UNIT-1

VECTOR ANALYSIS

INTRODUCTION:

Electromagnetics is a branch of Physics (or) electrical engineering which is used to study the electric and magnetic Phenomena.

what is a field ?

Consider a magnet. It has its own effect in a region surrounding it. The effect can be Placed by Placing another magnet near the first magnet. such an effect can be defined by a Particular Physical function.

In the region surrounding the magnet, there exists a Particular value ~~for~~ that Physical function, at every Point, describing the effect of magnet.

So field can be defined as the region in which, at each point there exists a corresponding value of some Physical function.

If the field ~~is~~ Produced is due to magnetic effects, it is called MAGNETIC FIELD.

There are two types of electric charges, Positive and negative. such an electric charge Produces a field around it which is called an ELECTRIC FIELD.

Moving charges produces current and current carrying conductor produces a magnetic field. In such case electric and magnetic fields are related to each other. such a field is called ELECTROMAGNETIC FIELD. such fields may be time varying or time independent.

2 It is seen that distribution of a quantity in a space is defined by a field. Hence to quantify the field, three dimensional representation plays an important ~~field~~ role. Such three dimensional representation can be made easy by the use of vector analysis.

SCALARS & VECTORS:

The various quantities involved in the study of engineering electromagnetics can be classified as

1. scalars &
2. vectors.

SCALAR:

The scalar is a quantity whose value may be represented by a single real number, which may be +ve (or) -ve. The direction is not at all required in describing a scalar. Thus

A scalar is a quantity which is wholly characterized by its magnitude.

eg: temperature, mass, volume, density, speed electric charge etc.

VECTOR:

A quantity which has both, a magnitude and a specific direction in space is called a vector.

In electromagnetics vectors defined in two and three dimensional spaces are required but vectors may be defined in n-dimensional space.

A vector is a quantity which is characterized by both, a magnitude and a direction.

eg: Force, velocity, displacement, electric field intensity, magnetic field intensity, acceleration etc.

~~VECTOR FIELDS~~

SCALAR FIELD:

The distribution of a scalar quantity with a definite position in a space is called SCALAR FIELD.

eg:- 1. Temperature of atmosphere.
   (It has a definite value in the atmosphere but no need of direction to specify).

2. Height of surface of earth above sea level
3. Sound intensity in an auditorium.
4. Light intensity in a room.
5. Atmospheric pressure in a given region etc.

VECTOR FIELD:

If a quantity which is specified in a region to define a field is a vector then the corresponding field is called a vector field.

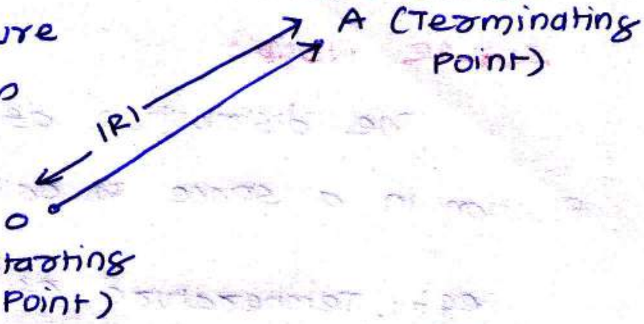
eg:- 1. Gravitational force on a mass in a space is a vector field. [This force has a value at various points in a space and always has a specific direction].

2. Velocity of particles in a moving fluid
3. wind velocity of atmosphere
4. Voltage gradient in a cable
5. Displacement of a flying bird in a space.
6. Magnetic field existing from north to south poles.

REPRESENTATION OF A VECTOR:

In two dimensional, a vector can be represented by a straight line with an arrow in a plane. The length of the ~~vector~~ segment is the magnitude of a vector while the arrow indicates the direction of the vector.

The vector shown in figure is symbolically denoted as $\overrightarrow{OA}$. It's length is called as magnitude, which is R for the vector OA.

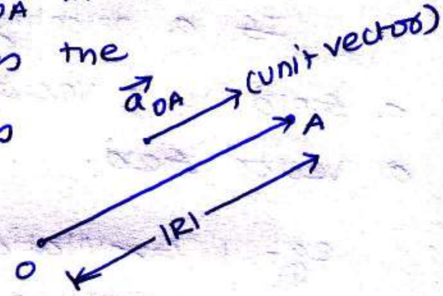


It is represented as $|\overrightarrow{OA}| = R$

UNIT VECTOR:

A unit vector has a function to indicate the direction. It's magnitude is always unity, irrespective of its direction. Thus for any vector, to indicate its direction a unit vector can be used.

Consider a unit vector $\vec{a}_{OA}$ in the direction of $\overrightarrow{OA}$ as shown in fig. This indicates the direction of $\overrightarrow{OA}$ but its magnitude is unity.



So vector $\overrightarrow{OA}$ can be represented completely as its magnitude R and the direction as indicated by the unit vector along its direction.

$$\overrightarrow{OA} = |\overrightarrow{OA}|\, \vec{a}_{OA} = R\vec{a}_{OA}$$

↳ unit vector along the direction OA and $|\vec{a}_{OA}| = 1$

(X)

letter $\vec{a}$ is used to indicate the unit vector

2 Mark Question:

1. Mention the purpose of unit vector in vector algebra.

In case if a vector is known then the unit vector along that vector can be obtained by dividing the vector by its magnitude. Thus unit vector can be expressed as,

$$\text{Unit vector } \vec{a}_{OA} = \frac{\overrightarrow{OA}}{|OA|}$$

2 mark question.

1. Express unit vector in terms of a vector and its magnitude.

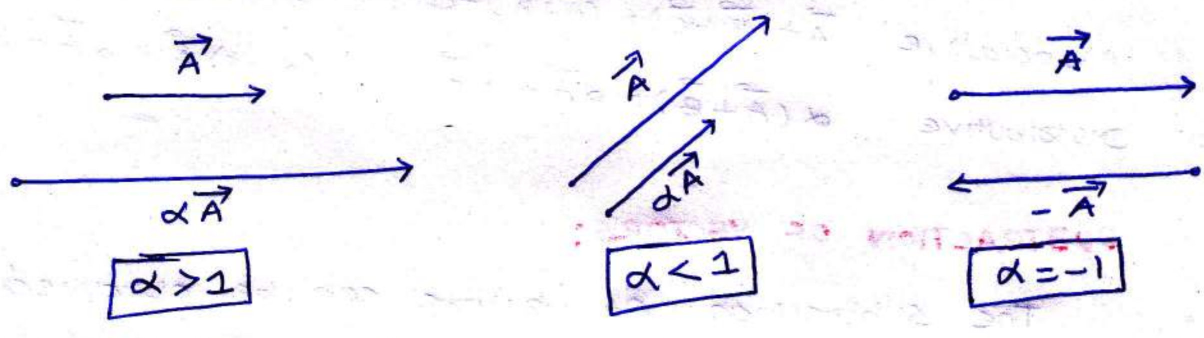
VECTOR ALGEBRA: [Scaling, Addition, subtraction].

SCALING OF VECTOR

→ This is multiplication by a scalar to a vector

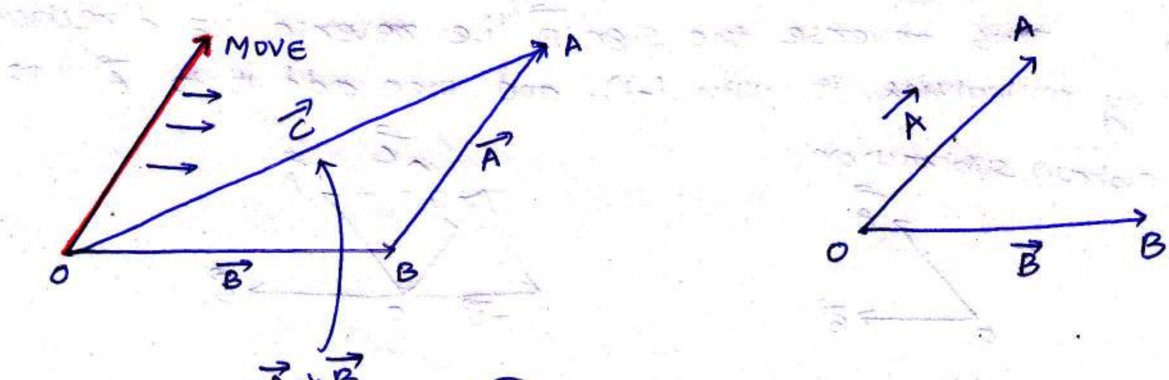
→ This changes the magnitude (length) of a vector but not its direction, when scalar is POSITIVE

→ When scalar = −1, the magnitude remains same but direction of the vector reverses.



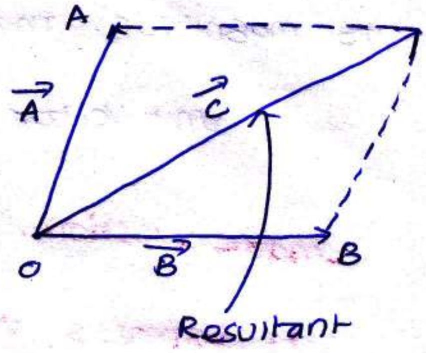
ADDITION OF VECTORS:

→ The vectors which lie on the same plane are called coplanar vectors.

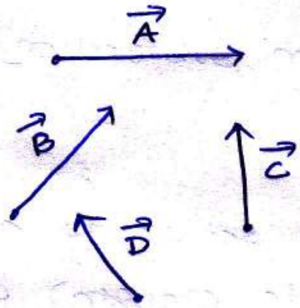


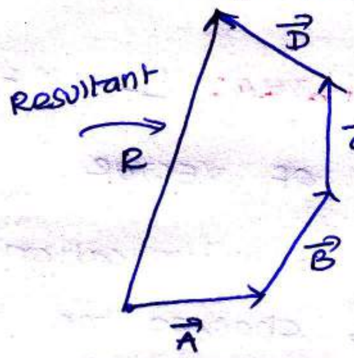
PARALLELOGRAM RULE:-

Complete the Parallelogram as shown in fig. Then the diagonal of the parallelogram represents the addition of the two vectors.



HEAD TO TRIAL RULE:-





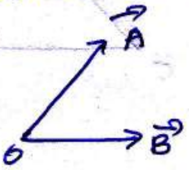
Law	Addition	Multiplication by scalar
commutative	$\vec{A}+\vec{B}=\vec{B}+\vec{A}$	$\alpha\vec{A}=\vec{A}\alpha$
Associative	$\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$	$\beta(\alpha\vec{A})=(\beta\alpha)\vec{A}$
Distributive	$\alpha(\vec{A}+\vec{B})=\alpha\vec{A}+\alpha\vec{B}$	$(\alpha+\beta)\vec{A}=\alpha\vec{A}+\beta\vec{A}$

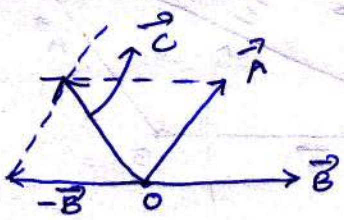
SUBTRACTION OF VECTORS:

The subtraction of vectors can be obtained from the rules of addition. If $\vec{B}$ is to be subtracted from $\vec{A}$ then based on addition it can be represented as

$$\vec{C}=\vec{A}+(-\vec{B}).$$

Thus reverse the sign $\vec{B}$ i.e reverse its direction by multiplying it with (-1). and then add it to $\vec{A}$ to obtain subtraction.





Identical vectors:

Two vectors are said to be identical if their difference is zero.

eg: $\vec{A} - \vec{B} = 0$ $\quad$ $\vec{A}$ & $\vec{B}$ are identical.

$\Rightarrow \vec{A} = \vec{B}$

VECTOR MULTIPLICATION:

consider two vectors $\vec{A}$ and $\vec{B}$. There are two types of products existing depending upon the result of the multiplication. These two types of products are

1. Scalar (or) DOT Product
2. Vector (or) cross product

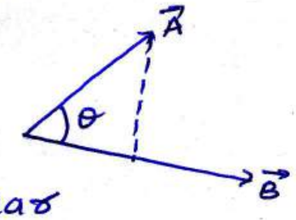
SCALAR (OR) DOT PRODUCT OF VECTORS:

→ It is denoted by $\vec{A} \cdot \vec{B}$

→ It is defined as the product of the magnitude of $\vec{A}$, the magnitude of $\vec{B}$ and the cosine of smaller angle b/w them.

→ It also can be defined as the product of magnitude of $\vec{B}$ and the projection of $\vec{A}$ onto $\vec{B}$ or vice versa

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta_{AB}}$$



The result of such a dot product is scalar hence it is also called as scalar product.

PROPERTIES OF DOT PRODUCT

1. If the two vectors are $\parallel^{el}$ to each other i.e $\theta = 0$ then $\cos\theta_{AB} = 1$ Thus

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \text{ for } \parallel^{el} \text{ vectors.}$$

2. If two vectors are $\perp^{r}$ to each other i.e $\theta = 90^\circ$ then $\cos\theta_{AB} = 0^\circ$ Thus

$$\vec{A} \cdot \vec{B} = 0 \text{ for } \perp^{r} \text{ vectors.}$$

3. If the dot product of vector with itself is performed, the result is square of the magnitude of that vector.

$$\vec{A}\cdot\vec{A} = |\vec{A}|\,|\vec{A}| \cos 0^\circ = |\vec{A}|^2$$

4. Any unit vector dotted with itself is unity.

$$\vec{a}_x \cdot \vec{a}_x = 1 = \vec{a}_y \cdot \vec{a}_y = \vec{a}_z \cdot \vec{a}_z$$

5. The dot product obeys commutative, & distributive law

(ie) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

2 mark question.

1. Given two vectors, How to identify whether they are $\perp^{re}$ or $\parallel^{el}$ to each other.

APPLICATION OF DOT PRODUCT:

1. To determine the angle b/w the two vectors.

$$\theta = \cos^{-1}\left\{\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}\right\}$$

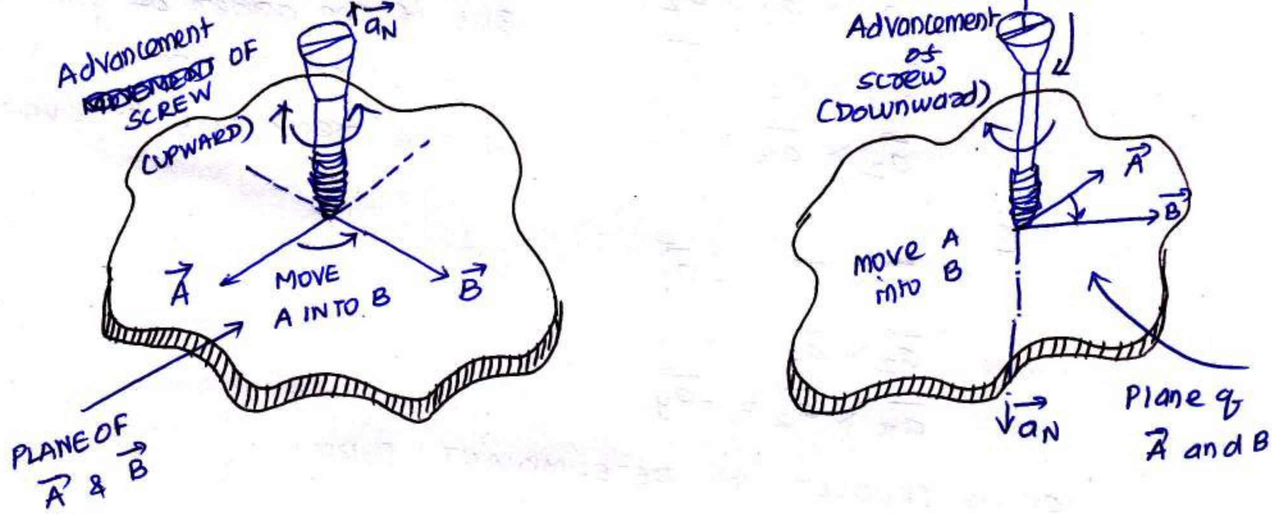
VECTOR (OR) CROSS PRODUCT OF VECTORS:

Consider two vectors $\vec{A}$ & $\vec{B}$ then the cross product is denoted as $\vec{A}\times\vec{B}$ and defined as the product of the magnitudes of $\vec{A}$ & $\vec{B}$ and the sine of the smaller angle between $\vec{A}$ and $\vec{B}$.

CROSS PRODUCT is a vector quantity and has a direction $\perp^{r}$ to the plane, containing the two vectors $\vec{A}$ and $\vec{B}$.

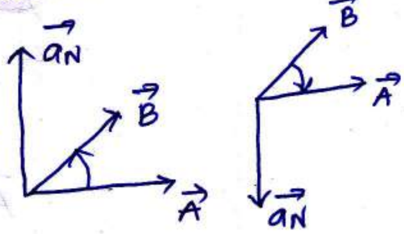
Mathematically cross product is expressed as

$$\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta_{AB}\,\vec{a}_N$$



PROPERTIES OF CROSS PRODUCT:

1. The commutative law is not applicable to the cross product. Thus $\boxed{\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}}$



2. Reversing the order of the vectors $\vec{A}$ and $\vec{B}$, a unit vector $\vec{a}_N$ reverses its direction hence we can write

$$\vec{A}\times\vec{B} = -[\vec{B}\times\vec{A}]$$ anticommutative

3. The cross product is not associative, thus

$$\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$$

4. With respect to addition cross product is distributive, thus

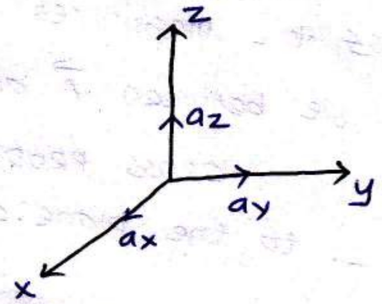
$$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}.$$

5. If two vectors are $\parallel^{el}$ to each other, (ie) they are in same direction then $\theta = 0^\circ$ & hence cross product of such two vectors is zero.

6. $\vec{A} \times \vec{A} = 0$ [cross product to itself].

7. cross product of unit vectors.

consider the unit vectors $\vec{a_x}$, $\vec{a_y}$ and $\vec{a_z}$ which are mutually $\perp^r$ to each other as shown in fig.



Then

$$\vec{a_x} \times \vec{a_y} = |\vec{a_x}||\vec{a_y}| \sin(90^\circ) \vec{a_N}$$

In this case $\vec{a_N} = \vec{a_z}$

and $|\vec{a_x}| = |\vec{a_y}| = \sin(90^\circ) = 1$

$$\therefore \quad \vec{a_x} \times \vec{a_y} = \vec{a_z}$$
$$\vec{a_y} \times \vec{a_z} = \vec{a_x}$$
$$\vec{a_z} \times \vec{a_x} = \vec{a_y}$$

But if the order of unit vectors is reversed, the result is -ve of the remaining third unit vector

Thus

$$\vec{a_y} \times \vec{a_x} = -\vec{a_z}$$
$$\vec{a_z} \times \vec{a_y} = -\vec{a_x}$$
$$\vec{a_x} \times \vec{a_z} = -\vec{a_y}$$

CROSS PRODUCT IN DETERMINANT FORM:

consider two vectors

$$\vec{A} = A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}$$
$$\vec{B} = B_x \vec{a_x} + B_y \vec{a_y} + B_z \vec{a_y}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\boxed{\vec{A} \times \vec{B} = [A_y B_z - B_y A_z]\vec{a_x} + [A_z B_x - A_x B_z]\vec{a_y} + [A_x B_y - A_y B_x]\vec{a_z}}$$

PRODUCTS OF THREE VECTORS ÷

Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ are the three given vectors. Then the product of these three vectors is classified into two ways called,

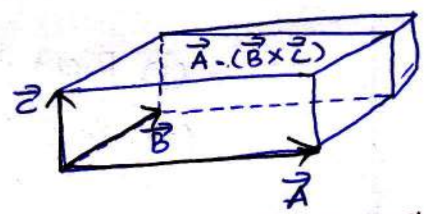
1. Scalar triple product
2. Vector triple product.

SCALAR TRIPLE PRODUCT: (scalar triple product of 3 vectors $\vec{A}, \vec{B}$ & $\vec{C}$).

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B}).$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}.$$

1. It represents the volume of parallelepiped.



2. If two (or) three vectors are equal then the result of the scalar triple product is zero.

3. cyclic order a, b, c is to be followed. If the order is changed, the sign is reversed.

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = -\vec{B}\cdot(\vec{A}\times\vec{C}).$$

VECTOR TRIPLE PRODUCT ÷

The vector triple product of the three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ is mathematically defined as

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$\boxed{\text{"bac-cab" RULE}}$

Problem

1. Three fields are given by $\vec{A} = 2\vec{a}_x - \vec{a}_z$, $\vec{B} = 2\vec{a}_x - \vec{a}_y + 2\vec{a}_z$, $\vec{C} = 2\vec{a}_x - 3\vec{a}_y + \vec{a}_z$.

Find the scalar and vector triple product.

Scalar triple product. (i)

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} 2 & 0 & 1 \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= 14.$$

Vector triple product.

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\vec{A}\cdot\vec{C} = (2)(2) + (0)(-3) + (-1)(1) = 3$$

$$\vec{A}\cdot\vec{B} = (2)(2) + (0)(-1) + (-1)(2) = 2$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = 3\vec{B} - 2\vec{C}$$
$$= 3[2\vec{a}_x - \vec{a}_y + 2\vec{a}_z] - 2[2\vec{a}_x - 3\vec{a}_y + \vec{a}_z]$$
$$= 2\vec{a}_x + 3\vec{a}_y + 4\vec{a}_z$$

CO-ORDINATE SYSTEM:

Three Types of co-ordinate systems are

(i) cartesian (or) Rectangular co-ordinate system

(ii) cylindrical co-ordinate system

(iii) spherical co-ordinate system.

CARTESIAN CO-ORDINATE SYSTEM:

- Also called Rectangular co-ordinate system
- Three co-ordinates x, y, z mutually $\perp^r$ to each other.
- Intersection of x, y, z is called origin.

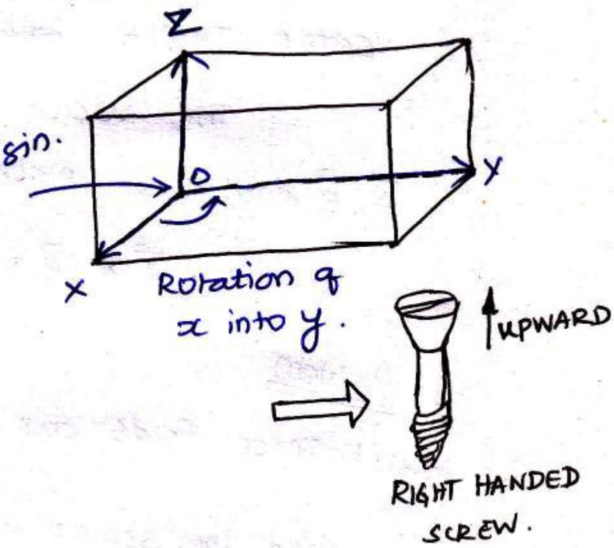
There are two types of such systems, they are

(i) Right handed system

(ii) Left handed system.

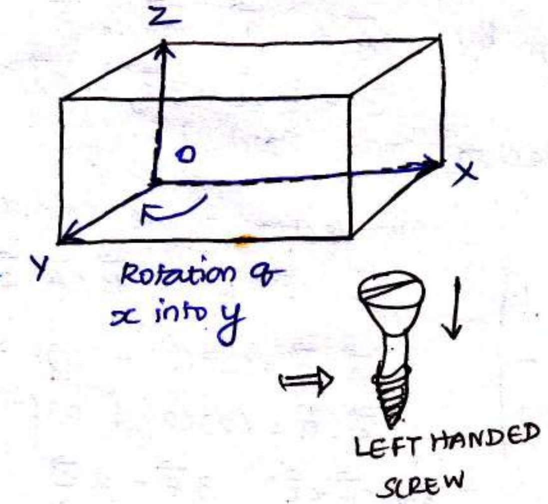
RIGHT HANDED SYSTEM:

If x axis is rotated towards y axis through a smaller angle, thus this rotation causes the upward movement of right handed screw in the 'z' axis direction.



LEFT HANDED SYSTEM:

Downward movement of screw.



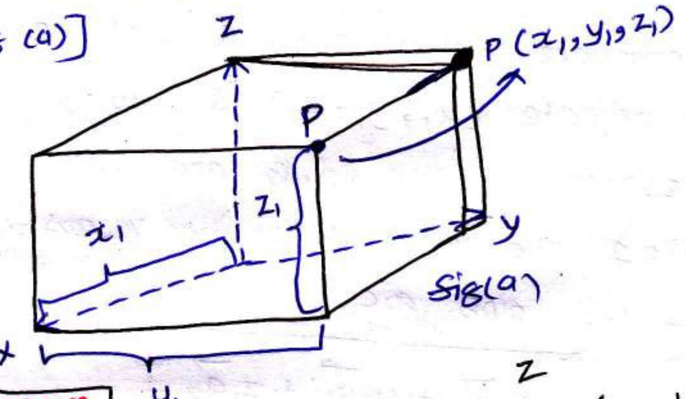
Note: RIGHT HANDED SYSTEM IS COMMONLY USED

REPRESENTING A POINT IN RECTANGULAR CO-ORDINATE SYSTEMS

→ A Point in rectangular co-ordinate system is located by three coordinates namely x, y and z co-ordinates.

→ The Point can be reached by moving from origin, the distance x in x direction, then the distance y in y direction and finally distance z in z direction.

→ Consider a Point P having co-ordinates x_1, y_1 and z_1. It is represented as $P(x_1, y_1, z_1)$. The co-ordinates x_1, y_1, z_1 may be +ve or -ve [fig (a)]



fig(a)

→ The Point Q (3, -1, 2) can be shown in this system as shown in fig (b)

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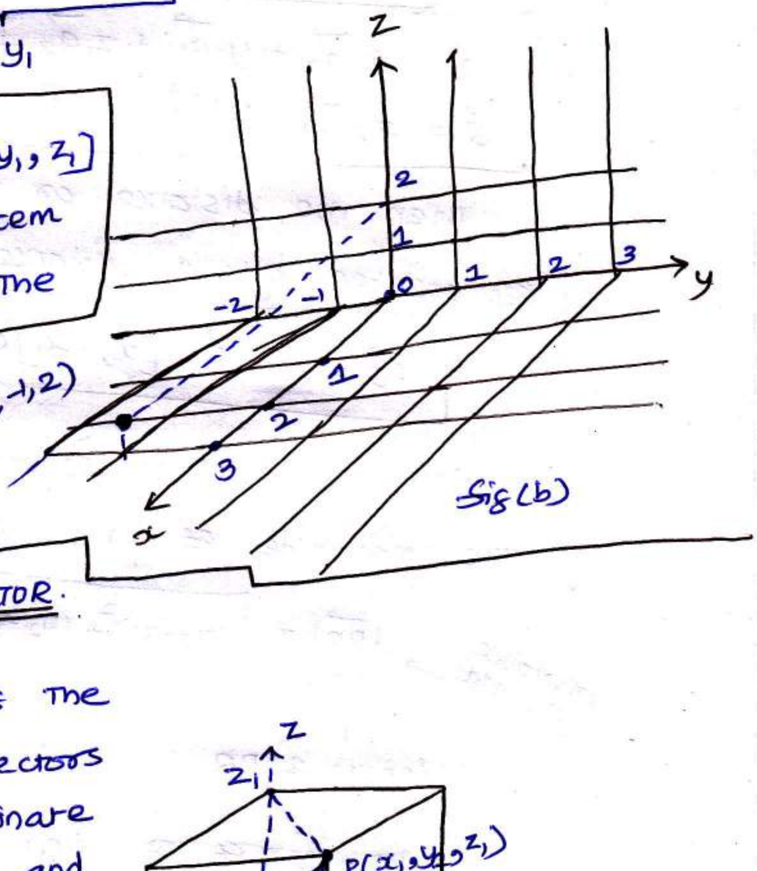


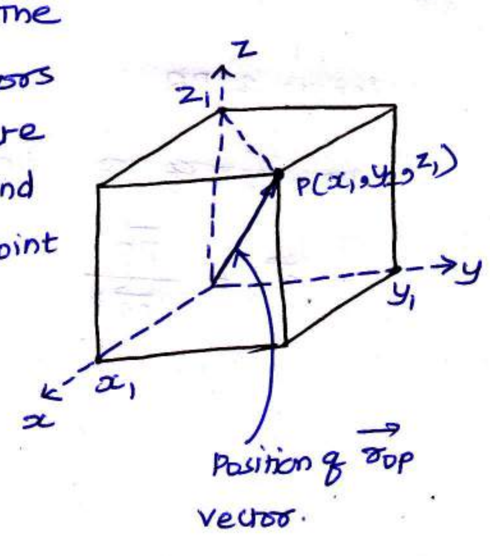
fig (b)

POSITION & DISTANCE VECTORS

Consider a point $P[x_1, y_1, z_1]$ in cartesian co-ordinate system as shown in fig (c). Then the Position vector of Point 'P' is represented by the distance of point P from the origin directed from origin to point P. This is also called RADIUS VECTOR.

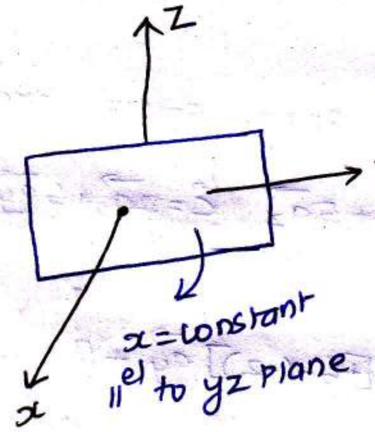
- The three components of the Position vector $\vec{r}_{op}$ are three vectors oriented along the three co-ordinate axes with the magnitudes x_1, y_1 and z_1. Thus the position vector of point P can be represented as

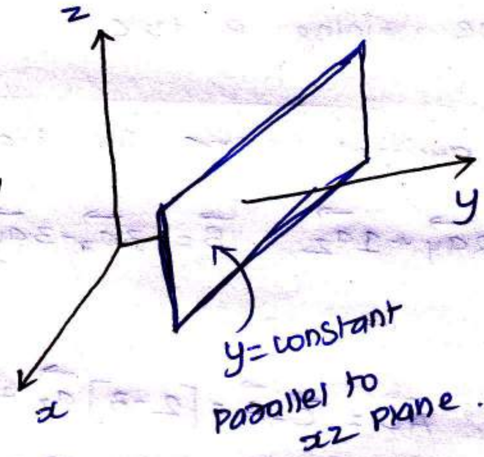
$$\boxed{\vec{r}_{op} = x_1 \vec{a}_x + y_1 \vec{a}_y + z_1 \vec{a}_z}$$

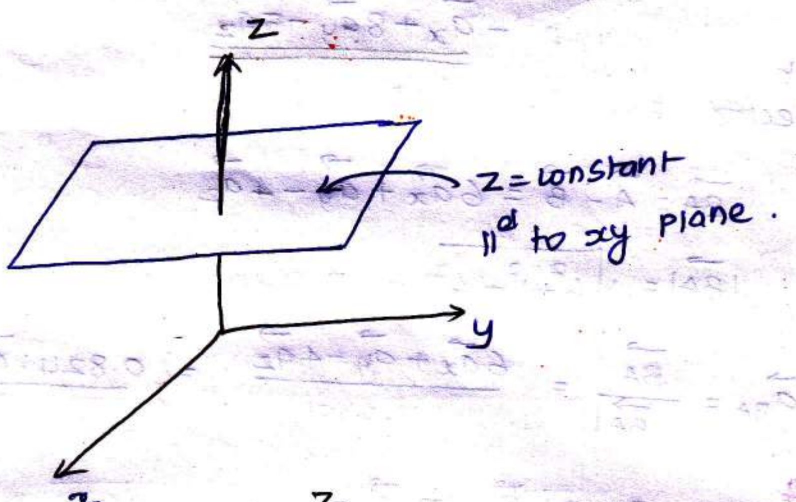


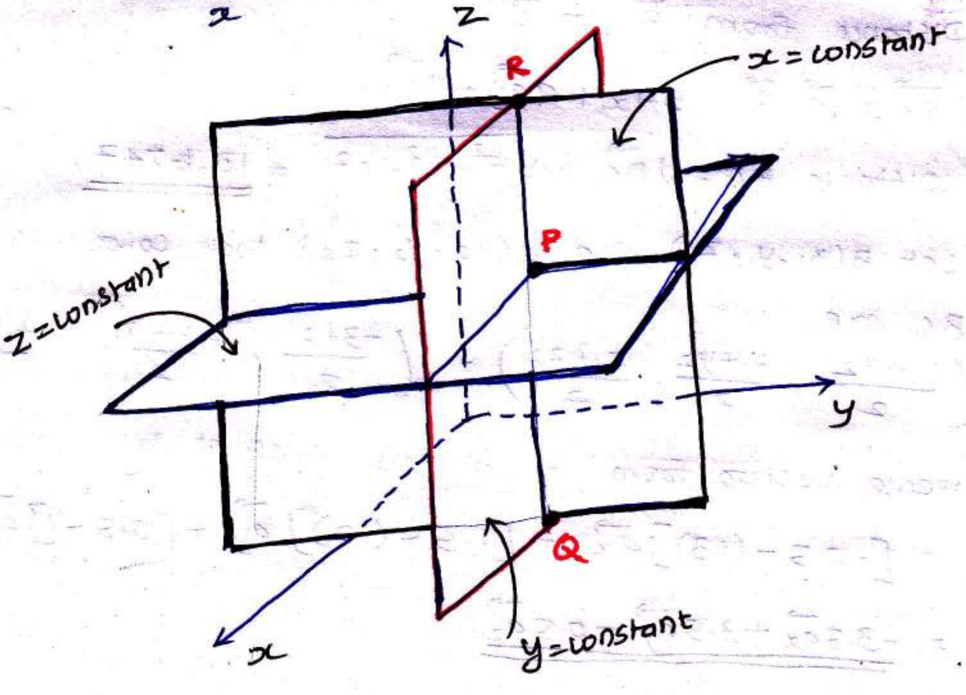
ALTERNATE METHOD TO DEFINE A POINT IN CARTESIAN SYSTEM:

Alternate method to consider three surfaces namely $x =$ constant, $y =$ constant and $z =$ constant. The common intersection of all these three surfaces is the point to be defined and the constants indicate the co-ordinates of that point. as shown in figures given below.









Problem

Given three Points in Cartesian co-ordinate system as $A(3,-2,1)$, $B(-3,-3,5)$ and $C(2,6,-4)$

Find (i) The vector from A to C

(ii) The unit vector from B to A

(iii) The distance from B to C.

(iv) The vector from A to the midpoint of the straight line joining B to C.

Solution:-

The Position vectors for the given points are

$\vec{A} = 3\vec{a_x} - 2\vec{a_y} + 1\vec{a_z}$ $\quad \vec{B} = -3\vec{a_x} - 3\vec{a_y} + 5\vec{a_z}$ $\quad \vec{C} = 2\vec{a_x} + 6\vec{a_y} - 4\vec{a_z}$

(i) vector from ~~vector~~ $\vec{A}$ to $\vec{C}$

$$\vec{AC} = \vec{C} - \vec{A} = [2-3]\vec{a_x} + [+6-(-2)]\vec{a_y} + [-4-(+1)]\vec{a_z}$$

$$= -\vec{a_x} + 8\vec{a_y} - 5\vec{a_z}$$

(ii) unit Vector ~~from~~ from $\vec{B}$ to $\vec{A}$

$$\vec{BA} = \vec{A} - \vec{B} = 6\vec{a_x} + \vec{a_y} - 4\vec{a_z}$$

$$|\vec{BA}| = \sqrt{6^2+1^2+4^2} = 7.2801.$$

$$\vec{a}_{BA} = \frac{\vec{BA}}{|\vec{BA}|} = \frac{6\vec{a_x} + \vec{a_y} - 4\vec{a_z}}{7.2801} = 0.8241\vec{a_x} + 0.1373\vec{a_y} - 0.5494\vec{a_z}$$

(iii) Distance from $\vec{B}$ to $\vec{C}$

$$\vec{BC} = \vec{C} - \vec{B} = 5\vec{a_x} + 9\vec{a_y} - 9\vec{a_z}$$

$$\text{Distance } BC = |\vec{BC}| = \sqrt{5^2+9^2+9^2} = 13.6747.$$

(iv) Let $B(x_1, y_1, z_1)$ and $C(x_2, y_2, z_2)$ then co-ordinates of midpoint of BC are

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = \left(\frac{-3+2}{2}, \frac{-3+6}{2}, \frac{+5-4}{2}\right) = (-0.5, 1.5, 0.5)$$

Hence vector from A to this midpoint is

$$= [-0.5-(+3)]\vec{a_x} + [1.5-(-2)]\vec{a_y} + [0.5-1]\vec{a_z}$$

$$= -3.5\vec{a_x} + 3.5\vec{a_y} - 0.5\vec{a_z}$$

The magnitude of vectors interms of three mutually ⊥r components are given by

$$|\vec{r}_{OP}| = \sqrt{(x_1)^2+(y_1)^2+(z_1)^2}$$

if point P has co-ordinates (1, 2, 3) then its position vector is

$$\boxed{\vec{r}_{OP} = 1\vec{a_x}+2\vec{a_y}+3\vec{a_z}} \qquad |\vec{r}_{OP}| = \sqrt{1^2+2^2+3^2} = \sqrt{14} = 3.7416.$$

Now consider two points in a cartesian coordinate system, P and Q with the co-ordinate (x_1, y_1, z_1) & (x_2, y_2, z_2) respectively. The points are shown in fig 1. The individual position vectors of the points are

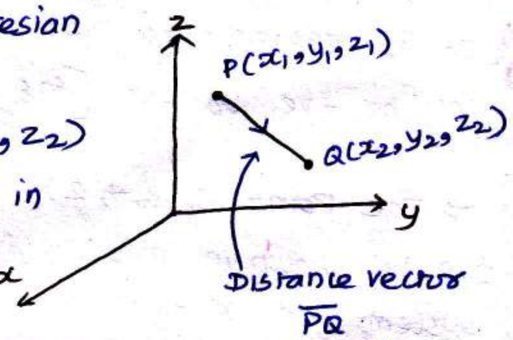


Fig 1

$$\boxed{\begin{aligned}\vec{P} &= x_1\vec{a_x}+y_1\vec{a_y}+z_1\vec{a_z}\\ \vec{Q} &= x_2\vec{a_x}+y_2\vec{a_y}+z_2\vec{a_z}\end{aligned}}$$

Then the distance or displacement from P to Q is represented by a distance vector $\vec{PQ}$ and is given by

$$\boxed{\vec{PQ} = \vec{Q}-\vec{P} = [x_2-x_1]\vec{a_x} + [y_2-y_1]\vec{a_y} + [z_2-z_1]\vec{a_z}}$$

This is also called seperation vectors.

The magnitude of this vector is given by

$$\underbrace{|\vec{PQ}|}_{\text{length of PQ}} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

distance formula

~~directions of the~~ unit vector along direction of PQ is

$$\vec{a}_{PQ} = \frac{\vec{PQ}}{|\vec{PQ}|}.$$

PROBLEM:

1. Obtain the unit vector in the direction from the origin towards the point P (3, -3, 2)

Solution:-

The origin O(0,0,0) while P(3,-3,2) hence the distance vector $\vec{OP}$ is

$$\vec{OP} = (3-0)\vec{a_x} + (-3-0)\vec{a_y} + (-2-0)\vec{a_z}$$

$$= 3\vec{a_x} - 3\vec{a_y} - 2\vec{a_z}$$

$$|\vec{OP}| = \sqrt{3^2 + (-3)^2 + (-2)^2} = 4.6904.$$

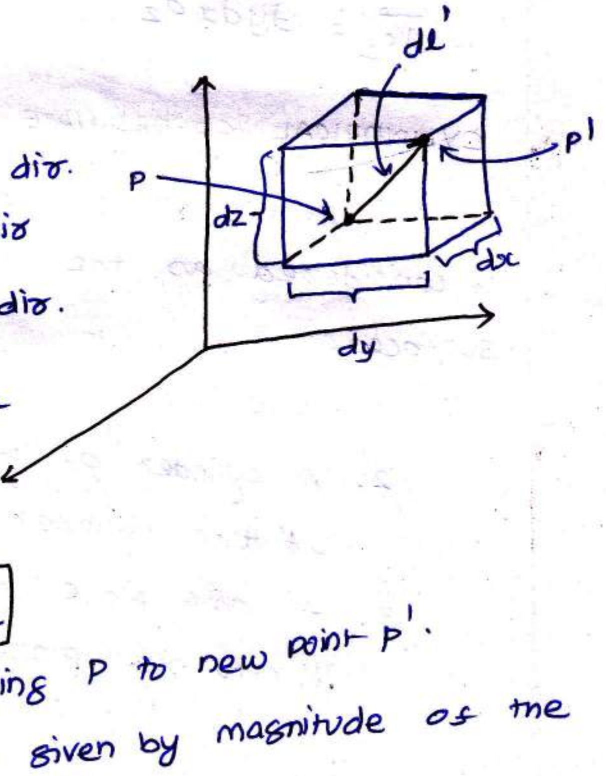
Hence the unit vector along the direction OP is

$$\vec{a}_{OP} = \frac{\vec{OP}}{|\vec{OP}|} = \frac{3\vec{a_x} - 3\vec{a_y} - 2\vec{a_z}}{4.6904}$$

$$= 0.6396\,\vec{a_x} - 0.6396\,\vec{a_y} - 0.4264\,\vec{a_z}$$

DIFFERENTIAL ELEMENTS IN CARTESIAN CO-ORDINATOR SYSTEM:

Consider a point $P(x, y, z)$ in the rectangular coordinate system. Let us increase each co-ordinate by differential amount. A new point p' will be obtained. having co-ordinates $(x+dx, y+dy, z+dz)$



dx = Differential length in x dir.

dy = Differential length in y dir

dz = Differential length in z dir.

Hence differential vector length also called elementary vector length can be represented as

$$\boxed{\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}}$$

↳ This is the vector joining P to new point P'.

The distance of P' from P is given by magnitude of the differential vector length.

$$\boxed{|\vec{dl}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}}$$

Hence the differential volume of the rectangular Parallelepiped is given by,

$$\boxed{dv = dx\,dy\,dz}$$

Note: $\vec{dl}$ is a vector but dv is a scalar.

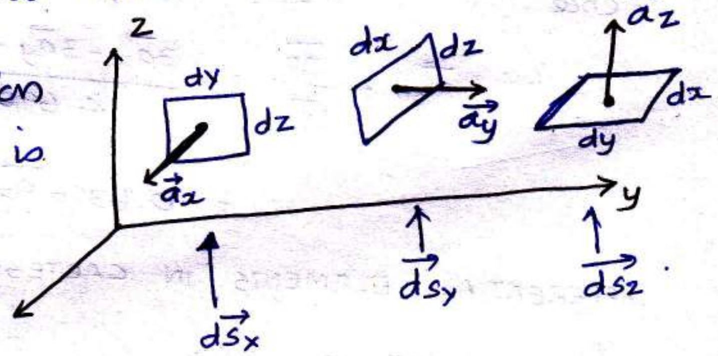
Let us define differential surface areas, the differential surface element $\vec{ds}$ is represented as

$$\vec{ds} = ds\,\vec{a}_n$$

where ds = Differential surface area of the element.

$\vec{a}_n$ = unit vector normal to surface ds.

The vector representation of these three elements is given as,



$$\vec{ds}_x = dy\,dz\,\vec{a}_x$$
$$\vec{ds}_y = dx\,dz\,\vec{a}_y$$
$$\vec{ds}_z = dy\,dx\,\vec{a}_z$$

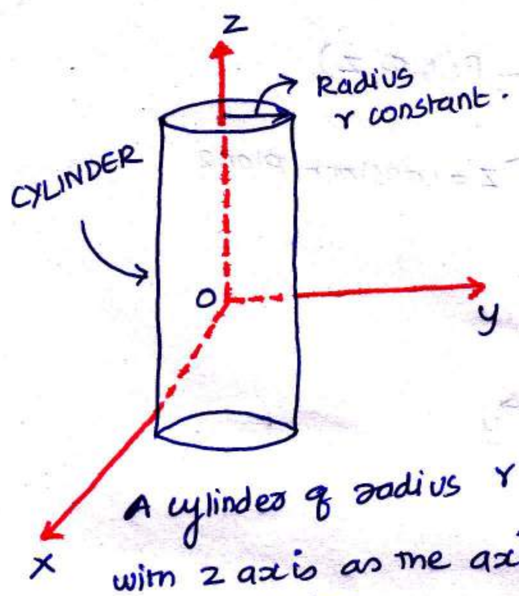
CYLINDRICAL CO-ORDINATE SYSTEM :-

In this system of co-ordinates, any point in a space is considered as the point of intersection of the following surfaces.

1. Plane of constant z which is $\parallel^{el}$ to xy plane
2. A cylinder of radius r with z axis as the axis of the cylinder
3. A half plane $\perp^r$ to xy plane and at an angle ϕ w.r.to xz plane. The angle ϕ is called azimuthal angle

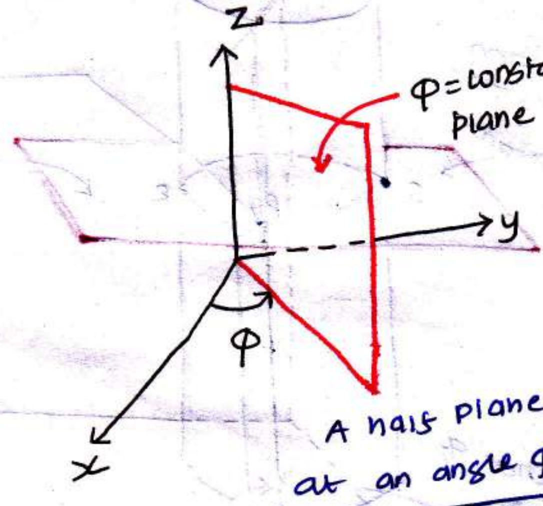
The range of variables are

$$\left.\begin{aligned} 0 \le r \le \infty \\ 0 \le \varphi \le 2\pi \\ -\infty \le z \le \infty \end{aligned}\right\} \quad -(1)$$



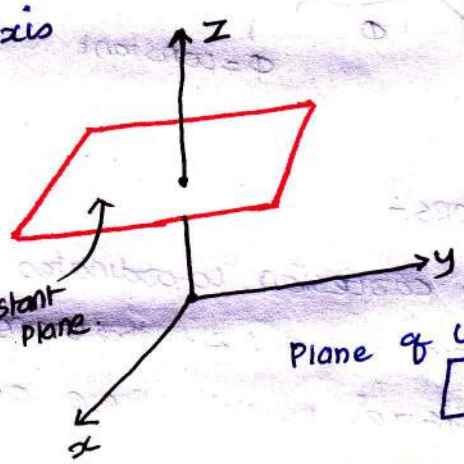
A cylinder of radius r with z axis as the axis of the cylinder

$\boxed{\text{fig 1}}$



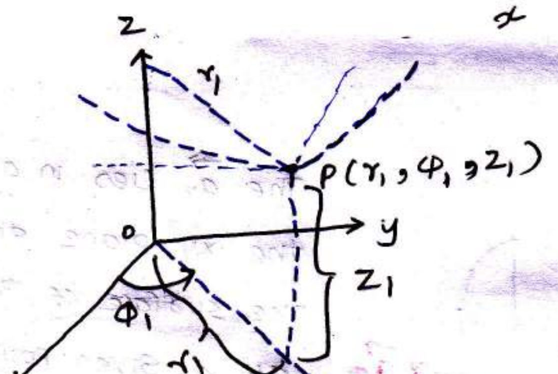
A half plane $\perp^r$ to xy plane at an angle φ w.r.to xz plane

$\boxed{\text{fig 2}}$



Plane of constant z

$\boxed{\text{fig 3}}$

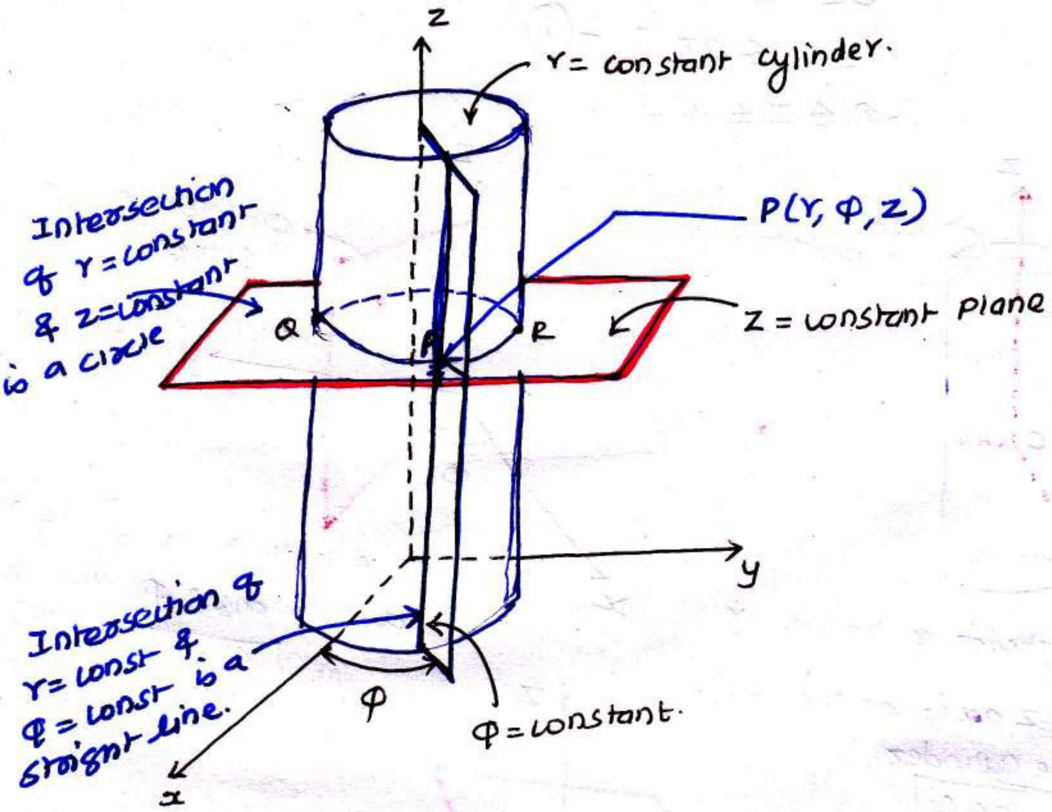


$\boxed{\text{fig 4}}$

The point P in cylindrical co-ordinate system has three co-ordinates r, φ and z whose values lie in the respective ranges as given in (1).

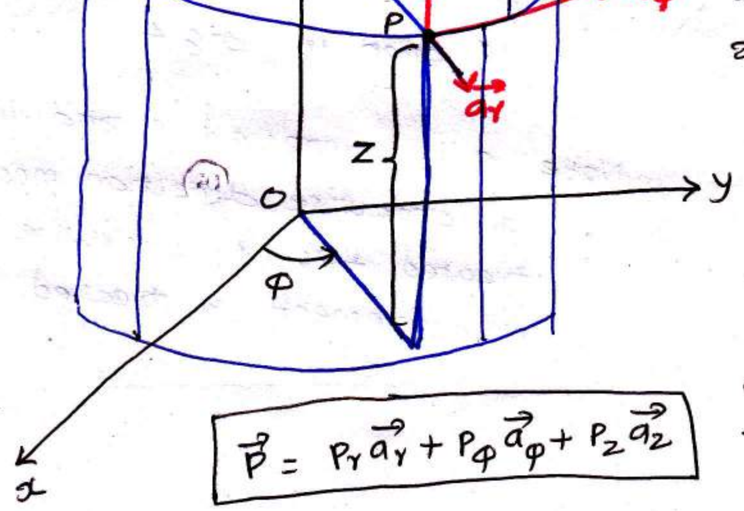
The point $P(r_1, \varphi_1, z_1)$ can be seen in fig 4

Note: φ is expressed in radians and for φ, anticlockwise direction measurement is treated +ve & clockwise direction measurement is treated −ve.



BASE VECTORS:-

$\|\|^r$ to cartesian co-ordinates system, there are three unit vectors in the r, φ and z directions denoted as $\vec{a}_r, \vec{a}_\varphi$ and $\vec{a}_z$ as shown in figure below. These are mutually $\perp^r$ to each other.



$$\boxed{\vec{P} = P_r\vec{a}_r + P_\varphi\vec{a}_\varphi + P_z\vec{a}_z}$$

The $\vec{a}_r$ lies in a plane $\|^{el}$ to the xy plane and is $\perp^r$ to the surface of the cylinder at a given point comming radially outward.

The unit vector $\vec{a}_\varphi$ also lies in a plane $\|^{el}$ to xy plane but it is tangent to the cylinder, pointing in the direction of increasing φ, at the given point.

The unit vector $\vec{a}_z$ is $\|^{el}$ to z axis and directed towards increasing 'z'.

$$\vec{P} = P_r \vec{a_r} + P_\phi \vec{a_\phi} + P_z \vec{a_z}$$

where P_r is radius r

P_ϕ is angle ϕ

P_z is co-ordinate of point P.

Key note:

In the cartesian co-ordinate system, the unit vectors are not dependent on the co-ordinates. But in cylindrical co-ordinate system $\vec{a_r}$ and $\vec{a_\phi}$ are functions of ϕ co-ordinate as their direction changes as ϕ changes. Hence the differentiation or integration w.r.to ϕ components in $\vec{a_r}$ and $\vec{a_\phi}$ should not be treated as constants.

DIFFERENTIAL ELEMENTS IN CYLINDRICAL CO-ORDINATE SYSTEM:

Consider a point $P(r, \phi, z)$ in a cylindrical co-ordinate system. Let each co-ordinate is increased by the differential amount. The differential increments in r, ϕ, z are $dr, d\phi$ and dz respectively.

→ Now there are two cylinders of radius r and $r+dr$

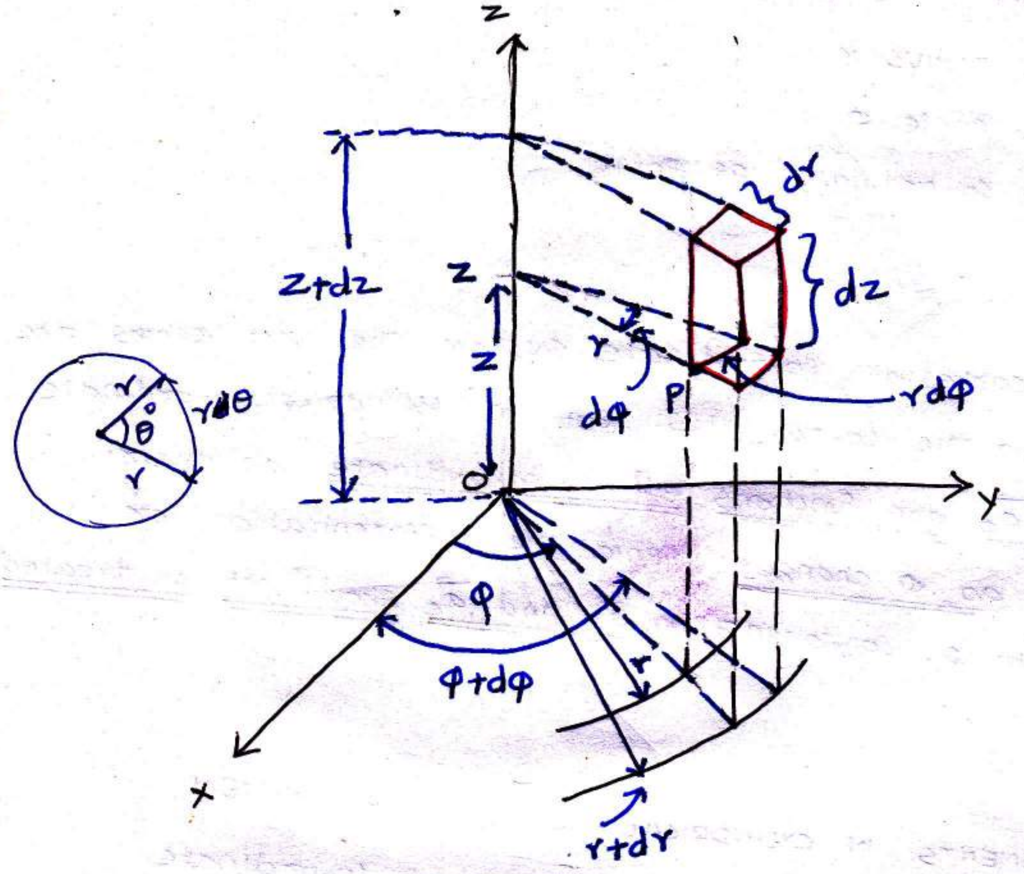
→ There are two radial planes at the angles ϕ and $\phi+d\phi$

→ Two Horizontal Planes at the heights z and $z+dz$.

→ Differential lengths in r and z directions are dr and dz respectively.

→ In ϕ direction, $d\phi$ is the change in angle ϕ and is not the differential length.

→ Due to this change $d\phi$, there exists a differential arc length in ϕ direction. This differential length, due to $d\phi$, in ϕ direction is $r\,d\phi$ as shown in fig.



Hence the differential vector length in cylindrical co-ordinate system is given by,

$$\boxed{\vec{dl} = dr\,\vec{a_r} + r\,d\varphi\,\vec{a_\varphi} + dz\,\vec{a_z}}$$

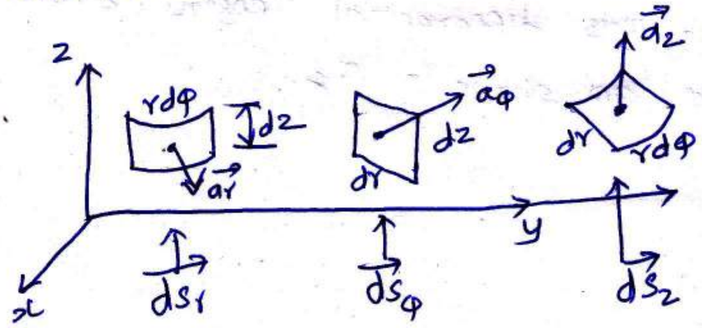
The magnitude of the differential length vector is given by,

$$\boxed{|\vec{dl}| = \sqrt{(\vec{dr})^2 + (r\,d\varphi)^2 + (dz)^2}}$$

The differential volume of the differential element formed is given by,

$$\boxed{dv = dr \times r\,d\varphi \times dz}$$

The differential surface areas in the three directions are shown in fig below.

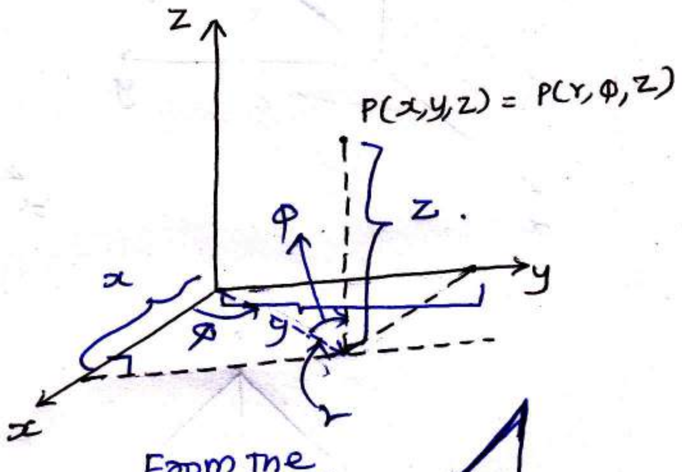


$$\vec{ds_r} = r\,d\varphi\,dz\,\vec{a_r}$$
$$\vec{ds_\varphi} = dr\,dz\,\vec{a_\varphi}$$
$$\vec{ds_z} = r\,d\varphi\,dr\,\vec{a_z}$$

RELATIONSHIP B/W CARTESIAN & CYLINDRICAL SYSTEMS:

Consider a point 'p' whose cartesian co-ordinates are x, y and z while the cylindrical co-ordinates are r, ϕ and z as shown in fig below.

$$\boxed{\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \\ z &= z \end{aligned}}$$

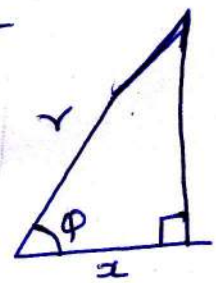


It can be seen that, r can be expressed interms of x and y as.

$$r = \sqrt{x^2 + y^2}$$

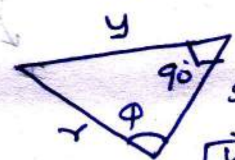
while $\boxed{\tan\phi = \frac{y}{x}}$

From the planes



$\cos\phi = \frac{x}{r}$

$\boxed{x = r\cos\phi}$



$\sin\phi = \frac{y}{r}$

$\boxed{y = r\sin\phi}$

Thus the transformation from cartesian to cylindrical can be obtained from the equations.

$$\boxed{r = \sqrt{x^2+y^2}, \quad \phi = \tan^{-1}\frac{y}{x} \quad \text{and} \quad z = z}$$

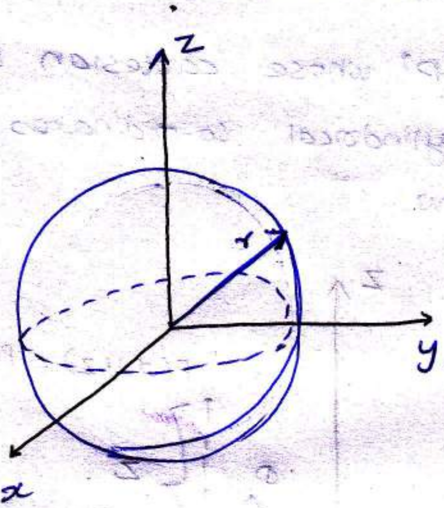
SPHERICAL CO-ORDINATE SYSTEM:

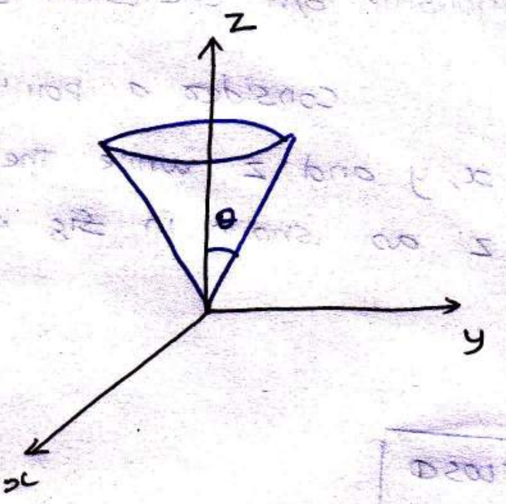
The surfaces which are used to define the spherical co-ordinate system on the three cartesian axes are

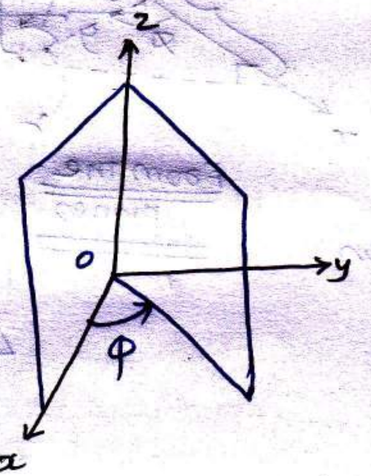
(i) sphere of radius r, origin as the centre of the sphere

(ii) A right circular cone with its apex at the origin and its axis as z axis. Its half angle is θ. It rotates about z axis and θ varies from $0°$ to $180°$

(iii) A half plane $\perp r$ to xy plane containing z axis, making an angle ϕ with the xz plane.





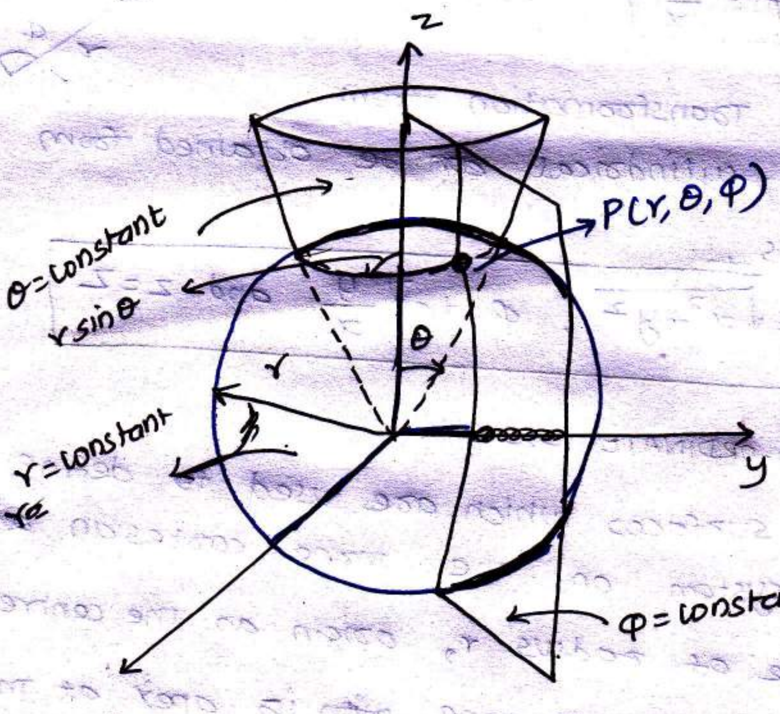


The range of the variables are

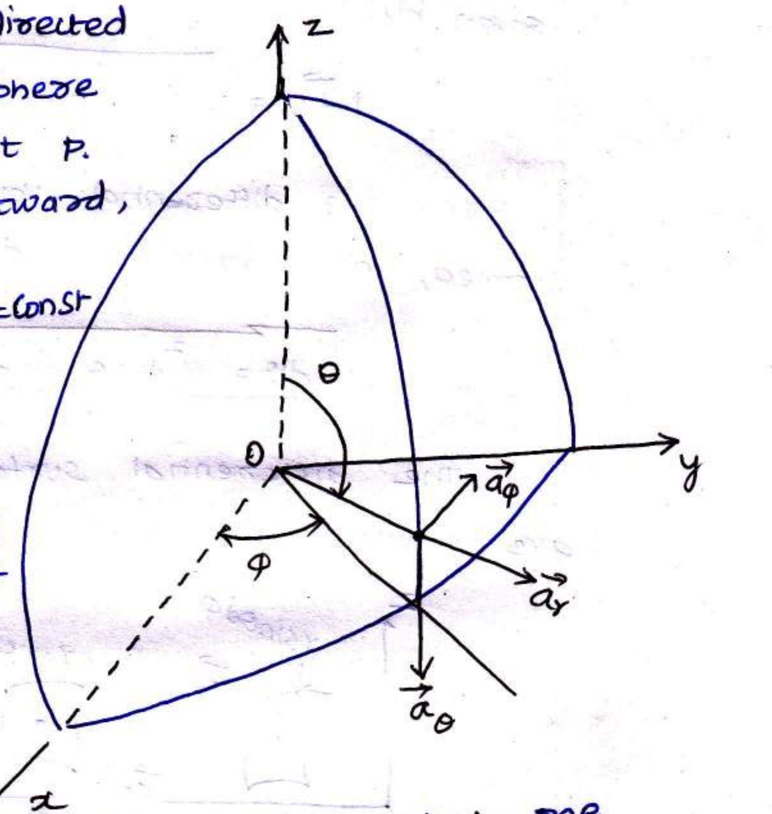
$0 \leq r \leq \infty$

$0 \leq \phi \leq 2\pi$

$0 \leq \theta \leq \pi$ as 1/2 angle.



BASE VECTORS :



→ The unit vector $\vec{a}_r$ is directed from the centre of the sphere i.e. origin to the given point P. It is directed radially outward, normal to the sphere.

It lies in the cone $\theta = const$ and plane $\phi = constant$.

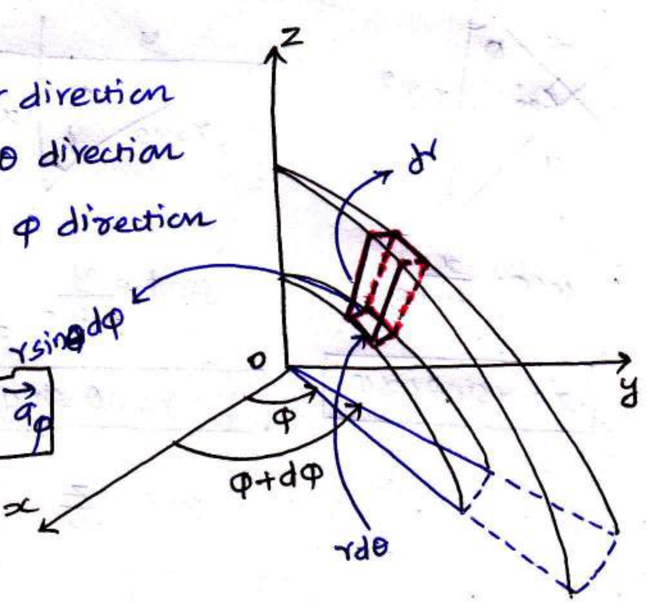
→ The unit vector $\vec{a}_\theta$ is tangent to the sphere and oriented in the direction of increasing θ. It is normal to the conical surface

→ The 3rd unit vector $\vec{a}_\phi$ is tangent to the sphere and also tangent to the conical surface. It is oriented in the direction of increasing ϕ. It is same as defined in the cylindrical co-ordinate system.

Here the vector of point 'p' can be represented as

$$\boxed{\vec{P} = P_r \vec{a}_r + P_\theta \vec{a}_\theta + P_\phi \vec{a}_\phi}$$

DIFFERENTIAL ELEMENTS IN SPHERICAL CO-ORDINATE SYSTEMS :



dr = Differential length in r direction

$rd\theta$ = Differential length in θ direction

$r\sin\theta d\phi$ = Differential length in ϕ direction

∴ differential vector length

$$\boxed{\vec{dl} = dr\vec{a}_r + rd\theta\,\vec{a}_\theta + r\sin\theta\, d\phi\,\vec{a}_\phi}$$

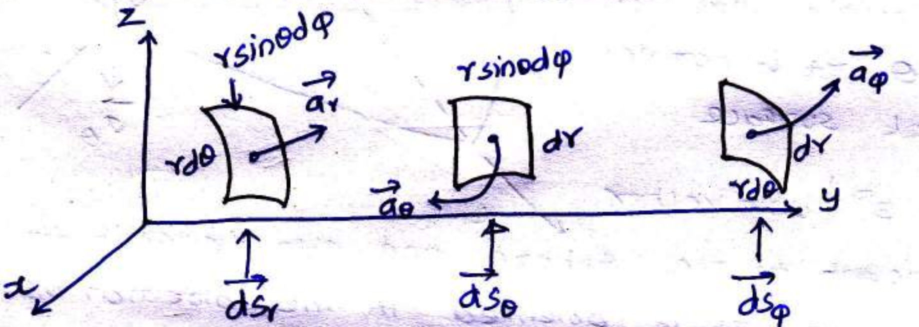
~~Hence~~ The magnitude of the differential length vector is given by,

$$|\vec{dl}| = \sqrt{(dr)^2 + (r d\theta)^2 + (r\sin\theta\, d\phi)^2}$$

Hence the differential volume of the differential element formed, in spherical co-ordinate system is given by,

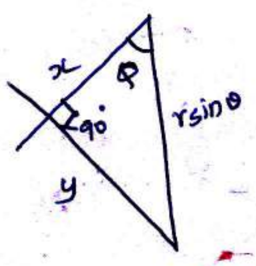
$$\boxed{dv = r^2 \sin\theta\, dr\, d\theta\, d\phi}$$

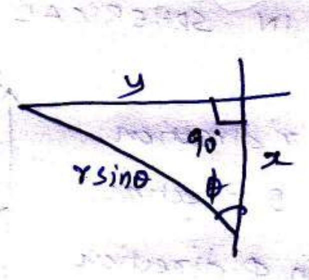
The differential surface areas in the three directions are shown in fig below.

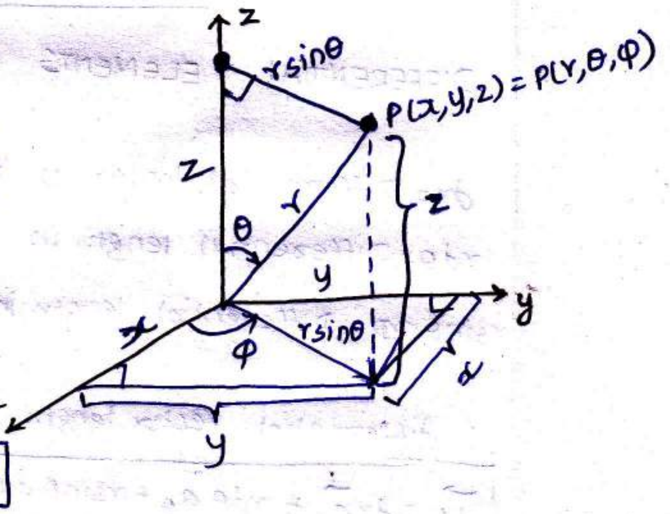


$$\vec{ds_r} = r^2 \sin\theta\, d\theta\, d\phi \qquad \vec{ds_\theta} = r\sin\theta\, dr\, d\phi \qquad \vec{ds_\phi} = r\, dr\, d\theta .$$

RELATIONSHIP B/W CARTESIAN & SPHERICAL SYSTEMS :-







$$\cos\phi = \frac{x}{r\sin\theta} \qquad \sin\phi = \frac{y}{r\sin\theta}$$

$$\boxed{x = r\sin\theta\cos\phi} \qquad \boxed{y = r\sin\theta\sin\phi}$$

$$\cos\theta = \frac{z}{r} \qquad \boxed{z = r\cos\theta}$$

Hence the transformation from spherical to cartesian can be obtained from equations,

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta . \end{aligned}}$$

Now r can be expressed as,

$$x^2+y^2+z^2 = r^2\sin^2\theta\cos^2\varphi + r^2\sin^2\theta\sin^2\varphi + r^2\cos^2\varphi$$

$$= r^2\sin^2\theta\left[\cos^2\varphi+\sin^2\varphi\right] + r^2\cos^2\varphi$$

$$= r^2\left[\sin^2\theta+\cos^2\theta\right]$$

$$= r^2$$

$$\boxed{r = \sqrt{x^2+y^2+z^2}}$$

While $\tan\varphi = \frac{y}{z}$ and $\cos\theta = \frac{z}{r}$.

As r is known, θ can be obtained.

Thus the Transformation from cartesian to spherical co-ordinates system can be obtained from the following equations.

$$r = \sqrt{x^2+y^2+z^2}, \quad \theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right] \text{ and } \varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

TRANSFORMATION OF VECTORS:

TRANSFORMATION OF VECTORS FROM CARTESIAN TO CYLINDRICAL

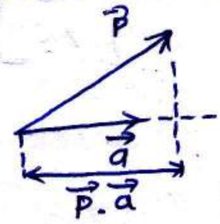
Consider a vector $\vec{A}$ in cartesian co-ordinate system as,

$$\vec{A} = A_x\vec{a_x} + A_y\vec{a_y} + A_z\vec{a_z}$$

While the same vector in cylindrical co-ordinate system can be represented as

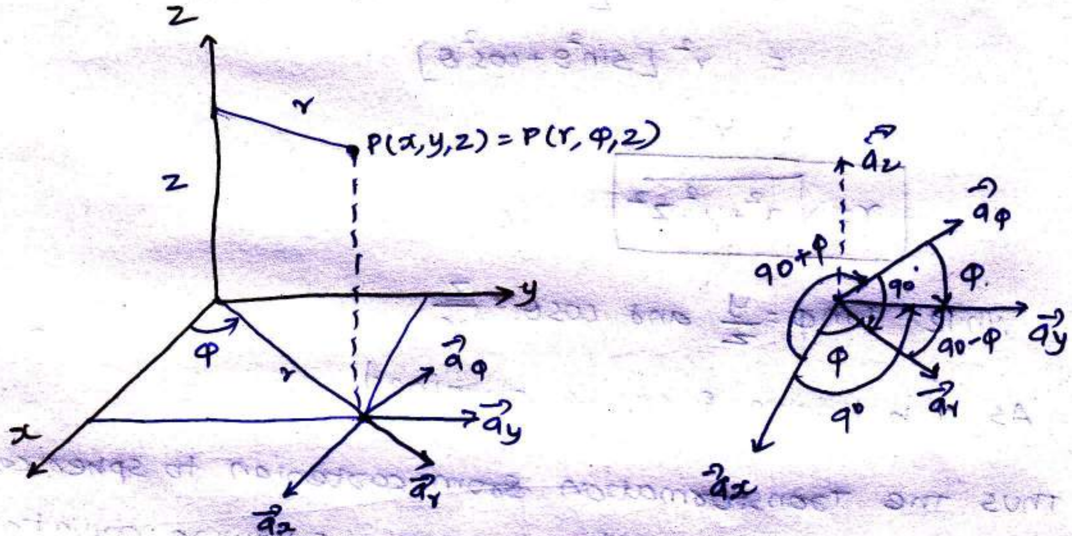
$$\vec{A} = A_r\vec{a_r} + A_\varphi\vec{a_\varphi} + A_z\vec{a_z}$$

From the dot product it is known that the component of vector in the direction of unit vector is it dot product with that unit vector.



Hence the component of $\vec{A}$ in the direction of $\vec{a_r}$ is the dot product of $\vec{A}$ with $\vec{a_r}$. This component is nothing but A_r.

$$\therefore A_r = \vec{A} \cdot \vec{a_r}$$
$$= [A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}] \cdot \vec{a_r}$$
$$= A_x \vec{a_x} \cdot \vec{a_r} + A_y \vec{a_y} \cdot \vec{a_r} + A_z \vec{a_z} \cdot \vec{a_r}$$



$$\vec{a_x} \cdot \vec{a_r} = (1)(1) \cos\varphi = \cos\varphi.$$
$$\vec{a_x} \cdot \vec{a_\varphi} = (1)(1) \cos(90+\varphi) = -\sin\varphi$$
$$\vec{a_y} \cdot \vec{a_r} = (1)(1) \cos(90-\varphi) = \sin\varphi$$
$$\vec{a_y} \cdot \vec{a_\varphi} = (1)(1) \cos\varphi = \cos\varphi.$$

$$\vec{a_z} \cdot \vec{a_r} = \vec{a_z} \cdot \vec{a_\varphi} = 0$$

$$\vec{a_z} \cdot \vec{a_z} = 1 \Rightarrow A_r = A_x \cos\varphi + A_y \sin\varphi.$$

IIIly
$$A_\varphi = \vec{A} \cdot \vec{a_\varphi} \Rightarrow \vec{A}_\varphi = -A_x \sin\varphi + A_y \cos\varphi$$
$$\vec{A}_z = \vec{A} \cdot \vec{a_z} = A_z \Rightarrow A_z = A_z$$

$$\therefore \begin{bmatrix} A_r \\ A_\varphi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

TRANSFORMATION OF VECTORS FROM CYLINDRICAL TO CARTESIAN

Now it is necessary to find the transformation from cylindrical to cartesian. Hence we assume $\vec{A}$ is known in cylindrical system.

Thus component of $\vec{A}$ in a_x direction is given by

$$\vec{A}_x = \vec{A} \cdot \vec{a}_x = [A_r \vec{a}_r + A_\phi \vec{a}_\phi + A_z \vec{a}_z] \cdot \vec{a}_x$$

$$= A_r \vec{a}_r \cdot \vec{a}_x + A_\phi \vec{a}_\phi \cdot \vec{a}_x + A_z \vec{a}_z \cdot \vec{a}_x$$

As dot product is commutative

$$\vec{a}_r \cdot \vec{a}_x = \vec{a}_x \cdot \vec{a}_r = \cos\phi$$

Hence

$$\vec{A}_x = A_r \cos\phi + (-\sin\phi) A_\phi$$

$$\vec{A}_x = A_r \cos\phi - A_\phi \sin\phi$$

|||ly

$$\vec{A}_y = \vec{A} \cdot \vec{a}_y = \sin\phi \, A_r + \cos\phi \, A_\phi$$

$$A_z = A_z$$

$$\therefore \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix}$$

TRANSFORMATION OF VECTORS FROM CARTESIAN TO SPHERICAL

Let

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$

$$A_r = \vec{A} \cdot \vec{a}_r = [A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z] \cdot \vec{a}_r$$

$$A_r = A_x \vec{a}_x \cdot \vec{a}_r + A_y \vec{a}_y \cdot \vec{a}_r + A_z \vec{a}_z \cdot \vec{a}_r$$

$$A_\theta = \vec{A} \cdot \vec{a}_\theta = [A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z] \cdot \vec{a}_\theta$$

$$A_\theta = A_x \vec{a}_x \cdot \vec{a}_\theta + A_y \vec{a}_y \cdot \vec{a}_\theta + A_z \vec{a}_z \cdot \vec{a}_\theta$$

$$A_\phi = \vec{A}\cdot\vec{a}_\phi = \left[A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z\right]\cdot\vec{a}_\phi$$

$$= A_x\,\vec{a}_x\cdot\vec{a}_\phi + A_y\,\vec{a}_y\cdot\vec{a}_\phi + A_z\,\vec{a}_z\cdot\vec{a}_\phi$$

The dot product of ~~eg~~ spherical unit vectors are given below.

$\cdot$	$\vec{a}_r$	$\vec{a}_\theta$	$\vec{a}_\phi$
$\vec{a}_x$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\vec{a}_y$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\vec{a}_z$	$\cos\theta$	$-\sin\theta$	0

$$\therefore \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

TRANSFORMATION OF VECTORS FROM SPHERICAL TO CARTESIAN

$$\vec{A} = A_r\vec{a}_r + A_\theta\vec{a}_\theta + A_\phi\vec{a}_\phi$$

$$A_x = \vec{A}\cdot\vec{a}_x = A_r\,\vec{a}_r\cdot\vec{a}_x + A_\theta\,\vec{a}_\theta\cdot\vec{a}_x + A_\phi\,\vec{a}_\phi\cdot\vec{a}_x$$

$$A_y = \vec{A}\cdot\vec{a}_y = A_r\,\vec{a}_r\cdot\vec{a}_y + A_\theta\,\vec{a}_\theta\cdot\vec{a}_y + A_\phi\,\vec{a}_\phi\cdot\vec{a}_y$$

$$A_z = \vec{A}\cdot\vec{a}_z = A_r\,\vec{a}_r\cdot\vec{a}_z + A_\theta\,\vec{a}_\theta\cdot\vec{a}_z + A_\phi\,\vec{a}_\phi\cdot\vec{a}_z$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\phi & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

DISTANCE OF ALL CO-ORDINATE SYSTEMS

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} \quad \text{- cartesian}$$

$$d = \sqrt{r_2^2 + r_1^2 - 2r_1r_2\cos(\phi_2-\phi_1) + (z_2-z_1)^2} \quad \text{- cylindrical}$$

$$d = \sqrt{r_2^2 + r_1^2 - 2r_1r_2\cos\theta_2\cos\theta_1 - 2r_1r_2\sin\theta_2\sin\theta_1\cos(\phi_2-\phi_1)}$$

- spherical.

TRANSFORMATION OF VECTORS FROM SPHERICAL TO CYLINDRICAL

Let

$$\vec{A} = A_r \vec{a}_r + A_\theta \vec{a}_\theta + A_\phi \vec{a}_\phi$$

$$A_\rho = A_r \vec{a}_r \cdot \vec{a}_\rho + A_\theta \vec{a}_\theta \cdot \vec{a}_\rho + A_\phi \vec{a}_\phi \cdot \vec{a}_\rho$$

$$A_\phi = A_r \vec{a}_r \cdot \vec{a}_\phi + A_\theta \vec{a}_\theta \cdot \vec{a}_\phi + A_\phi \vec{a}_\phi \cdot \vec{a}_\phi$$

$$A_z = A_r \vec{a}_r \cdot \vec{a}_z + A_\theta \vec{a}_\theta \cdot \vec{a}_z + A_\phi \vec{a}_\phi \cdot \vec{a}_z$$

$\vec{a}_r \cdot \vec{a}_\rho = \sin\theta \qquad \vec{a}_\theta \cdot \vec{a}_\rho = \cos\theta \qquad \vec{a}_\phi \cdot \vec{a}_\rho = 0$

$\vec{a}_r \cdot \vec{a}_\phi = 0 \qquad \vec{a}_\theta \cdot \vec{a}_\phi = 0 \qquad \vec{a}_\phi \cdot \vec{a}_\phi = 1$

$\vec{a}_r \cdot \vec{a}_z = \cos\theta \qquad \vec{a}_\theta \cdot \vec{a}_z = -\sin\theta \qquad \vec{a}_\phi \cdot \vec{a}_z = 0.$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

\# TRANSFORMATION OF VECTORS FROM CYLINDRICAL TO SPHERICAL

Let

$$\vec{A} = A_\rho \vec{a}_\rho + A_\phi \vec{a}_\phi + A_z \vec{a}_z$$

$$A_r = A_\rho \vec{a}_\rho \cdot \vec{a}_r + A_\phi \vec{a}_\phi \cdot \vec{a}_r + A_z \vec{a}_z \cdot \vec{a}_r$$

$$A_\theta = A_\rho \vec{a}_\rho \cdot \vec{a}_\theta + A_\phi \vec{a}_\phi \cdot \vec{a}_\theta + A_z \vec{a}_z \cdot \vec{a}_\theta$$

$$A_\phi = A_\rho \vec{a}_\rho \cdot \vec{a}_\phi + A_\phi \vec{a}_\phi \cdot \vec{a}_\phi + A_z \vec{a}_z \cdot \vec{a}_\phi$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

TYPES OF INTEGRAL RELATED TO ELECTROMAGNETIC THEORY

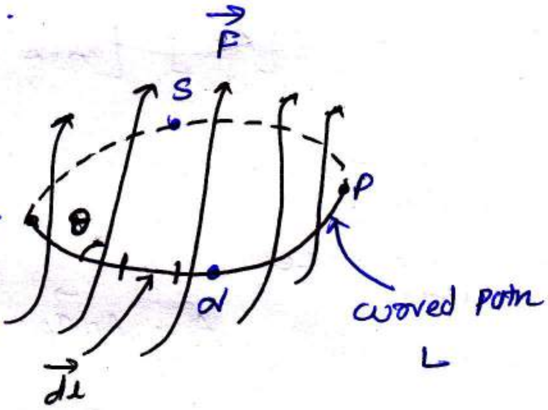
In electromagnetic theory a charge can exist in Point form, line form, surface form ~~and~~ or volume form.

Hence for charge distribution analysis, the following types of integrals are required.

1. Line integral
2. surface integral
3. Volume integral.

LINE INTEGRAL

- A line can exist as a straight line or it can be distance travelled along a curve
- ~~The~~ From mathematical point of view, a line is a curved path in a space.



Consider a vector field $\vec{F}$ shown in fig. The curved path shown in the field is P-r. This is called Path of integration and corresponding integral can be defined as.

$$\int_L \vec{F}.\vec{dl} = \int_P^r |F|\, dl \cos\theta \quad \text{[using dot product definition]}.$$

where

$dl \rightarrow$ Elementary length.

This is called line integral of $\vec{F}$ around the curved path L ~~P-r~~.

The curved path can be of two types.

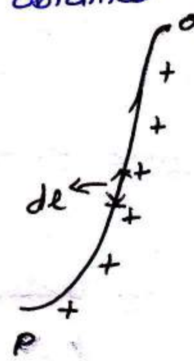
(i) open Path as P-r shown in fig

(ii) closed path as P-q-r-s-P.

The closed path is also called contour. The corresponding integral is called contour integral, closed integral (or) circular integral, and mathematicalled defined as.

$$\oint_L \vec{F}.\vec{dl} = \text{circular integral.}$$

If there exists a charge along a ~~straight~~ line as shown in figure, then the total charge is obtained by calculating a line integral.



$$Q = \int_L \rho_L \cdot dl$$

ρ_L = Line charge density (or) charge per unit length (C/m).

SURFACE INTEGRAL:

In electromagnetic theory a charge may exist in a distributed form. It may be spreaded over a surface as shown in figure(a) below.

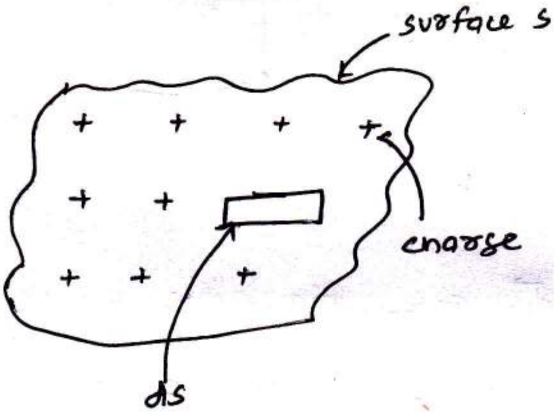
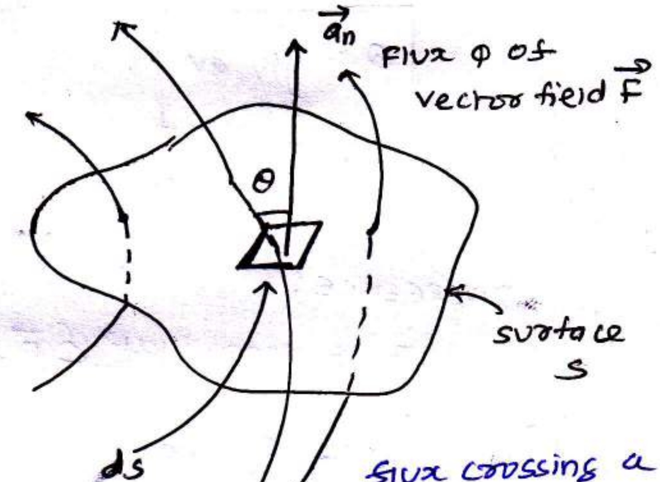


fig (a)



flux crossing a surface.

fig (b)

similarly a flux φ may pass through a surface as shown in fig(b). While doing analysis of such cases an integral is required called <u>SURFACE INTEGRAL</u>, to be carried out over a surface related to a vector field.

For a charge distribution shown in fig (a), we can write total charge existing on the surface as

$$Q = \int_S \rho_s \, ds. \quad \rho_s \to \text{surface charge density in } C/m^2$$

$ds \to$ Elementary surface area.

IIIly from fig (b), the total flux crossing the surface S can be expressed as

$$\varphi = \int_S \vec{F} \cdot \vec{ds} = \int_S |\vec{F}| \, ds \cos\theta$$

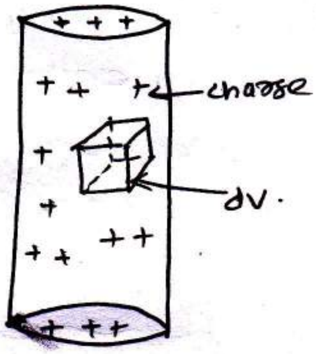
If the surface is closed, then it defines a volume and corresponding surface integral is given by,

$$\varphi = \oint_S \vec{F} \cdot \vec{ds}.$$

VOLUME INTEGRAL

If the charge distribution exists in a three dimensional volume form as shown in figure below, then a volume integral is required to calculate the total charge.

Thus if ρ_v is the volume charge density over volume V. then the volume integral is defined as



$$\boxed{Q = \int_V \rho_v \cdot dv}$$

dv = Elementary volume.

DIVERGENCE:

It is seen that $\oint_S \vec{F} \cdot \vec{ds}$ gives the flux flowing across the surface S. Then mathematically divergence is defined as the net outward flow of the flux per unit volume over a closed incremental surface. It is denoted as $\text{div}\,\vec{F}$ and given by

$$\text{div}\,\vec{F} = \lim_{\Delta v \to 0} \frac{\oint \vec{F} \cdot \vec{ds}}{\Delta v} = \text{Divergence of } \vec{F}.$$

symbolically it is denoted as

$$\nabla \cdot \vec{F} = \text{Divergence of } \vec{F}$$

where ∇ = vector operator $= \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}$

But $\vec{F} = F_x \vec{a_x} + F_y \vec{a_y} + F_z \vec{a_z}$

$$\therefore \boxed{\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x}\cancel{a_x} + \frac{\partial F_y}{\partial y}\cancel{a_y} + \frac{\partial F_z}{\partial z}\cancel{a_z}}$$ → In cartesian form

III^{rly} Divergence in other co-ordinates.

$$\nabla \cdot \vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(r F_r) + \frac{1}{r}\frac{\partial F_\varphi}{\partial \varphi} + \frac{\partial F_z}{\partial z} \quad \text{cylindrical}$$

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial \varphi} \quad \text{spherical.}$$

(X) The vector field having its divergence zero is called solenoidal field

$$\boxed{\nabla \cdot \vec{A} = 0 \text{ for } \vec{A} \text{ to be solenoidal}}$$

GRADIENT OF A SCALAR:-

consider that in space let W be the unique function of x, y and z co-ordinates. in the cartesian system. This is the scalar function and denoted as $W(x,y,z)$. consider a vector operator in cartesian system denoted as ∇ (called del). It is defined as.

$$\nabla(\text{del}) = \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}$$

The operation of the vector operator del(∇) on a scalar function is called gradient of a scalar.

$$\text{Grad } W = \nabla W = \left(\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right) W$$

$$\boxed{\nabla W = \frac{\partial W}{\partial x}\vec{a_x} + \frac{\partial W}{\partial y}\vec{a_y} + \frac{\partial W}{\partial z}\vec{a_z}} \quad \text{cartesian.}$$

In cylindrical co-ordinates.

$$\nabla W = \frac{\partial W}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial W}{\partial \varphi}\vec{a_\varphi} + \frac{\partial W}{\partial z}\vec{a_z}$$

spherical co-ordinates

$$\nabla W = \frac{\partial W}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial W}{\partial \theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial W}{\partial \varphi}\vec{a_\varphi}$$

CURL OF A VECTOR:

$\nabla \times \vec{F}$ = curl of $\vec{F}$.

curl indicates the rotational property of vector field.

If curl of vector is zero, the vector field is irrotational.

$$\nabla \times \vec{F} = \left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right]\vec{a_x} + \left[\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right]\vec{a_y} + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]\vec{a_z}$$

$$\nabla \times \vec{F} = \begin{bmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{bmatrix}$$ CARTESIAN

$$\nabla \times \vec{F} = \begin{bmatrix} \vec{a_r} & r\vec{a_\phi} & \vec{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_r & F_\phi & F_z \end{bmatrix}$$ CYLINDRICAL

$$\nabla \times \vec{F} = \begin{bmatrix} \vec{a_r} & r\vec{a_\theta} & r\sin\theta\,\vec{a_\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & rF_\theta & r\sin\theta\,F_\phi \end{bmatrix}$$ SPHERICAL

ELECTROSTATICS:

→ Electrostatics is a science related to the electric charges which are static i.e are at rest.

→ An Electric charge has an effect in a region or a space around it. This region is called an electric field of that charge.

→ such an Electric field Produced due to stationary electric charge doesnot vary with time. It is time invarient and called static electric field.

COULOMB'S LAW:

→ study of electrostatics started from french army engineer, colonel charles coulomb.

→ The experiments are related to the force exerted b/w the two point charges, which are placed near each other.

→ The force exerted is due to the electric fields produced by the point charges.

POINT CHARGE:

- A Point charge means that the electric charge which is spreaded on a surface or space whose geometrical dimensions are very very small compared to the other dimensions, in which the effect of its electric field is to be studied. Thus a point charge has a location but not the dimensions.

- A charge can be +ve or -ve
- A charge is actually the defficiency or excess of electrons in the atoms of a particle.
- An electron Passesses a -ve charge. so the deficiency of an electron produces +ve charge while excess of an electron produces -ve charge.

- The charge is measured in COULOMBS (C).

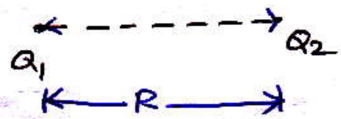
~~- one smallest possible charge~~

STATEMENT OF COULOMB's LAW:

The coulomb's law states that force b/w the two point charges Q_1 and Q_2

1. Acts along the line joining the two point charges.
2. Is directly proportional to the product (Q_1, Q_2) of the two charges
3. Is inversely proportional to the square of distance b/w them.

— consider two point charges Q_1 & Q_2 seperated by distance R.



→ The charge Q_1 exerts a force on Q_2 while Q_2 exerts a force on Q_1.

→ The force acts along the line joining Q_1 and Q_2

→ The force exerted b/w them is repulsive if the charges are of same polarity, while it is attractive if charges are of different polarity.

Force F b/w two charges is expressed as

$$F \propto \frac{Q_1 Q_2}{R^2}$$

The coulomb's law also states that this force depends on the medium in which the point charge are located. This effect is included as constant of proportionality

$$\therefore \; F = K \frac{Q_1 Q_2}{R^2}$$

$K = \frac{1}{4\pi\epsilon}$ $\quad$ ϵ → Permittivity of the medium in which charges are located.

units of ϵ Farads/meter (F/m).

In general ϵ is expressed as

$$\epsilon = \epsilon_0 \epsilon_r$$

$\epsilon_0 \rightarrow$ Permittivity of free space or Vaccum.

$\epsilon_r \rightarrow$ Relative Permittivity (or) dielectric constant of the medium w.r.to free space.

$\epsilon \rightarrow$ Absolute permittivity

For the free space or vaccum, $\epsilon_r = 1$

$$\therefore \epsilon = \epsilon_0$$

$$\therefore F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R^2}$$

$$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} = 8.854 \times 10^{-12} \text{ F/m.}$$

$$k = \frac{1}{4\pi\epsilon_0} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 8.98 \times 10^{9} \simeq 9 \times 10^{9} \text{ m/F.}$$

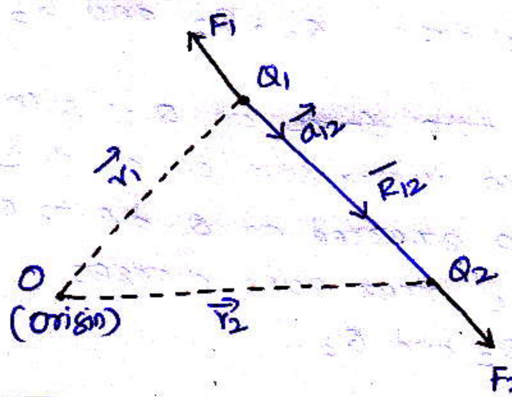
$$\boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}}$$

$\leftarrow$ This is the force b/w two Point charges located in free space or Vaccum.

VECTOR FORM OF COULOMB'S LAW:

The force exerted b/w The two Point charges has a fixed direction which is a straight line joining the two charges. Hence the force exerted b/w the two charges can be expressed in a vector form.

Consider two point charges Q_1 and Q_2 located at the points having position vectors $\vec{r_1}$ and $\vec{r_2}$ as shown in fig below



Then the force exerted by Q_1 on Q_2 acts along the direction $\vec{R}_{12}$ where $\vec{a}_{12}$ is unit vector along $\vec{R}_{12}$. Hence the force in the vector form can be expressed as.

$$\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \vec{a}_{12}$$

$$\vec{a}_{12} = \text{unit vector along } \vec{R}_{12} = \frac{\text{Vector}}{\text{Magnitude of vector}}$$

$$\vec{a}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{R}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}.$$

where $|\vec{R}_{12}|$ is the distance b/w the two charges.

|||ly the force f_1 is exerted on Q_1 due to Q_2. It can be expressed as

$$\vec{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0^2 R_{21}^2} \vec{a}_{21}$$

$$\vec{a}_{21} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} \quad \text{But} \quad \vec{r}_1 - \vec{r}_2 = -[\vec{r}_2 - \vec{r}_1]$$

$$\vec{a}_{21} = -\vec{a}_{12}$$

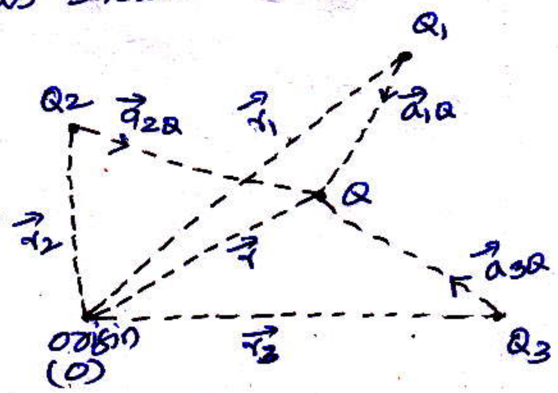
$$\therefore \vec{F}_1 = -\vec{F}_2$$

PRINCIPLE OF SUPERPOSITION :

If there are more than two point charges, then each will exert on the other, then the net force on any charge can be obtained by the principle of super position.

Consider a point charge Q surrounded by three other point charges Q_1, Q_2 and Q_3 as shown in figure below.

The total force on Q in such a case is vector sum of all the forces exerted on Q due to each of the other point charges Q_1, Q_2 and Q_3.



consider force exerted on Q due to Q_1. At this time, according to principle of superposition effects of Q_2 & Q_3 are to be suppressed.

$$\vec{F}_{Q1Q} = \frac{Q_1 Q}{4\pi\epsilon_0 R_{1Q}^2}\vec{a}_{1Q} \quad \text{where } \vec{a}_{1Q} = \frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|}$$

$||^{ly}$ force exerted due to Q_2 on Q is

$$\vec{F}_{Q2Q} = \frac{Q_2 Q}{4\pi\epsilon_0 R_{2Q}^2}\vec{a}_{2Q} \quad \text{where } \vec{a}_{2Q} = \frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|}$$

and force exerted due to Q_3 on Q is

$$\vec{F}_{Q3Q} = \frac{Q_3 Q}{4\pi\epsilon_0 R_{3Q}^2}\vec{a}_{3Q} \quad \text{where } \vec{a}_{3Q} = \frac{\vec{r}-\vec{r}_3}{|\vec{r}-\vec{r}_3|}.$$

Hence the total force on Q is

$$\vec{F}_Q = \vec{F}_{Q1Q} + \vec{F}_{Q2Q} + \vec{F}_{Q3Q}$$

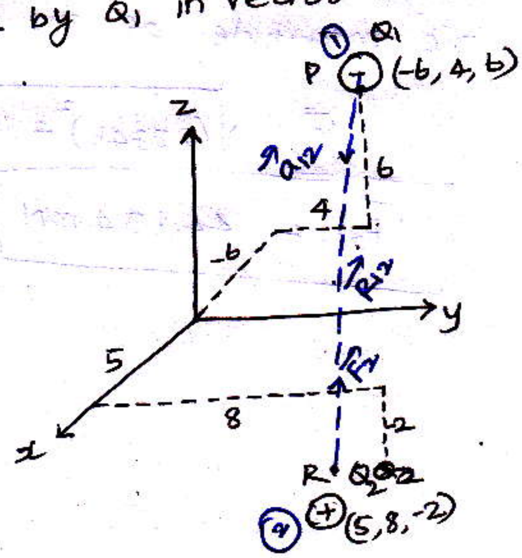
In general is there are n other charges then force exerted on Q due to all other charges is,

$$\vec{F}_Q = \vec{F}_{Q1Q} + \vec{F}_{Q2Q} + \cdots + \vec{F}_{QnQ}$$

$$\boxed{\vec{F}_Q = \frac{Q}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{Q_i}{R_{iQ}^2}\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|}}$$

Problems:

A charge $Q_1 = -20\mu C$ is located at P(-6, 4, 6) and a charge $Q_2 = 50\mu C$ is located at R(5, 8, -2) in a free space. Find the force exerted on Q_2 by Q_1 in vector form. The distance given are in meters.



From the co-ordinates of P and R, the respective position vectors are

$$\vec{P} = -6\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z \ (Q_1)$$

and $$\vec{R} = 5\vec{a}_x + 8\vec{a}_y - 2\vec{a}_z \ (Q_2)$$

The force on Q_2 is given by,

$$\vec{F}_{Q_2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \vec{a}_{12}$$

$$\vec{R}_{12} = \vec{R}_{PR} = \vec{R} - \vec{P} = 11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z$$

$$|\vec{R}_{12}| = \sqrt{11^2 + 4^2 + 8^2} = 14.1774$$

$$\therefore \ \vec{a}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z}{14.1774}$$

$$\boxed{\vec{a}_{12} = 0.7758\vec{a}_x + 0.2821\vec{a}_y - 0.5642\vec{a}_z}$$

$$\therefore \ \vec{F}_2 = \frac{-20\times10^{-6} \times 50\times10^{-6}}{4\pi\times8.854\times10^{-12} (14.1774)^2} [\vec{a}_{12}]$$

$$= -0.0447[0.7758\vec{a}_x + 0.2821\vec{a}_y - 0.5642\vec{a}_z]$$

$$\boxed{\vec{F}_2 = -0.0346\vec{a}_x - 0.01261\vec{a}_y + 0.02522\vec{a}_z \ N}$$

This is the required force exerted on Q_2 by Q_1.

The magnitude of the force is,

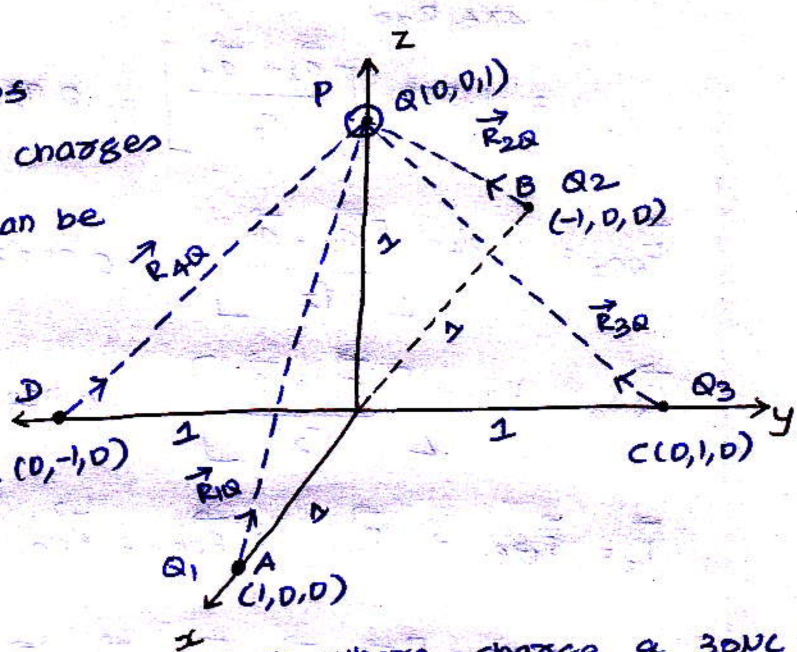
$$|\vec{F}_2| = \sqrt{(0.0346)^2 + (0.01261)^2 + (-0.02522)^2}$$

$$\boxed{|\vec{F}_2| = 44.634 \ mN}$$

Problem 2:

Four point charges each of 10 nC are placed in free space at the points (1,0,0), (-1,0,0), (0,1,0) and (0,-1,0) m respectively. Determine the force on a point charge of 30 nC located at a point (0,0,1) m.

The position vectors of 4 points at which the charges Q_1 to Q_4 are located can be obtained as,



$\vec{A} = \vec{a_x}$

$B = -\vec{a_x}$

$C = \vec{a_y}$

$D = -\vec{a_y}$

While the position vector of point P where charge q 30 nC is situated is

$\vec{P} = \vec{a_z}$

Consider force on Q due to Q_1 is

$$\vec{F}_{QQ_1} = \frac{Q Q_1}{4\pi\epsilon_0 R_{1Q}^2} \vec{a}_{1Q}$$

$$\vec{R}_{1Q} = \vec{R}_{AP} = \vec{P} - \vec{A} = \vec{a_z} - (\vec{a_x}) = \vec{a_z} - \vec{a_x}$$

$$|\vec{R}_{1Q}| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\therefore \vec{F}_{QQ_1} = \frac{10\times10^{-6} \times 30\times10^{-6}}{4\pi 8.854\times10^{-12}\times(\sqrt{2})^2} \frac{[\vec{a_z} - \vec{a_x}]}{\sqrt{2}}$$

$$\boxed{\vec{F}_{QQ_1} = 0.9533(\vec{a_z} - \vec{a_x})}$$

Due to symmetry from the figure

$$|\vec{R}_{2Q}| = |\vec{R}_{3Q}| = |\vec{R}_{4Q}| = |\vec{R}_{1Q}| = \sqrt{2}.$$

$$\vec{R}_{2Q} = \vec{P} - \vec{B} = \vec{a_z} + \vec{a_x} \qquad \therefore \vec{a}_{2Q} = \frac{\vec{a_z} + \vec{a_x}}{\sqrt{2}}$$

$$\vec{R}_{3Q} = \vec{P} - \vec{C} = \vec{a}_z - \vec{a}_y \qquad \vec{a}_{3Q} = \frac{\vec{a}_z - \vec{a}_y}{\sqrt{2}}$$

$$\vec{R}_{4Q} = \vec{P} - \vec{D} = \vec{a}_z + \vec{a}_y \qquad \vec{a}_{4Q} = \frac{\vec{a}_z + \vec{a}_y}{\sqrt{2}}.$$

$$\vec{F}_{QQ_2} = \frac{Q\,Q_2}{4\pi\epsilon_0 R_{2Q}^2}\,\vec{a}_{2Q}$$

$$= 1.3481\left[\frac{\vec{a}_z + \vec{a}_x}{\sqrt{2}}\right] = 0.9533\,(\vec{a}_z + \vec{a}_x)$$

$$\vec{F}_{QQ_3} = 1.3481\left[\frac{\vec{a}_z - \vec{a}_y}{\sqrt{2}}\right] = 0.9533\,(\vec{a}_z - \vec{a}_y)$$

$$\vec{F}_{QQ_4} = 1.3481\left[\frac{\vec{a}_z + \vec{a}_y}{\sqrt{2}}\right] = 0.9533\,(\vec{a}_z + \vec{a}_y)$$

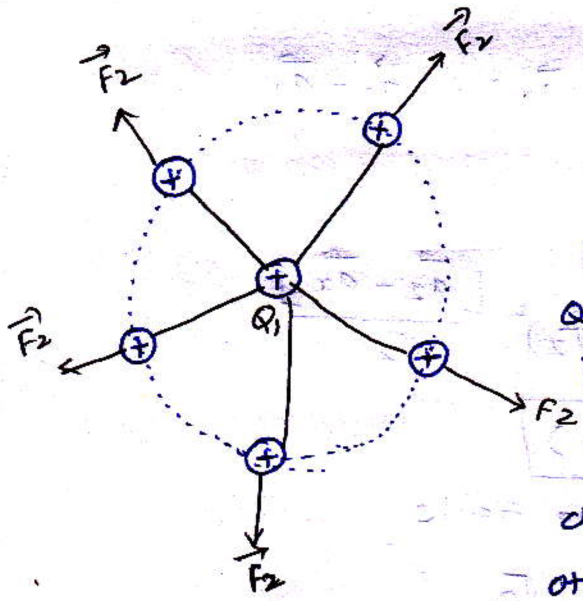
$$\vec{F}_t = \vec{F}_{QQ_1} + \vec{F}_{QQ_2} + \vec{F}_{QQ_3} + \vec{F}_{QQ_4}$$

$$= 0.9533\left[\vec{a}_z - \vec{a}_x + \vec{a}_z + \vec{a}_x + \vec{a}_z - \vec{a}_y + \vec{a}_z + \vec{a}_y\right]$$

$$\boxed{\vec{F}_t = 3.813\,\vec{a}_z \text{ N}}$$

ELECTRIC FIELD INTENSITY:

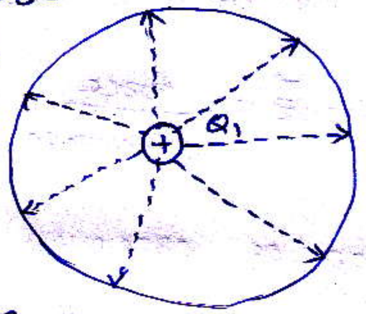
Consider a Point charge Q_1 as shown in fig.



If any other similar charge Q_2 is brought near it, Q_2 experiences a force. Infact if Q_2 is moved around Q_1, still Q_2 experiences a force as shown in fig.

Thus there exists a region around a charge in which it exerts a force on any other charge. This region where a particular charge exerts a force on any other charge located in that region is called ELECTRIC FIELD of that charge.

The electric field of Q_1 is shown in fig below.



The force experienced by the charge Q_2 due to Q_1 is given by coulomb's law as

$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \vec{a}_{12}$$

Thus force per unit charge can be written as

$$\frac{\vec{F_2}}{Q_2} = \frac{Q_1}{4\pi\epsilon_0 R_{12}^2} \vec{a}_{12}$$

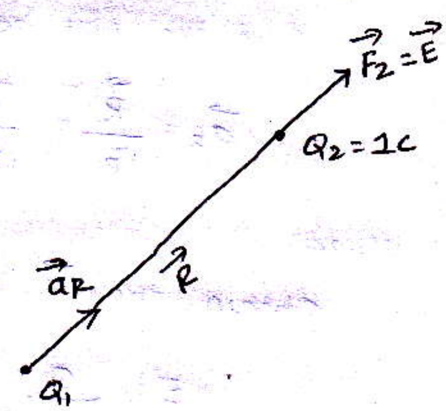
2marks

(This force exerted by unit charge is called ELECTRIC FIELD INTENSITY. (or) ELECTRIC STRENGTH. It is a vector quantity and is directed along a segment ~~from~~ the charge Q_1 to the position of any other charge. It is denoted by E.

$$\boxed{\vec{E} = \frac{Q_1}{4\pi\epsilon_0 R_{1P}^2} \vec{a}_{1P}}$$ where P = Position of any other charge around Q_1)

Consider a charge Q_1 as shown in figure below. The unit positive charge $Q_2 = 1C$ is placed at a distance of R from Q_1. Then the force acting on Q_2 due to Q_1 is along the unit vector $\vec{a}_R$. As the charge Q_2 is unit charge, the force exerted on Q_2 is nothing but electric field intensity $\vec{E}$ of Q_1 at the point where unit charge is placed.

$$\vec{E} = \frac{Q_1}{4\pi\epsilon_0 R^2} \vec{a}_R$$



$$E = \frac{Q_1}{4\pi\epsilon_0 r^2} \vec{a}_r$$ – spherical system.

$r \rightarrow$ radius of sphere 'r'.

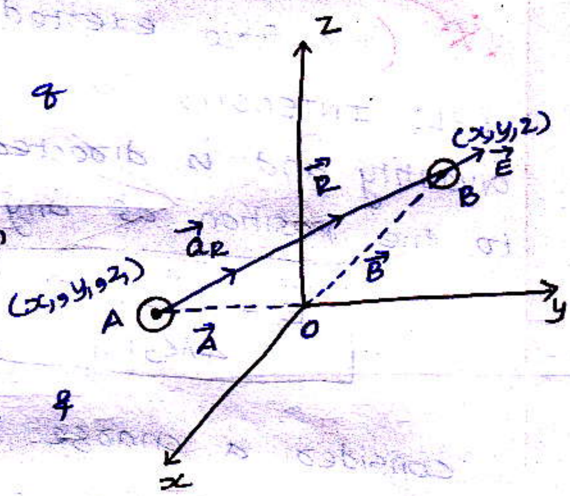
UNITS OF $\vec{E}$:

The definition of Electric field intensity is,

$$\vec{E} = \frac{\text{force}}{\text{unit charge}} = \frac{N \text{ (Newtons)}}{C \text{ (coulomb)}}.$$

METHOD OF OBTAINING $\vec{E}$ in CARTESIAN SYSTEM:

consider a charge Q_1 located at point $A(x_1, y_1, z_1)$ as shown in figure. It is required to obtain $\vec{E}$ at any point $B(x,y,z)$ in the cartesian system. then $\vec{E}$ at point B can be obtained using following steps:



STEP 1:

Obtain the position vectors of Points A & B

$\therefore \vec{r}_A = \vec{A}$ while $\vec{r}_B = \vec{B}$ from their co-ordinates.

$$\therefore \vec{A} = x_1 \vec{a}_x + y_1 \vec{a}_y + z_1 \vec{a}_z \quad \&$$

$$\vec{B} = x \vec{a}_x + y \vec{a}_y + z \vec{a}_z$$

STEP 2: Find the distance vector $\vec{R}$ directed from A to B.

$$\vec{R} = \vec{B} - \vec{A} = (x - x_1)\vec{a}_x + (y - y_1)\vec{a}_y + (z - z_1)\vec{a}_z$$

STEP 3:

Find the unit vector $\vec{a}_R$ along the direction from A to B.

$$\vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{\vec{B} - \vec{A}}{|\vec{B} - \vec{A}|}.$$

step 4:

obtain the $\vec{E}$ at the point B as,

$$E = \frac{Q}{4\pi\epsilon_0 R^2} \vec{a}_R = \frac{Q}{4\pi\epsilon_0 R^2} \frac{\bar{R}}{|\bar{R}|} \text{ V/m}.$$

where $R^2 = |\bar{R}|^2$ = Distance b/w the points A & B.

Step 5:

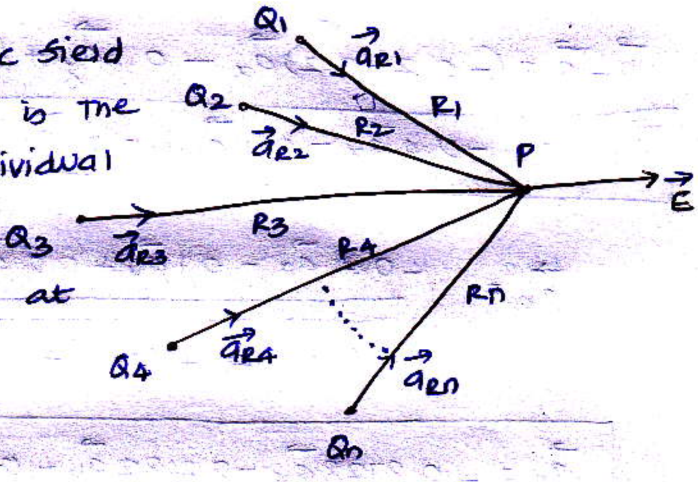
Magnitude of $\vec{E}$ is given by,

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0 R^2} \text{ V/m.}$$

ELECTRIC FIELD INTENSITY DUE TO DISCRETE CHARGES:

Consider 'n' charges $Q_1, Q_2, ----, Q_n$ as shown in figure given below. The combined electric field intensity is to be obtained at point P. The distances of point P from $Q_1, Q_2, ----, Q_n$ are $R_1, R_2, ----, R_n$ respectively. The unit vectors along these directions are $\vec{a}_{R1}, \vec{a}_{R2}, ----, \vec{a}_{Rn}$ respectively.

Then the total electric field intensity at point P is the vector sum of the individual field intensities produced by the various charges at the point P.



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + --- + \vec{E}_n$$

$$= \frac{Q_1}{4\pi\epsilon_0 R_1^2}\vec{a}_{R1} + \frac{Q_2}{4\pi\epsilon_0 R_2^2}\vec{a}_{R2} + ---- + \frac{Q_n}{4\pi\epsilon_0 R_n^2}\vec{a}_{Rn}$$

$$\boxed{\therefore \quad \vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{Q_i}{R_i^2}\vec{a}_{Ri}}$$

Each unit vector can be obtained by using the method discussed earlier

$$\vec{a}_{Ri} = \frac{\vec{r}_p - \vec{r}_i}{|\vec{r}_p - \vec{r}_i|}$$

$r_p \rightarrow$ Position vector of point P

$r_i \rightarrow$ Position vector of point where charge Q_i is placed.

Problems :-

1. Determine the electric field intensity at $P(-0.2, 0, -2.3)$ m due to a point charge of $+5nC$ at $Q(0.2, 0.1, -2.5)$ m in air.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\vec{a}_R$$

$$\vec{a}_R = \frac{\vec{R}_{QP}}{|\vec{R}_{QP}|} = \frac{\vec{P}-\vec{Q}}{|\vec{P}-\vec{Q}|}$$

$$\vec{P} = -0.2\vec{a}_x + 0\vec{a}_y - 2.3\vec{a}_z$$

$$\vec{Q} = 0.2\vec{a}_x + 0.1\vec{a}_y - 2.5\vec{a}_z$$

$$\vec{P}-\vec{Q} = (-0.2-0.2)\vec{a}_x - 0.1\vec{a}_y + (-2.3+2.5)\vec{a}_z$$

$$\boxed{\vec{R}_{QP} = -0.4\vec{a}_x - 0.1\vec{a}_y + 0.2\vec{a}_z}$$

$$\vec{a}_R = \frac{-0.4\vec{a}_x - 0.1\vec{a}_y + 0.2\vec{a}_z}{\sqrt{(-0.4)^2 + (0.1)^2 + (0.2)^2}} = \frac{-0.4\vec{a}_x - 0.1\vec{a}_y + 0.2\vec{a}_z}{0.45825}$$

$$\boxed{\vec{a}_R = -0.8728\vec{a}_x - 0.2182\vec{a}_y + 0.4364\vec{a}_z}$$

$$\therefore E = \frac{5\times10^{-9}}{4\pi\times8.854\times10^{-12}\times(0.45825)^2}\left[-0.8728\vec{a}_x - 0.2182\vec{a}_y + 0.4364\vec{a}_z\right]$$

$$\boxed{\vec{E} = -186.779\vec{a}_x - 46.694\vec{a}_y + 93.389\vec{a}_z \ \text{V/m}}$$

2. A charge of 1C is at $(2,0,0)$. What charge must be placed at $(-2,0,0)$ which will make y component of total $\vec{E}$ zero at the point $(1,2,2)$?

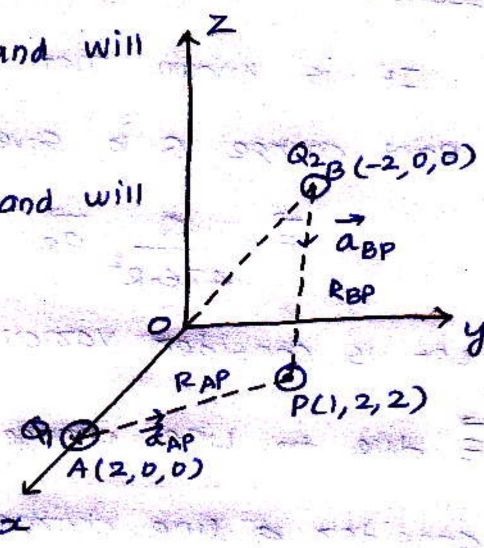
The position vectors of points A, B and P are

$$\vec{A} = 2\vec{a}_x \qquad \vec{P} = \vec{a}_x + 2\vec{a}_y + 2\vec{a}_z$$

$$\vec{B} = -2\vec{a}_y$$

$\vec{E}_A$ is field at P due to Q_1, and will act along $\vec{a}_{AP}$.

$\vec{E}_B$ is field at P due to Q_2 and will act along $\vec{a}_{BP}$



$$\therefore \vec{E}_A = \frac{Q_1}{4\pi\epsilon_0 R_{AP}^2}\vec{a}_{AP}$$

$$= \frac{Q_1}{4\pi\epsilon_0 R_{AP}^2}\left|\frac{\vec{P}-\vec{A}}{|\vec{P}-\vec{A}|}\right.$$

$$\vec{E}_B = \frac{Q_2}{4\pi\epsilon_0 R_{BP}^2}\vec{a}_{BP} = \frac{Q_2}{4\pi\epsilon_0 R_{BP}^2}\frac{\vec{P}-\vec{B}}{|\vec{P}-\vec{A}|}$$

$$\vec{E} \text{ at } P = \vec{E}_A + \vec{E}_B$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1}{R_{AP}^2}\frac{\vec{P}-\vec{A}}{|\vec{P}-\vec{A}|} + \frac{Q_2}{R_{BP}^2}\frac{\vec{P}-\vec{B}}{|\vec{P}-\vec{B}|}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{1[-\vec{a}_x+2\vec{a}_y+2\vec{a}_z]}{(\sqrt{9})^2\sqrt{9}} + \frac{Q_2(3\vec{a}_x+2\vec{a}_y+2\vec{a}_z)}{(\sqrt{17})^2(\sqrt{17})}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{-\vec{a}_x+2\vec{a}_y+2\vec{a}_z}{27} + \frac{Q_2[3\vec{a}_x+2\vec{a}_y+2\vec{a}_z]}{70.0927}\right]$$

The y component of $\vec{E}$ must be zero

$$\therefore \frac{2}{27} + \frac{2Q_2}{70.0927} = 0$$

$$Q_2 = -\frac{2}{27}\times\frac{70.0927}{2} = -2.596\,C$$

This is required charge Q_2 to be placed at (-2, 0, 0) which will make y component of $\vec{E}$ zero at point P.

ELECTRIC FIELD INTENSITY DUE TO VARIOUS CHARGE DISTRIBUTION

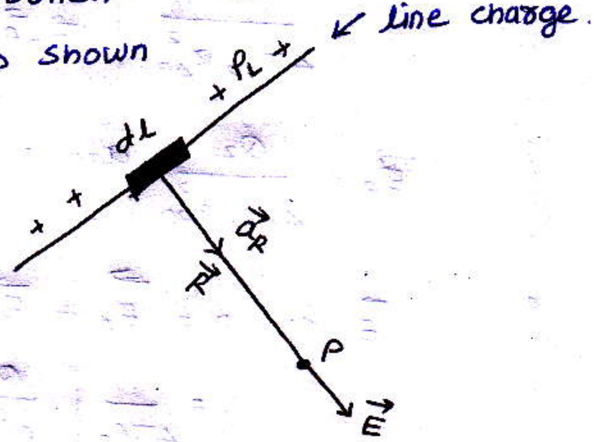
It is known that the electric field intensity due to a point charge Q is given by

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\vec{a_R}$$

Let us consider various charge distributions.

$\vec{E}$ due to line charge:

Consider a line charge distribution having a charge density ρ_L as shown in figure.



The charge dQ on the differential length dL is

$$\boxed{dQ = \rho_L \, dL}$$

Hence the differential electric field $\vec{dE}$ at point P due to dQ is given by.

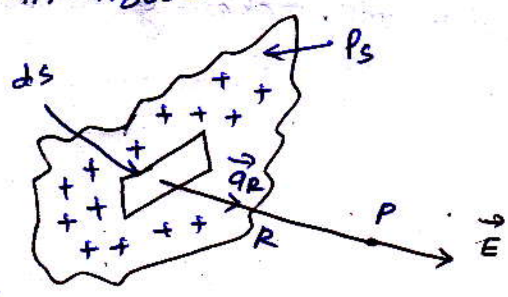
$$\vec{dE} = \frac{dQ}{4\pi\epsilon_0 R^2}\vec{a_R} = \frac{\rho_L \, dL}{4\pi\epsilon_0 R^2}\vec{a_R}$$

Hence the total $\vec{E}$ at a point P due to line charge can be obtained by integrating $\vec{dE}$ over the length of the charge

$$\boxed{\vec{E} = \int_L \frac{\rho_L \, dL}{4\pi\epsilon_0 R^2}\vec{a_R}}$$

$\vec{E}$ due to SURFACE CHARGE:

Consider a surface charge ~~density~~ distribution having a charge density ρ_s as shown in figure.



The charge dQ ~~do~~ on the differential surface area ds is

$$\boxed{dQ = \rho_s \, ds}$$

Hence the differential electric field $\vec{dE}$ at point P due to dQ is given by

$$\vec{dE} = \frac{dQ}{4\pi\epsilon_0 R^2}\vec{a_R} = \frac{\rho_s\,ds}{4\pi\epsilon_0 R^2}\vec{a_R}$$

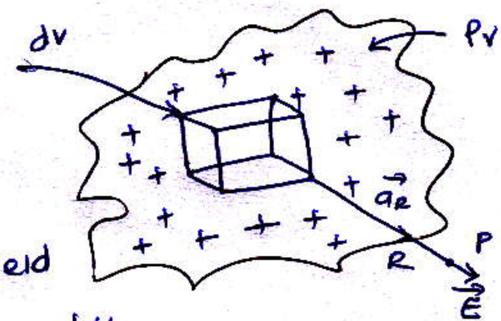
Hence the total $\vec{E}$ at a point P is to be obtained by integrating $\vec{dE}$ over the surface area on which charge is distributed. Note that this will be a double integration.

$$\boxed{\therefore\ \vec{E} = \int_S \frac{\rho_s\,ds}{4\pi\epsilon_0 R^2}\vec{a_R}}$$

The $\vec{a_R}$ and ds to be obtained according to the position of the sheet of charge and the co-ordinate system used.

$\vec{E}$ due to VOLUME CHARGE:

Consider a volume charge distribution having a charge density ρ_v as shown in figure.



The charge dQ on differential volume dv is

$$dQ = \rho_v\,dv$$

Hence the differential electric field $\vec{dE}$ at a point P due to dQ is given by

$$\vec{dE} = \frac{dQ}{4\pi\epsilon_0 R^2}\vec{a_R} = \frac{\rho_v\,dv}{4\pi\epsilon_0 R^2}\vec{a_R}$$

Hence the total $\vec{E}$ at a point P is to be obtained by integrating $\vec{dE}$ over the volume in which charge is accumulated. Note that this integration will be a triple integration.

$$\vec{E} = \int_{Vol} \frac{\rho_v\,dv}{4\pi\epsilon_0 R^2}\vec{a_R}$$

$\vec{a_R}$ & dv must be obtained according to the co-ordinate system used.