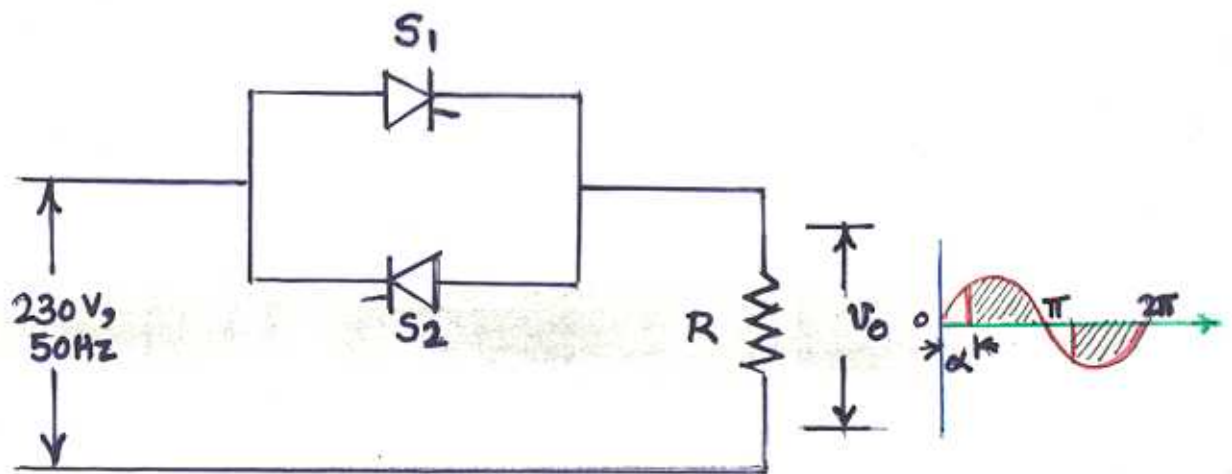


# Single-phase voltage controller



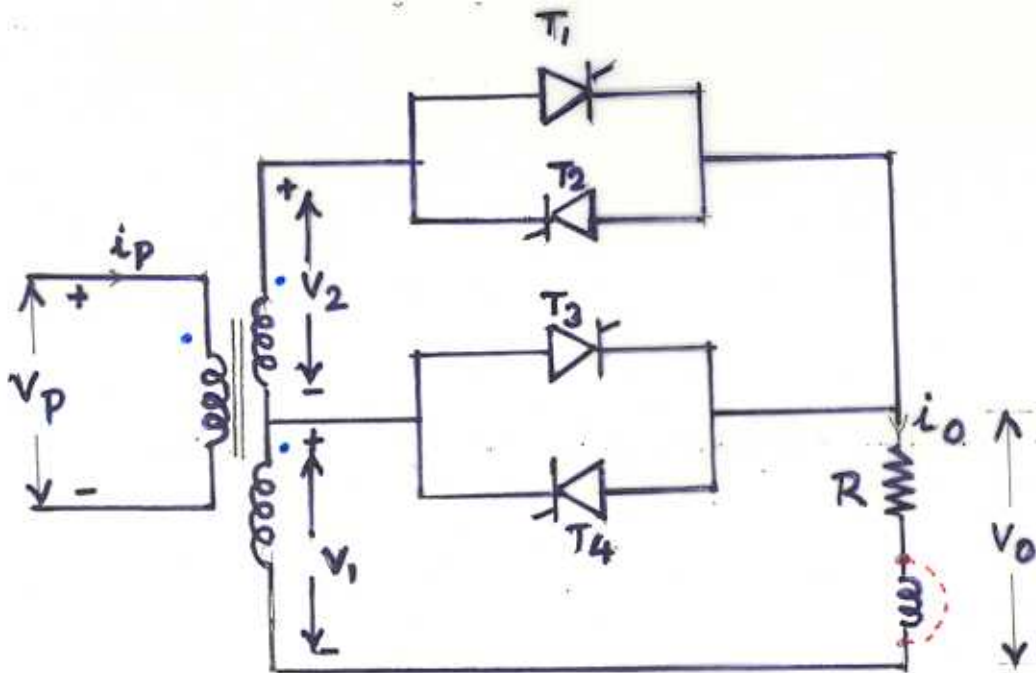
Expression for  $V_o$  (rms) :

$$\begin{aligned}
 V_o^2(\text{rms}) &= \frac{1}{\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \theta \, d\theta \\
 &= \frac{V_m^2}{\pi} \int_{\alpha}^{\pi} \left[ \frac{1 - \cos 2\theta}{2} \right] d\theta \\
 &= \frac{V_m^2}{\pi} \left[ \frac{1}{2} \theta - \frac{\sin 2\theta}{2 \times 2} \right]_{\alpha}^{\pi} \\
 &= \frac{V_m^2}{\pi} \left[ \frac{1}{2} (\pi - \alpha) - \frac{\sin 2\pi}{4} + \frac{\sin 2\alpha}{4} \right] \\
 &= \frac{V_m^2}{2\pi} \left[ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \right] \\
 &= \frac{V_m^2}{2} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right]
 \end{aligned}$$

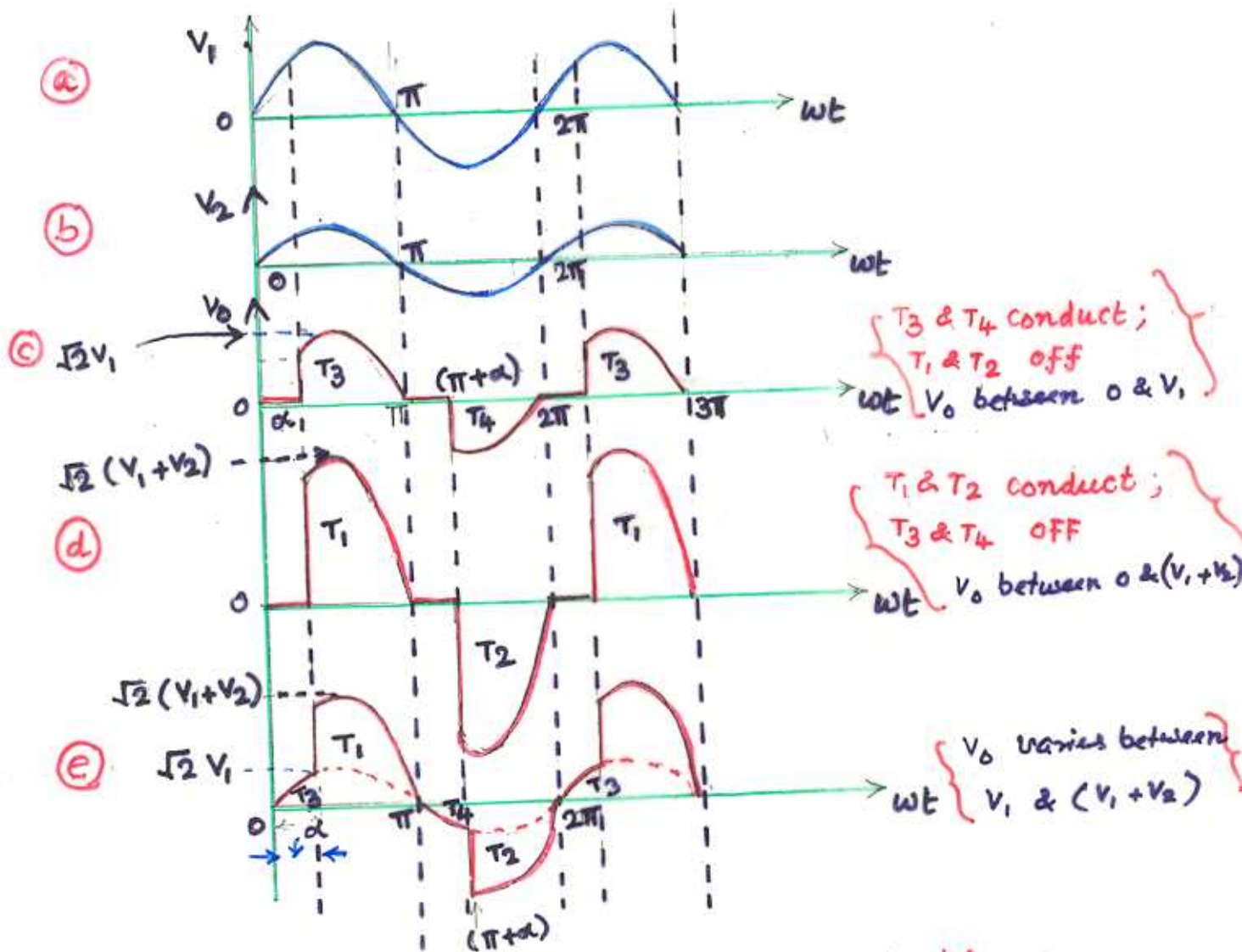
$$V_o = V \left[ \frac{1}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2}$$

$$\begin{aligned}
 \therefore V_o(\text{rms}) &= \sqrt{\frac{V_m^2}{2} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right]} \\
 &= \frac{V_m}{\sqrt{2}} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right]^{1/2}
 \end{aligned}$$

i.e.,  $V_o(\text{rms}) = V \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right]^{1/2}$  where  $V$  is the rms line voltage



Power circuit



Solid state Transformer Tap changing - single-phase  
(For resistive load)

The secondary instantaneous voltages are:

$$\begin{aligned} v_1 &= \sqrt{2} V_1 \sin \omega t \\ \text{and } v_2 &= \sqrt{2} V_2 \sin \omega t \end{aligned} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} V_1 \text{ and } V_2 \text{ are the} \\ \text{RMS secondary voltages} \end{array}$$

A tap changer is most commonly used for resistive heating loads.

When only  $T_3$  &  $T_4$  are alternately fired with  $\alpha = 0^\circ$ :

$$v_o = V_1$$

When only  $T_1$  &  $T_2$  are alternately fired with  $\alpha = 0^\circ$ :

$$v_o = (V_1 + V_2)$$

When  $\alpha$  is varied, the ranges of output voltage:

(i)  $0 \leq v_o \leq V_1$  :

To vary the load voltage within this range,  $T_1$  &  $T_2$  are turned off.

$T_3$  &  $T_4$  can operate as a single-phase voltage controller. The instantaneous load voltage waveform is shown in Fig. c for a resistive load.

The rms load voltage which can be determined

is 
$$V_o = V_1 \left[ \frac{1}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \quad \text{and} \quad \dots (1)$$

the range of delay angle is  $0 \leq \alpha \leq \pi$

The rms current of  $T_3$  &  $T_4 = I_{R3} = \frac{V_1}{R} \left[ \frac{1}{\pi} \left( \alpha - \frac{\sin 2\alpha}{2} \right) \right]^{1/2} ?$

(ii)  $0 \leq V_o \leq (V_1 + V_2)$  :

$T_3$  &  $T_4$  are turned off.  $T_1$  &  $T_2$  operate as a single-phase voltage controller.

Fig. d shows the load voltage  $V_o$  (which is same as  $V_1$ ) for a resistive load.

The rms load voltage can be obtained as

$$V_o = (V_1 + V_2) \left[ \frac{1}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \dots (2)$$

$$\left. \begin{array}{l} \text{RMS current of} \\ T_1 \text{ \& } T_2 \end{array} \right\} = I_{R1} = \frac{V_1 + V_2}{R} \left[ \frac{1}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \dots (3)$$

(iii)  $V_1 \leq V_o \leq (V_1 + V_2)$  :

$T_3$  is turned on at  $\omega t = 0$  and the secondary voltage  $V_1$  appears across the load.

If  $T_1$  is turned on at  $\omega t = \alpha$ , it applies a reverse bias ( $V_2$ ) across  $T_3$  and hence  $T_3$  is turned off.

The voltage appearing across the load is

$$V_o = (V_1 + V_2).$$

At  $\omega t = \pi$ ,  $T_1$  is self commutated and  $T_4$  is turned on.

The secondary voltage  $V_1$  appears across the load until  $T_2$  is fired at  $\omega t = (\pi + \alpha)$ .

When  $T_2$  is turned on at  $\omega t = (\pi + \alpha)$ ,  $T_4$  is turned off due to reverse voltage  $V_2$  and the load voltage is

$$V_o = (V_1 + V_2).$$

At  $\omega t = 2\pi$ ,  $T_2$  is self commutated.  $T_3$  is turned on again and the cycle is repeated.

The instantaneous load voltage  $v_o$  (& hence  $i_o$ ) is shown in Fig. e for a resistive load.

A tap changer with this type of control is also known as a synchronous tap changer, which uses two step control.

A part of secondary voltage  $v_2$  is super-imposed on a sinusoidal voltage,  $v_1$ .

As a result, the harmonic contents are less than that would be obtained by a normal <sup>phase</sup> delay as discussed for control range 2.

The rms load voltage can be obtained from

$$\begin{aligned}
 V_o &= \left[ \frac{1}{2\pi} \int_0^{2\pi} v_o^2 d(\omega t) \right]^{1/2} \\
 &= \left\{ \frac{2}{2\pi} \left[ \int_0^\alpha 2V_1^2 \sin^2 \omega t d(\omega t) + \int_\alpha^\pi 2(V_1+V_2)^2 \sin^2 \omega t d(\omega t) \right] \right\}^{1/2} \\
 &= \left[ \frac{V_1^2}{\pi} \left( \alpha - \frac{\sin 2\alpha}{2} \right) + \frac{(V_1+V_2)^2}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \\
 &= \left[ \frac{V_1^2}{\pi} \left( \alpha - \frac{\sin 2\alpha}{2} \right) + \frac{(V_1+V_2)^2}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2}
 \end{aligned}$$

(OR)

$$V_o = \frac{1}{\pi} \left[ \int_0^\alpha \underbrace{(\sqrt{2} V_1 \sin \omega t)^2}_{V_1^2 \left( \alpha - \frac{\sin 2\alpha}{2} \right)} d\omega t + \int_\alpha^\pi \underbrace{\{ \sqrt{2} (V_1+V_2) \sin \omega t \}^2}_{(V_1+V_2)^2 \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right)} d\omega t \right]^{1/2}$$