^r$ to each other.

consider that the electric field propagates in free space given by

$$\vec{E} = [E_m \cos(wt - \beta z)] \vec{a_x}$$

In the medium, the ratio of magnitudes of $\vec{E}$ & $\vec{H}$ depends on its intrinsic impedance η, for free space.

$$\eta = \eta_0 = \frac{E_m}{H_m} = 120\pi = 377\Omega.$$

Moreover in free space, electromagnetic wave travels at a speed of light,

thus we can write,

$$\vec{H} = [H_m \cos(wt - \beta z)] \vec{a_y}$$

$$= \left[\frac{E_m}{\eta_0} \cos(wt - \beta z)\right] \vec{a_y}$$

According to Poynting Theorem.

$$\vec{P} = \vec{H} \times \vec{E}$$

$$= \left[E_m \cos(\omega t - \beta z)\right]\vec{a}_x \times \left[\frac{E_m}{\eta_0}\cos(\omega t - \beta z)\right]\vec{a}_y$$

$$\vec{P} = \frac{E_m^2}{\eta_0}\cos^2(\omega t - \beta z)\,\vec{a}_z \quad W/m^2$$

This is nothing but the power density measured in watt/m^2. Thus the power passing particular area is given by,

Power = Powerdensity x Area.

AVERAGE POWER DENSITY (P_{avg}).

To find average power density, let us integrate the power density in z-direction over one cycle and divide by the period T of one cycle.

$$\therefore P_{avg} = \frac{1}{T}\int_0^T \frac{E_m^2}{\eta}\cos^2(\omega t - \beta z)\,dt.$$

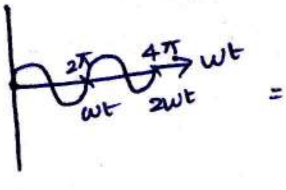
$$= \frac{E_m^2}{\eta T}\int_0^T \cos^2(\omega t - \beta z)\,dt$$

$$= \frac{E_m^2}{\eta T}\int_0^T \frac{1 + \cos 2(\omega t - \beta z)}{2}\,dt$$

$$= \frac{E_m^2}{\eta T}\left[\frac{t}{2} + \frac{\sin 2(\omega t - \beta z)}{(2\omega)2}\right]_0^T$$

$$= \frac{E_m^2}{\eta T}\left[\frac{t}{2} + \frac{\sin(2\omega t - 2\beta z)}{4\omega}\right]_0^T$$

$$= \frac{E_m^2}{\eta T}\left[\frac{T}{2} + \frac{\sin(2\omega T - 2\beta z)}{4\omega} - \frac{\sin(-2\beta z)}{4\omega}\right]$$



$$= \frac{E_m^2}{T\eta}\left[\frac{T}{2} + \frac{\sin(4\pi - 2\beta z)}{4\omega} + \frac{\sin(2\beta z)}{4\omega}\right]$$

$$= \frac{E_m^2}{T\eta}\left[\frac{T}{2} - \frac{\sin 2\beta z}{4\omega} + \frac{\sin 2\beta z}{4\omega}\right] = \frac{E_m^2 \cdot \cancel{T}}{2\cancel{T}\eta} = \frac{E_m^2}{2\eta}$$

Hence the average power is given by,

$$P_{avg} = \frac{1}{2} \frac{E_m^2}{\eta} \, W/m^2$$

INTEGRAL & POINT FORMS OF POYNTING THEOREM:

Consider Maxwell's equations as given below,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{----} \; (1)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \text{---} \; (2)$$

Doting both the sides of eqn (2) with $\vec{E}$, we get

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E} \cdot (\sigma \vec{E}) + \vec{E} \cdot \left(\varepsilon \frac{\partial \vec{E}}{\partial t}\right) \quad \text{---} \; (3)$$

Let us make use of vector identity as given below

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

applying the above identity ~~to eqn (3) we get~~ to equation (3) with $\vec{A} = \vec{E}$ & $\vec{B} = \vec{H}$ we get

$$\vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H}) = \nabla \cdot (\vec{E} \times \vec{H}).$$

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot (\nabla \times \vec{H})$$

sub (3) in above equation we get

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot (\sigma \vec{E}) + \vec{E} \cdot \left(\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \vec{E} \cdot \left(\varepsilon \frac{\partial \vec{E}}{\partial t}\right) \quad \text{---} \; (4)$$

consider the first term of (4) we get & by substituting (1) we get

$$\vec{H} \cdot (\nabla \times \vec{E}) = \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t}\right)$$

$$\vec{H} \cdot (\nabla \times \vec{E}) = -\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \quad \text{-----} \; (i)$$

Now consider term,

$$\frac{\partial}{\partial t}(\vec{H}\cdot\vec{H}) = \vec{H}\cdot\frac{\partial\vec{H}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{H}}{\partial t}$$

$$\frac{\partial}{\partial t}H^2 = 2\vec{H}\cdot\frac{\partial\vec{H}}{\partial t}$$

$$\frac{1}{2}\frac{\partial}{\partial t}\vec{H}^2 = \vec{H}\cdot\frac{\partial\vec{H}}{\partial t} \quad \text{--- (ii)}$$

III^{rly} we can write

$$\frac{1}{2}\frac{\partial}{\partial t}E^2 = \vec{E}\cdot\frac{\partial\vec{E}}{\partial t} \quad \text{---- (iii)}$$

Substituting (i), (ii) & (iii) in (4) we get

$$\vec{H}\cdot(\nabla\times\vec{E}) - \nabla\cdot(\vec{E}\times\vec{H}) = \sigma E^2 + \vec{E}\cdot\left(\varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$-\mu\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} - \nabla\cdot(\vec{E}\times\vec{H}) = \sigma E^2 + \varepsilon\left[\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}\right]$$

$$-\frac{\mu}{2}\frac{\partial}{\partial t}H^2 - \nabla\cdot(\vec{E}\times\vec{H}) = \sigma E^2 + \frac{\varepsilon}{2}\frac{\partial}{\partial t}E^2$$

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \sigma E^2 + \frac{\varepsilon}{2}\frac{\partial}{\partial t}E^2 + \frac{\mu}{2}\frac{\partial}{\partial t}H^2$$

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \sigma E^2 + \frac{1}{2}\frac{\partial}{\partial t}\left[\mu H^2 + \varepsilon E^2\right]$$

But $\vec{E}\times\vec{H} = \vec{P}$

$$\boxed{\therefore\ -\nabla\cdot(\vec{P}) = \sigma E^2 + \frac{1}{2}\frac{\partial}{\partial t}\left[\mu H^2 + \varepsilon E^2\right]} \quad \text{--- (5)}$$

The above equation represents Poynting Theorem in Point form.

If we integrate this Power over a volume, we can get energy distribution as,

$$-\int_V \nabla\cdot\vec{P}\,dV = \int_V \sigma E^2\,dV + \frac{\partial}{\partial t}\int_V \frac{1}{2}\left[\mu H^2 + \varepsilon E^2\right]dV$$

Applying divergence theorem to left of above equation we get

$$\boxed{-\oint \vec{P}\cdot\vec{ds} = \int_V \sigma E^2\,dV + \frac{\partial}{\partial t}\int_V \frac{1}{2}\left[\mu H^2 + \varepsilon E^2\right]dV.}$$

Above equation represents Poynting theorem in integral form..

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REFLECTION OF UNIFORM PLANE WAVES:

We have so far, studied the uniform plane waves travelling in unbounded and homogeneous media. But practically, very often, wave propagates in boundary regions consisting several media of different constitutive parameters such as $\varepsilon, \mu, \sigma, \eta$ etc.

Before we actually start with the reflection of the uniform plane wave, let us consider simple example of a transmission line.

Consider a transmission line having a characteristics impedance Z_0. Assume that the line is terminated in load impedance Z_L.

If the load impedance Z_L equals the characteristic impedance Z_0 (i.e $Z_L = Z_0$), then the line is said to be properly terminated.

If $Z_L \neq Z_0$, then there is a mismatch b/w the two impedances and the line is not properly terminated. Consider that the wave travelling along the line incidents at the load. The part of the wave gets absorbed by the load, while the other part is reflected back to the generator.

So we can say reflection occurs at the load if $Z_L \neq Z_0$. If there are two waves, one incident in forward direction, while other reflected back in backward direction, then the standing waves are said to be produced along the line.

When a uniform plane wave travels from one medium to other having different intrinsic impedances, the reflection takes place at the boundaries.

The part of the wave is transmitted in medium 2 and remaining part is reflected back to medium 1, depending upon the consecutive parameters of media.

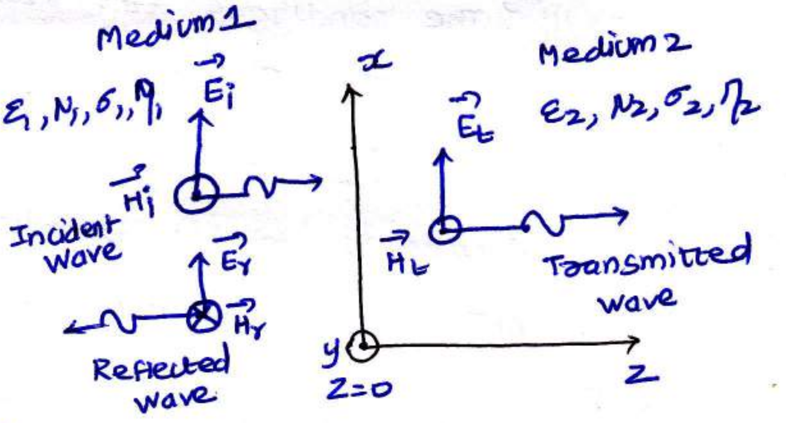
Depending upon the manner in which the uniform plane wave is incident on the boundary, there are two cases of incidence.

(i) Normal incidence

(ii) Oblique incidence.

NORMAL INCIDENCE AT PLANE DIELECTRIC BOUNDARY:

Consider a uniform plane wave striking the interface b/w the two dielectrics at right angles as shown in the figure.



Assume that the uniform plane wave travels along +z direction and incidence at right angles at the boundary b/w two dielectric media i.e. at $z=0$.

Below $z=0$, let the properties of medium 1 be $\varepsilon_1, \mu_1, \sigma_1, \eta_1$ and above $z=0$, the properties of medium 2 be $\varepsilon_2, \mu_2, \sigma_2, \eta_2$

So depending upon the properties of two media, part of the wave will be transmitted in medium 2 while other part will be reflected back in medium 1

Let E_i & H_i be the field strengths of the incident wave striking at the boundary.

E_t & H_t be the field strengths of the transmitted wave in the medium 2.

E_r & H_r be the field strengths of the reflected wave in the medium 1 returning back from the interface.

From figure it is clear that in medium 1, the total field composes of both the incident and reflected fields. But in medium 2 only transmitted field gives the total field.

So the conditions for the total field in medium 1 are given by,

$$\vec{E_1} = \vec{E_i} + \vec{E_r} \quad \&$$

$$\vec{H_1} = \vec{H_i} + \vec{H_r}$$

|||ly The conditions for total field in medium 2 is given by,

$$\vec{E_2} = \vec{E_t} \quad \&$$

$$\vec{H_2} = \vec{H_t}$$

According to the boundary condition, the tangential components of $\vec{E}$ & $\vec{H}$ must be continuous at the interface $z=0$.

$$\therefore \quad \vec{E_1}\tan = \vec{E_2}\tan$$

$$\vec{H_1}\tan = \vec{H_2}\tan$$

Thus at interface $z=0$, we can write

$$\vec{E_i} + \vec{E_r} = \vec{E_t} \quad \&$$

$$\vec{H_i} + \vec{H_r} = \vec{H_t}$$

The relationships b/w the magnitude of $\vec{E}$ & $\vec{H}$ at $z=0$ are given by the following expressions

$E_i = \eta_1 H_i$

$E_r = -\eta_1 H_r$ .... as direction of reflected wave is opposite to that of incident wave

$E_t = \eta_2 H_t$

Interms of magnitudes of the fields $\vec{E}$ & $\vec{H}$ at the interface, we can write

$$E_i + E_r = E_t \quad \& \quad \text{----} (1)$$

$$H_i + H_r = H_t \quad \text{----} (2)$$

In eqn (2), putting the values of H_i, H_r and H_t interms of E_i, E_r & E_t we get

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$\therefore E_i - E_r = \frac{\eta_1}{\eta_2} E_t \quad \text{----} (3)$$

Adding eqn (1) & (3), we get

$$2E_i = \left(1 + \frac{\eta_1}{\eta_2}\right) E_t$$

$$2E_i = \left(\frac{\eta_1 + \eta_2}{\eta_2}\right) E_t$$

$$\boxed{E_t = \frac{2\eta_2}{\eta_1 + \eta_2} E_i} \quad \text{---} (4)$$

The Transmission co-efficient is denoted by τ and it is given by,

$$\boxed{\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}} \quad \text{---} (5)$$

Eliminating E_t from eqn (1) & (3), we get

$$\frac{(1)}{(3)} \Rightarrow \frac{E_i + E_r}{E_i - E_r} = \frac{\eta_2}{\eta_1} \Rightarrow E_i + E_r = \frac{\eta_2}{\eta_1} E_i - E_r$$

$$\eta_2(E_i + E_r) = \eta_2(E_i - E_r)$$

$$\eta_1 E_i + \eta_2 E_r = \eta_2 E_i - \eta_2 E_r$$

$$(\eta_1 - \eta_2) E_i = -E_r(\eta_1 + \eta_2)$$

$$E_i(\eta_1 - \eta_2) = -E_r(\eta_1 + \eta_2)$$

$$E_r = \frac{\eta_1 - \eta_2}{-(\eta_1 + \eta_2)} E_i$$

$$\boxed{E_r = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} E_i} \quad --- \text{ (6)}$$

The reflection co-efficient is denoted as Γ and it is given by,

$$\boxed{\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}} \quad --- \text{ (7)}$$

From eqn (5) and (7), we can draw some important results as

(a) $1 + \Gamma = \tau$

(b) $0 \leq |\Gamma| \leq 1$

(c) Both the co-efficients ; Γ and τ are dimensionless and may be complex in nature.

According to Poynting theorem, the average power density is given by,

$$P_{avg} = \frac{1}{2} \frac{E_m^2}{\eta} \ W/m^2$$

where $E_m \rightarrow$ Amplitude of the Electric field intensity

$\eta \rightarrow$ Intrinsic impedance of a medium

The average power ~~transmitted~~ incident in medium-1 is given by

$$P_{iavg} = \frac{1}{2} \frac{E_i^2}{\eta_1} \ W/m^2$$

The average power reflected in medium-1 is given by

$$P_{ravg} = \frac{1}{2} \frac{E_r^2}{\eta_1} \ W/m^2$$

The ratio of power transmitted to power incident is given by

$$\frac{P_{tavg}}{P_{iavg}} = \frac{1/2\ E_t^2/\eta_2}{1/2\ E_i^2/\eta_1} = \frac{\eta_1}{\eta_2}\left[\frac{E_t}{E_i}\right]^2 = \frac{\eta_1}{\eta_2}\left[\frac{2\eta_2}{\eta_2+\eta_1}\right]^2 = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2}$$

Arranging terms we can write,

$$P_{tavg} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2} P_{iavg} \quad \text{--- (1)}$$

$$= \frac{\eta_1^2 + 2\eta_1\eta_2 + \eta_2^2 - (\eta_2^2 - 2\eta_1\eta_2 + \eta_1^2)}{(\eta_1+\eta_2)^2} P_{iavg}$$

$$= \frac{(\eta_1+\eta_2)^2 - (\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2} P_{iavg}$$

$$= \left[\frac{(\eta_1+\eta_2)^2}{(\eta_1+\eta_2)^2} - \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2}\right] P_{iavg}$$

$$\boxed{P_{tavg} = \left[1 - |\Gamma|^2\right] P_{iavg}} \quad \text{--- (2)}$$

The ratio of power reflected to power incident is given by,

$$\frac{P_{ravg}}{P_{iavg}} = \frac{1/2\ E_r^2/\eta_1}{1/2\ E_i^2/\eta_1} = \left[\frac{E_r}{E_i}\right]^2 = \left[\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right]^2 = \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2} \quad \text{--- (3)}$$

Rearranging terms we get

$$P_{ravg} = \frac{(\eta_2-\eta_1)^2}{(\eta_2+\eta_1)^2} P_{iavg}.$$

$$\therefore \boxed{P_{ravg} = (\Gamma)^2 P_{iavg}} \quad --- (4)$$

Adding (1) and (3) we get

$$\frac{P_{tavg}}{P_{iavg}} + \frac{P_{ravg}}{P_{iavg}} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2} + \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2}$$

$$= \frac{\eta_1^2+\eta_2^2+2\eta_1\eta_2}{(\eta_1+\eta_2)^2} = \frac{(\eta_1+\eta_2)^2}{(\eta_1+\eta_2)^2}$$

$$= 1$$

$$\therefore \boxed{P_{tavg} + P_{ravg} = P_{iavg}} \quad --- (5).$$

OBLIQUE INCIDENCE:-

When a uniform plane wave strikes obliquely on the surface (either conductor or dielectric), the behaviour of the reflected wave is decided by the polarization of the incident wave. There are two cases for the oblique incidence as given below.

case (i): The electric field vector $\perp^r$ to the plane of incidence. In other words, The Electric field vector is aligned $\parallel^{el}$ to the boundary surface as shown below. This is called Horizontal Polarization.

According to Poynting Theorem, The average Power density is given by,

$$P_{avg} = \frac{1}{2} \frac{E_m^2}{\eta} \text{ W/m}^2$$

where $E_m \rightarrow$ Amplitude of the Electric field intensity

$\eta \rightarrow$ Intrinsic impedance of a medium

The average Power ~~transmitted~~ incident in medium-1 is given by

$$P_{iavg} = \frac{1}{2} \frac{E_i^2}{\eta_1} \text{ W/m}^2$$

The average Power reflected in medium-1 is given by

$$P_{ravg} = \frac{1}{2} \frac{E_r^2}{\eta_1} \text{ W/m}^2$$

The ratio of Power transmitted to Power incident is given by

$$\frac{P_{tavg}}{P_{iavg}} = \frac{\frac{1}{2} E_t^2/\eta_2}{\frac{1}{2} E_i^2/\eta_1} = \frac{\eta_1}{\eta_2}\left[\frac{E_t}{E_i}\right]^2 = \frac{\eta_1}{\eta_2}\left[\frac{2\eta_2}{\eta_2+\eta_1}\right]^2 = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2}$$

Arranging terms we can write,

$$P_{tavg} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2} P_{iavg} \quad \text{--- (1)}$$

$$= \frac{\eta_1^2 + 2\eta_1\eta_2 + \eta_2^2 - (\eta_2^2 + 2\eta_1\eta_2 + \eta_1^2)}{(\eta_1+\eta_2)^2} P_{iavg}$$

$$= \frac{(\eta_1+\eta_2)^2 - (\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2} P_{iavg}$$

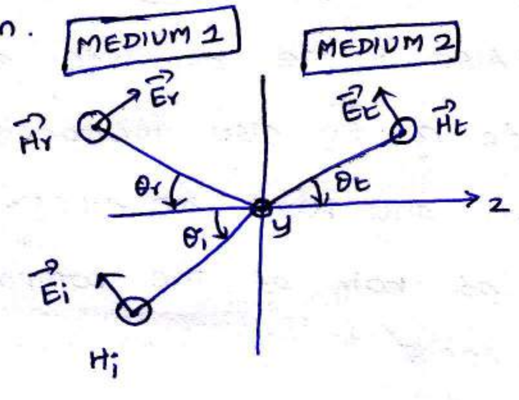
$$= \left[\frac{(\eta_1+\eta_2)^2}{(\eta_1+\eta_2)^2} - \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2}\right] P_{iavg}$$

$$\boxed{P_{tavg} = \left[1-|\Gamma|^2\right] P_{iavg}} \quad \text{--- (2)}$$

The ratio of Power reflected to Power incident is given by,

$$\frac{P_{ravg}}{P_{iavg}} = \frac{\frac{1}{2} E_r^2/\eta_1}{\frac{1}{2} E_i^2/\eta_1} = \left[\frac{E_r}{E_i}\right]^2 = \left[\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right]^2 = \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2} \quad \text{--- (3)}$$

CASE (ii) : The magnetic field vector is aligned $\parallel^{el}$ to the boundary surface. In other words, the magnetic field vector is $\perp r$ to the plane of incidence while electric field vector is aligned $\parallel^{el}$ to the plane of incidence as shown below. This is called vertical polarization.



PLANE OF INCIDENCE:-

A plane of incidence is a plane containing the vector in the direction of propagation of the incident wave and the normal to the boundary surface.

POLARIZATION OF ELECTROMAGNETIC WAVES:

The polarization of uniform plane waves is defined as time varying behaviour of the Electric field intensity vector $\vec{E}$ at some fixed point in space, along the direction of propagation.

There are three different types of polarization of a uniform plane wave as given below

(a) Linear polarization
(b) Elliptical polarization
(c) Circular polarization.

In other words polarization is nothing but the way in which the magnitude and direction of the electric field varies.

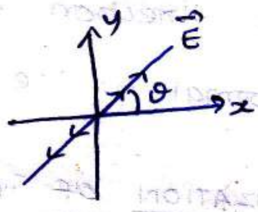
LINEAR POLARIZATION:

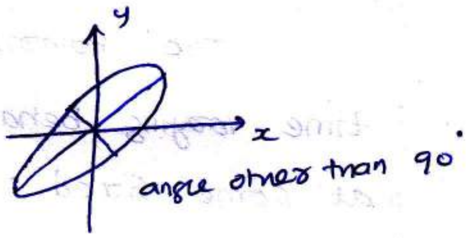
Let the components of $\vec{E}$ be $\vec{E_x}$ and $\vec{E_y}$ along x and y-direction respectively. Both these components are in phase having different amplitudes. As $\vec{E_x}$ and $\vec{E_y}$ are in phase they will have their amplitudes reaching max or min value simultaneously. Also if the amplitude of $\vec{E_x}$ increases or decreases the amplitude of $\vec{E_y}$ also increases or decreases.

In other words, at any point along +ve z-axis the ratio of amplitudes of both of the components is constant as both of them are in phase having same wavelength

The electric field $\vec{E}$ is the resultant of $\vec{E_x}$ & $\vec{E_y}$ and the direction of it depends on the relative magnitude of $\vec{E_x}$ & $\vec{E_y}$. Thus the angle made by $\vec{E}$ with x-axis is given by,

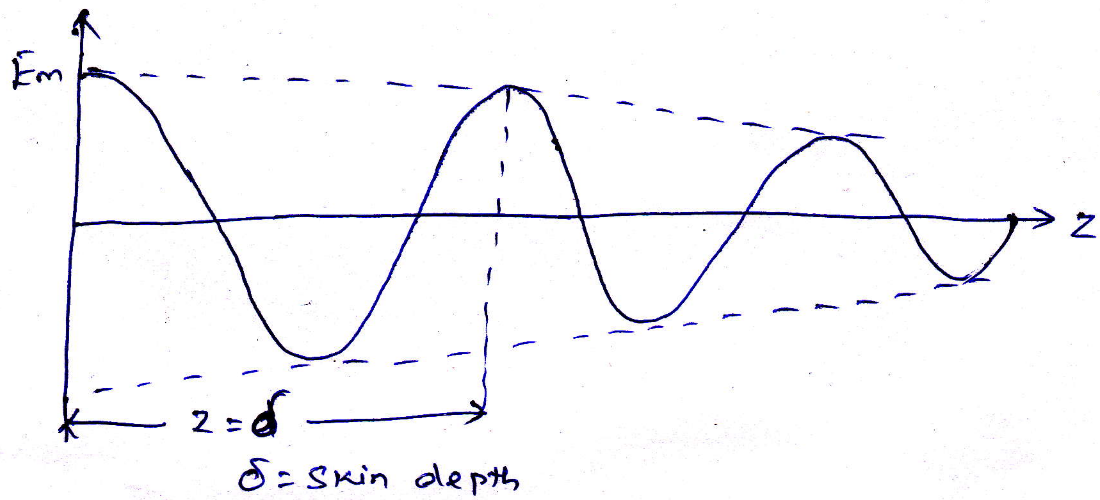
$$\theta = \tan^{-1} \frac{E_y}{E_x}.$$





SKIN DEPTH:

The skin depth is defined as the depth in which the wave has attenuated to $1/e$ ie, approximately 37 % of its original value. It is also called as depth of penetration.



Problem:

① A 300 Hz uniform plane wave propogate through fresh water for which sigma = 0, $\mu_r = 1$, $\varepsilon_r = 78$. Calculate wavelength.

Given $f = 300\,Hz$, sigma $= 0$, $\mu_r = 1$, $\varepsilon_r = 78$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{(\mu_0\mu_r)(\varepsilon_0\varepsilon_r)}$$

$$\omega = 2\pi f$$

$$\beta = (2 \times \pi \times 300) \times \sqrt{(4\pi \times 10^{-7})(1)(8.854 \times 10^{-12})(78)}$$

$$\cancel{\beta = 5.54 \times 10^{-4}\ rad/m}$$

$$\boxed{\beta = 5.54 \times 10^{-4}\ rad/m}$$

$$\cancel{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.54 \times 10^{-4}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.54 \times 10^{-4}}$$

$$\boxed{\lambda = 1.13 \times 10^{-4}\ metre}$$