UNIT-I
Principles of Electromechanical Energy Conversion

Topics to be cover

1) Introduction
2) EMF in Electromechanical Systems
3) Force and Torque on a Conductor
4) Force and Torque Calculation from Energy and Co energy
5) Model of Electromechanical Systems

Introduction

For energy conversion between electrical and mechanical forms, electromechanical devices are developed. In general, electromechanical energy conversion devices can be divided into three categories:

(1) ***Transducers (for measurement and control)***

These devices transform the signals of different forms. Examples are microphones, pickups, and speakers.

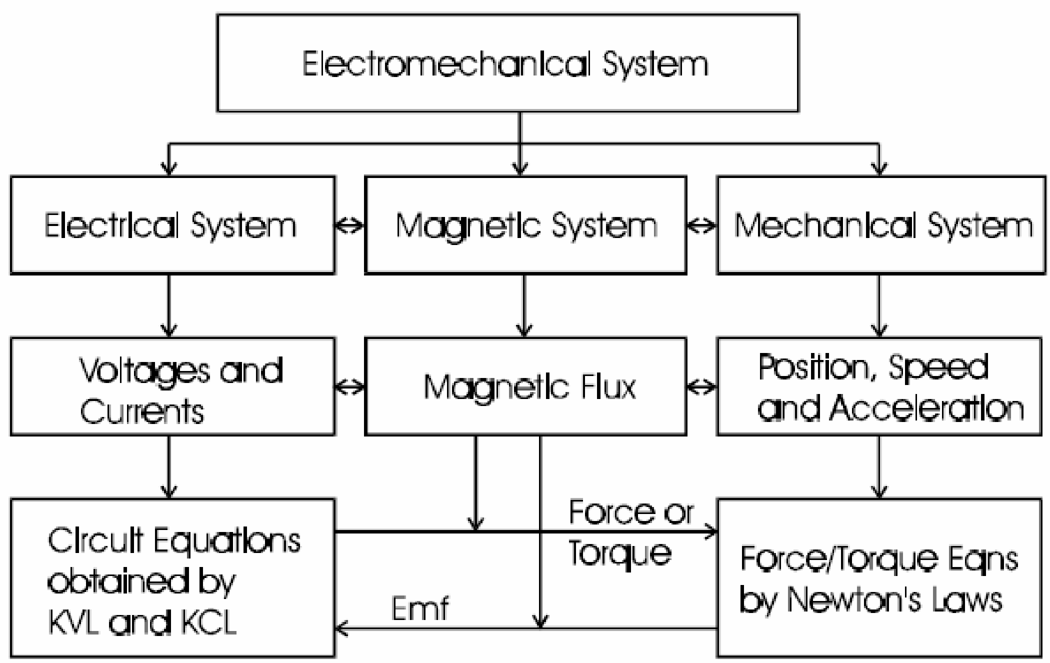
(2) ***Force producing devices (linear motion devices)***

These type of devices produce forces mostly for linear motion drives, such as relays, Solenoids (linear actuators), and electromagnets.

(3) ***Continuous energy conversion equipment***

These devices operate in rotating mode. A device would be known as a generator if it convert mechanical energy into electrical energy, or as a motor if it does the other way around (from electrical to mechanical).

Since the permeability of ferromagnetic materials is much larger than the permittivity of dielectric materials, it is more advantageous to use electromagnetic field as the medium for electromechanical energy conversion. As illustrated in the following diagram, an electromechanical system consists of an electrical subsystem (electric circuits such as windings), a magnetic subsystem (magnetic field in the magnetic cores and air gaps), and a mechanical subsystem (mechanically movable parts such as a plunger in a linear actuator and a rotor in a rotating electrical machine). Voltages and currents are used to describe the state of the electrical subsystem and they are governed by the basic circuital laws: Ohm's law, KCL and KVL. The state of the mechanical subsystem can be described in terms of positions, velocities, and accelerations, and is governed by the Newton's laws. The magnetic subsystem or magnetic field fits between the electrical and mechanical subsystems and acting as a "ferry" in energy transform and conversion. The field quantities such as magnetic flux, flux density, and field strength, are governed by the Maxwell's equations. When coupled with an electric circuit, the magnetic flux interacting with the current in the circuit would produce a force or torque on a mechanically movable part. On the other hand, the movement of the moving part will could variation of the magnetic flux linking the electric circuit and induces an electromotive force (*emf*) in the circuit. The product of the torque and speed (the mechanical power) equals the active component of the product of the *emf* and current. Therefore, the electrical energy and the mechanical energy are inter converted via the magnetic field.



Concept map of electromechanical system modeling

In this chapter, the methods for determining the induced *emf* in an electrical circuit and force/torque experienced by a movable part will be discussed. The general concept of electromechanical system modeling will also be illustrated by a singly excited rotating system.

Induced emf in Electromechanical Systems

The diagram below shows a conductor of length l placed in a uniform magnetic field of flux density **B**. When the conductor moves at a speed **v**, the induced *emf* in the conductor can be determined by

$$\mathbf{e} = l\mathbf{v} \times \mathbf{B}$$

The direction of the *emf* can be determined by the "right hand rule" for cross products. In a coil of N turns, the induced *emf* can be calculated by

$$e = -\frac{d\lambda}{dt}$$

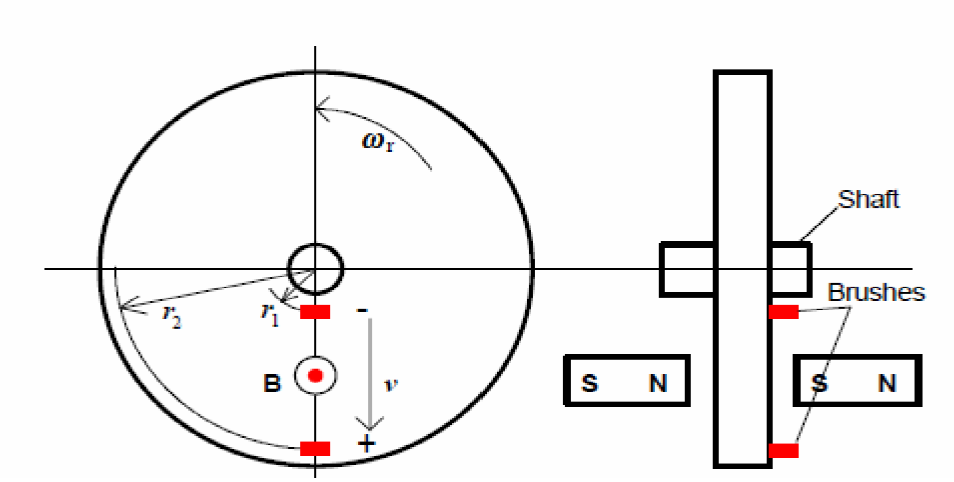
Where λ is the flux linkage of the coil and the minus sign indicates that the induced current opposes the variation of the field. It makes no difference whether the variation of the flux linkage is a result of the field variation or coil movement. In practice, it would convenient if we treat the *emf* as a voltage. The above express can then be rewritten as

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

if the system is magnetically linear, i.e. the self inductance is independent of the current. It should be noted that the self **inductance is a function of the displacement x** since there is a moving part in the system.

Example:

Calculate the open circuit voltage between the brushes on a Faraday's disc as shown schematically in the diagram below.



Solution:

Choose a small line segment of length dr at position $r(r_1 \le r \le r_2)$ from the center of the disc between the brushes. The induced emf in this elemental length is then

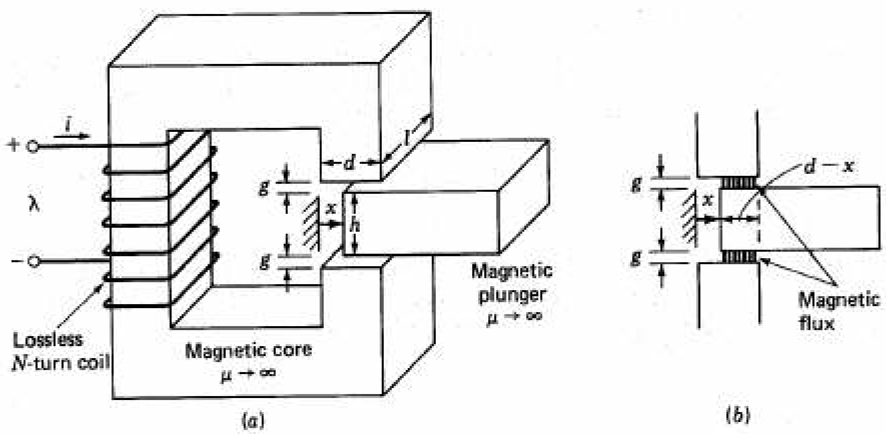
$$de = Bvdr = B\omega_r rdr$$

Where $V = r\omega_r$ *Therefore*

$$e = \int de = \int_{r_1}^{r_2} B\omega_r rdr = \omega_r B \frac{r^2}{2}\bigg|_{r_1}^{r_2} = \omega_r B \frac{r_2^2 - r_1^2}{2}$$

Example:

Sketch L(x) and calculate the induced emf in the excitation coil for a linear actuator shown below.



A singly excited linear actuator

Solution:

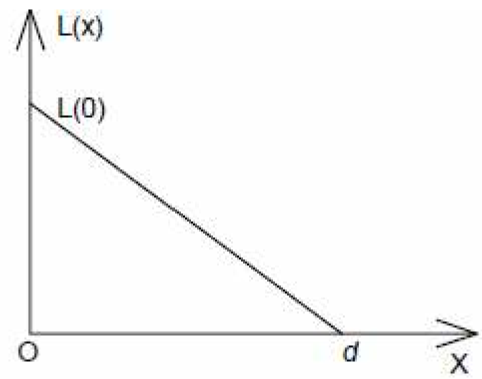
$$L(x) = \frac{N^2}{R_g(x)}$$

$$R_g(x) = \frac{2g}{\mu_o (d-x) l}$$

$$\therefore L(x) = \frac{\mu_o N^2 l}{2g}(d-x)$$

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

$$= L(x)\frac{di}{dt} - i\frac{\mu_o N^2 l}{2g} v$$



Inductance vs. displacement

If $i = I_{dc}$,

$$e = -I_{dc}\frac{\mu_o N^2 l}{2g}v$$

If $i = I_m \sin\omega t$,

$$e = \frac{\mu_o N^2 l}{2g}(d-x)\omega I_m \cos\omega t - vI_m \sin\omega t\frac{\mu_o N^2 l}{2g}$$

$$= I_m\frac{\mu_o N^2 l}{2g}\left[(d-x)\omega\cos\omega t - v\sin\omega t\right]$$

$$= I_m\frac{\mu_o N^2 l}{2g}\sqrt{(d-x)^2\omega^2 + v^2}\cos\left[\omega t + \arctan\left(\frac{v}{(d-x)\omega}\right)\right]$$

Force and Torque on a Current Carrying Conductor:

The force on a moving particle of electric charge q in a magnetic field is given by the Lorentz's force law:

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$

The force acting on a current carrying conductor can be directly derived from the equation as

$$\mathbf{F} = I\int_C dl\times\mathbf{B}$$

where C is the contour of the conductor. For a homogeneous conductor of length $\boldsymbol{l}$ carrying current I in a uniform magnetic field, the above expression can be reduced to

$$\mathbf{F} = I(\boldsymbol{l}\times\mathbf{B})$$

In a rotating system, the torque about an axis can be calculated by

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

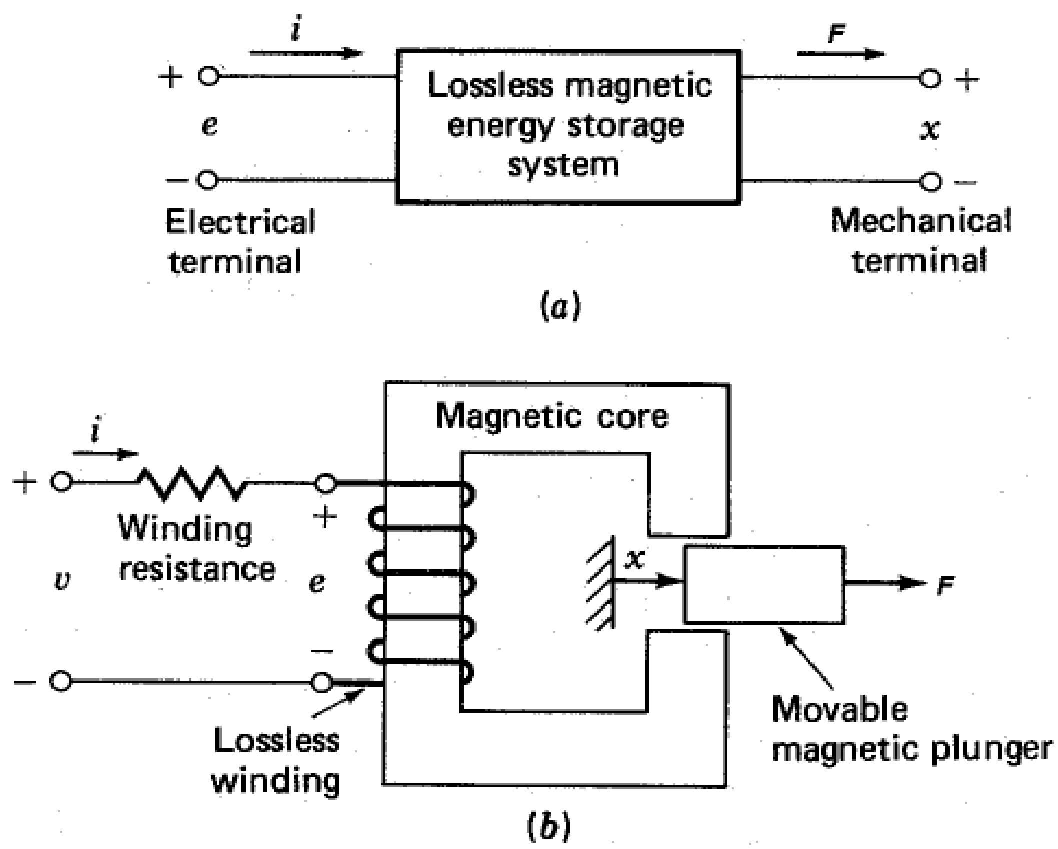
Where **r** is the radius vector from the axis towards the conductor

Force and Torque Calculation from Energy and Co energy:

A Singly Excited Linear Actuator

Consider a singly excited linear actuator as shown below. The winding resistance is R. At a certain time instant *t*, we record that the terminal voltage applied to the excitation winding is *v*, the excitation winding current *i*, the position of the movable plunger *x*, and the force acting on the plunger **F** with the reference direction chosen in the positive direction of the *x* axis, as shown in the diagram. After a time interval *dt*, we notice that the plunger has moved for a distance *dx* under the action of the force **F**. The mechanical done by the force acting on the plunger during this time interval is thus

$$dW_m = Fdx$$



A singly excited linear actuator

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the winding resistance from the total power fed into the excitation winding as

$$dW_e = dW_f + dW_m = vidt - Ri^2dt$$

Because

$$e = \frac{d\lambda}{dt} = v - Ri$$

we can write

$$dW_f = dW_e - dW_m = eidt - Fdx$$
$$= id\lambda - Fdx$$

From the above equation, we know that the energy stored in the magnetic field is a function of the flux linkage of the excitation winding and the position of the plunger. Mathematically, we can also write

$$dW_f(\lambda, x) = \frac{\partial W_f(\lambda, x)}{\partial \lambda} d\lambda + \frac{\partial W_f(\lambda, x)}{\partial x} dx$$

Therefore, by comparing the above two equations, we conclude

$$i = \frac{\partial W_f(\lambda, x)}{\partial \lambda} \quad \text{and} \quad F = -\frac{\partial W_f(\lambda, x)}{\partial x}$$

From the knowledge of electromagnetic, the energy stored in a magnetic field can be expressed as

$$W_f(\lambda, x) = \int_0^{\lambda} i(\lambda, x) d\lambda$$

For a magnetically linear (with a constant permeability or a straight line magnetization curve such that the inductance of the coil is independent of the excitation current) system, the above expression becomes

$$W_f(\lambda, x) = \frac{1}{2} \frac{\lambda^2}{L(x)}$$

and the force acting on the plunger is then

$$F = -\frac{\partial W_f(\lambda, x)}{\partial x} = \frac{1}{2}\left[\frac{\lambda}{L(x)}\right]^2 \frac{dL(x)}{dx} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

In the diagram below, it is shown that the magnetic energy is equivalent to the area above the magnetization or λ-i curve. Mathematically, if we define the area underneath the magnetization curve as the ***co energy*** (which does not exist physically), i.e.

$$W_f'(i, x) = i\lambda - W_f(\lambda, x)$$

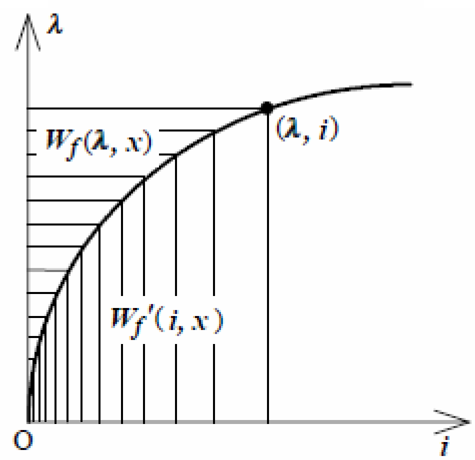
we can obtain

$$\begin{aligned} dW_f'(i, x) &= \lambda di + i d\lambda - dW_f(\lambda, x) \\ &= \lambda di + F dx \\ &= \frac{\partial W_f'(i, x)}{\partial i} di + \frac{\partial W_f'(i, x)}{\partial x} dx \end{aligned}$$

Therefore

$$\lambda = \frac{\partial W_f'(i, x)}{\partial i}$$

and $$F = \frac{\partial W_f'(i, x)}{\partial x}$$



Energy and coenergy

From the above diagram, the co energy or the area underneath the magnetization curve can be calculated by

$$W_f'(i, x) = \int_0^i \lambda(i, x) di$$

For a magnetically linear system, the above expression becomes

$$W_f'(i,x) = \frac{1}{2} i^2 L(x)$$

and the force acting on the plunger is then

$$F = \frac{\partial W_f'(i,x)}{\partial x} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

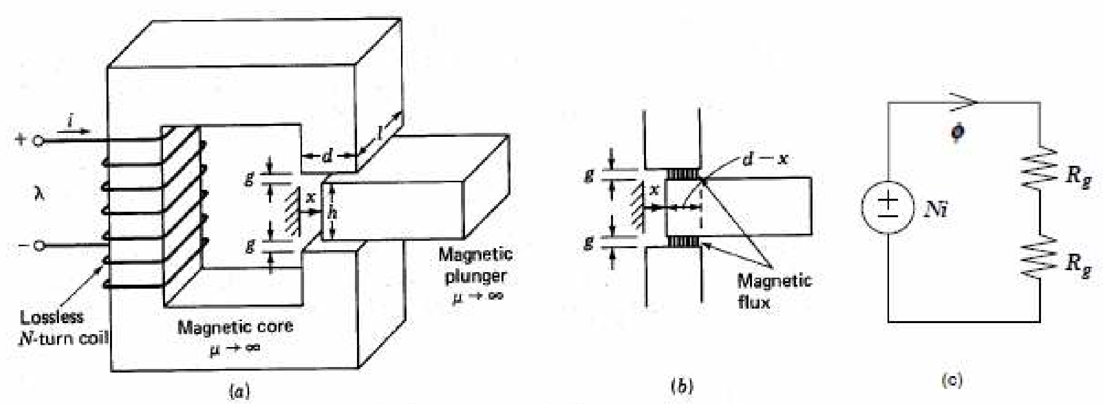
Example:

Calculate the force acting on the plunger of a linear actuator discussed in this section.

Solution:

Assume the permeability of the magnetic core of the actuator is infinite, and hence the system can be treated as magnetically linear. From the equivalent magnetic circuit of the actuator shown in figure (c) above, one can readily find the self inductance of the excitation winding as

$$L(x) = \frac{N^2}{2R_g} = \frac{\mu_o N^2 l(d-x)}{2g}$$



A singly excited linear actuator

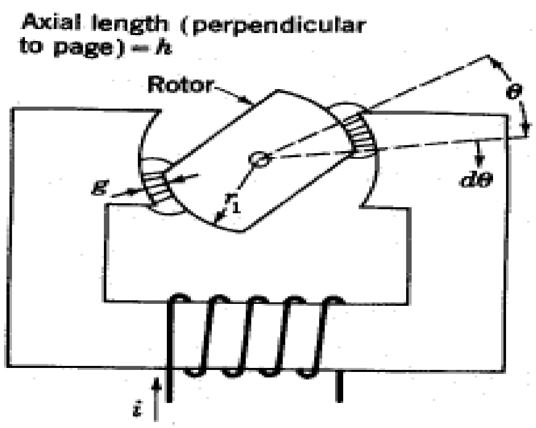
Therefore, the force acting on the plunger is

$$F = \frac{1}{2} i^2 \frac{dL(x)}{dx} = -\frac{\mu_o l}{4g}(Ni)^2$$

The minus sign of the force indicates that the direction of the force is to reduce the displacement so as to reduce the reluctance of the air gaps. Since this force is caused by the variation of magnetic reluctance of the magnetic circuit, it is known as the **reluctance force**.

Singly Excited Rotating Actuator

The singly excited linear actuator mentioned above becomes a singly excited rotating actuator if the linearly movable plunger is replaced by a rotor, as illustrated in the diagram below. Through a derivation similar to that for a singly excited linear actuator, one can readily obtain that the torque acting on the rotor can be expressed as the negative partial derivative of the energy stored in the magnetic field against the angular displacement or as the positive partial derivative of the co energy against the angular displacement, as summarized in the following table.



Energy	Coenergy
In general, $dW_f = id\lambda - Td\theta$	$dW_f' = \lambda di + Td\theta$
$W_f(\lambda,\theta) = \int_0^{\lambda} i(\lambda,\theta)d\lambda$	$W_f'(i,\theta) = \int_0^{i} \lambda(i,\theta)di$
$i = \frac{\partial W_f(\lambda,\theta)}{\partial \lambda}$	$\lambda = \frac{\partial W_f'(i,\theta)}{\partial i}$
$T = -\frac{\partial W_f(\lambda,\theta)}{\partial \theta}$	$T = \frac{\partial W_f'(i,\theta)}{\partial \theta}$

If the permeability is a constant,

$$W_f(\lambda,\theta)=\frac{1}{2}\frac{\lambda^2}{L(\theta)} \qquad W_f'(i,\theta)=\frac{1}{2}i^2L(\theta)$$

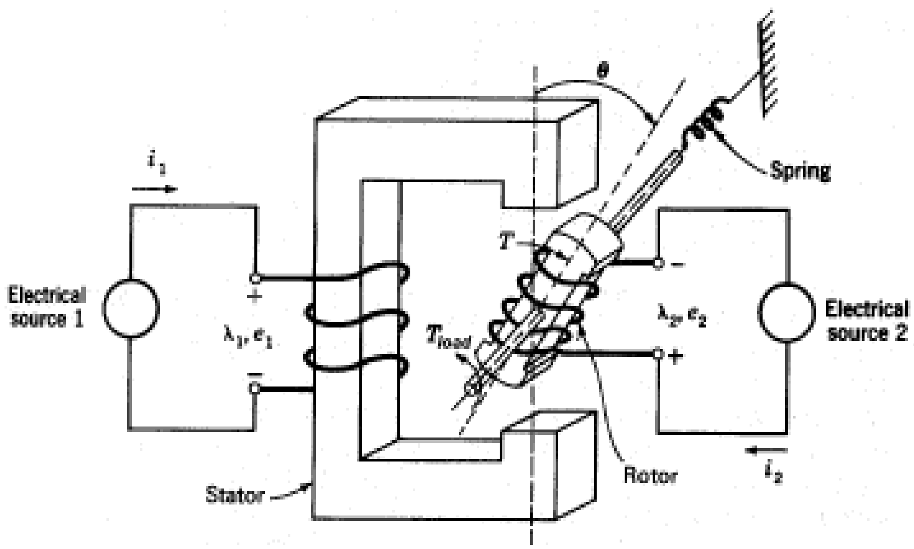
$$T=\frac{1}{2}\left[\frac{\lambda}{L(\theta)}\right]^2\frac{dL(\theta)}{d\theta}=\frac{1}{2}i^2\frac{dL(\theta)}{d\theta} \qquad T=\frac{1}{2}i^2\frac{dL(\theta)}{d\theta}$$

Doubly Excited Rotating Actuator

The general principle for force and torque calculation discussed above is equally applicable to multi-excited systems. Consider a doubly excited rotating actuator shown Schematically in the diagram below as an example. The differential energy and co energy functions can be derived as following:

$$dW_f=dW_e-dW_m$$

$$dW_e=e_1i_1dt+e_2i_2dt$$



A doubly excited actuator

$$e_1=\frac{d\lambda_1}{dt}, \qquad e_2=\frac{d\lambda_2}{dt}$$

and $$dW_m=Td\theta$$

Hence,

$$dW_f(\lambda_1,\lambda_2,\theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - T d\theta$$
$$= \frac{\partial W_f(\lambda_1,\lambda_2,\theta)}{\partial \lambda_1} d\lambda_1 + \frac{\partial W_f(\lambda_1,\lambda_2,\theta)}{\partial \lambda_2} d\lambda_2$$
$$+ \frac{\partial W_f(\lambda_1,\lambda_2,\theta)}{\partial \theta} d\theta$$

and

$$dW_f'(i_1,i_2,\theta) = d\left[i_1\lambda_1 + i_2\lambda_2 - W_f(\lambda_1,\lambda_2,\theta)\right]$$
$$= \lambda_1 di_1 + \lambda_2 di_2 + T d\theta$$
$$= \frac{\partial W_f'(i_1,i_2,\theta)}{\partial i_1} di_1 + \frac{\partial W_f'(i_1,i_2,\theta)}{\partial i_2} di_2$$
$$+ \frac{\partial W_f'(i_1,i_2,\theta)}{\partial \theta} d\theta$$

Therefore, comparing the corresponding differential terms, we obtain

$$T = -\frac{\partial W_f(\lambda_1,\lambda_2,\theta)}{\partial \theta}$$

or

$$T = \frac{\partial W_f'(i_1,i_2,\theta)}{\partial \theta}$$

For magnetically linear systems, currents and flux linkages can be related by constant inductances as following

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

Where $L_{12}=L_{21}$, $\Gamma_{11}=L_{22}/\Delta$, $\Gamma_{12}=\Gamma_{21}=-L_{12}/\Delta$, $\Gamma_{22}=L_{11}/\Delta$, and $\Delta=L_{11}L_{22}-L_{12}^2$.

The magnetic energy and co energy can then be expressed as

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\Gamma_{11}\lambda_1^2 + \frac{1}{2}\Gamma_{22}\lambda_2^2 + \Gamma_{12}\lambda_1\lambda_2$$

and

$$W_f'(i_1, i_2, \theta) = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1 i_2$$

Respectively, and it can be shown that they are equal.

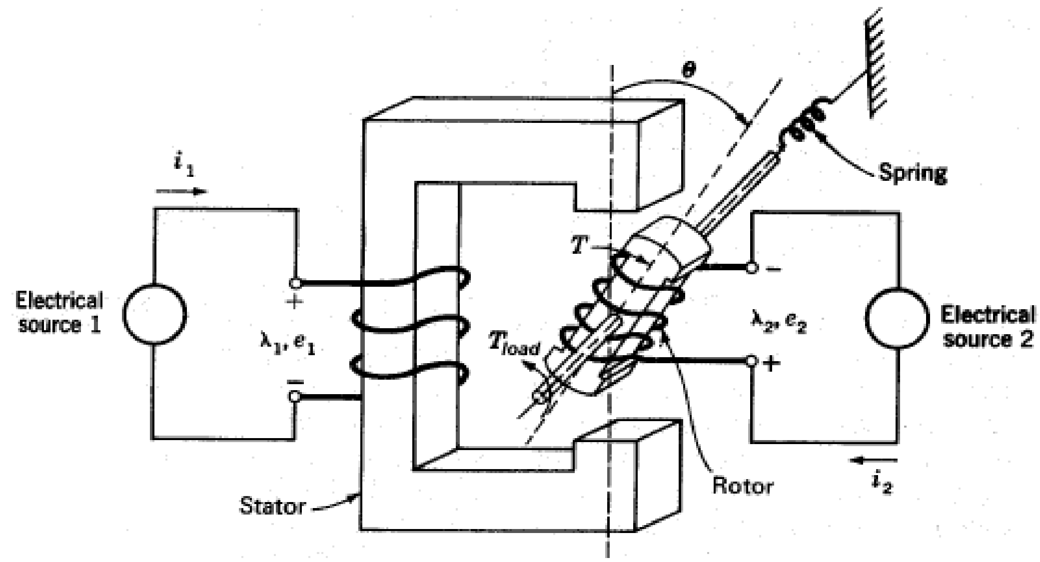
Therefore, the torque acting on the rotor can be calculated as

$$T = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$
$$= \frac{1}{2}i_1^2 \frac{dL_{11}(\theta)}{\partial \theta} + \frac{1}{2}i_2^2 \frac{dL_{22}(\theta)}{\partial \theta} + i_1 i_2 \frac{dL_{12}(\theta)}{\partial \theta}$$

Because of the salient (not round) structure of the rotor, the self inductance of the stator is a function of the rotor position and the first term on the right hand side of the above torque expression is nonzero for that $\frac{dL_{11}}{d\theta} \neq 0$. Similarly, the second term on the right hand side of the above torque express is nonzero because of the salient structure of the stator. Therefore, these two terms are known as the reluctance torque component. The last term in the torque expression, however, is only related to the relative position of the stator and rotor and is independent of the shape of the stator and rotor poles.

Model of Electromechanical Systems

To illustrate the general principle for modeling of an electromechanical system, we still use the doubly excited rotating actuator discussed above as an example. For convenience, we plot it here again. As discussed in the introduction, the mathematical model of an electromechanical system consists of circuit equations for the electrical subsystem and force or torque balance equations for the mechanical subsystem, whereas the interactions between the two subsystems via the magnetic field can be expressed in terms of the *emf*'s and the electromagnetic force or torque. Thus, for the doubly excited rotating actuator, we can write



A doubly excited actuator

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt} = R_1 i_1 + \frac{d(\lambda_{11} + \lambda_{12})}{dt}$$

$$= R_1 i_1 + L_{11}\frac{di_1}{dt} + i_1\frac{dL_{11}(\theta)}{d\theta}\frac{d\theta}{dt} + L_{12}\frac{di_2}{dt} + i_2\frac{dL_{12}(\theta)}{d\theta}\frac{d\theta}{dt}$$

$$= \left[R_1 + \omega_r\frac{dL_{11}(\theta)}{d\theta}\right]i_1 + \omega_r\frac{dL_{12}(\theta)}{d\theta}i_2 + L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

$$v_2 = R_2 i_2 + \frac{d\lambda_2}{dt} = R_2 i_2 + \frac{d(\lambda_{21} + \lambda_{22})}{dt}$$

$$= R_2 i_2 + L_{12}\frac{di_1}{dt} + i_1\frac{dL_{12}(\theta)}{d\theta}\frac{d\theta}{dt} + L_{22}\frac{di_2}{dt} + i_2\frac{dL_{22}(\theta)}{d\theta}\frac{d\theta}{dt}$$

$$= \omega_r\frac{dL_{12}(\theta)}{d\theta}i_1 + \left[R_2 + \omega_r\frac{dL_{22}(\theta)}{d\theta}\right]i_2 + L_{12}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

and $$T - T_{load} = J\frac{d\omega_r}{dt}$$

where $$\omega_r = \frac{d\theta}{dt}$$

is the angular speed of the rotor, T_{load} the load torque, and J the inertia of the rotor and the mechanical load which is coupled to the rotor shaft. The above equations are nonlinear differential equations which can only be solved numerically. In the format of state equations, the above equations can be rewritten as

$$\frac{di_1}{dt} = -\frac{1}{L_{11}}\left[R_1 + \frac{dL_{11}(\theta)}{d\theta}\omega_r\right]i_1 - \frac{1}{L_{11}}\frac{dL_{12}(\theta)}{d\theta}\omega_r i_2 - \frac{L_{12}}{L_{11}}\frac{di_2}{dt} + \frac{1}{L_{11}}v_1$$

$$\frac{di_2}{dt} = -\frac{1}{L_{22}}\frac{dL_{12}(\theta)}{d\theta}\omega_r i_1 - \frac{1}{L_{22}}\left[R_2 + \frac{dL_{22}(\theta)}{d\theta}\omega_r\right]i_2 - \frac{L_{12}}{L_{22}}\frac{di_1}{dt} + \frac{1}{L_{22}}v_2$$

$$\frac{d\omega_r}{dt} = \frac{1}{J}T - \frac{1}{J}T_{load}$$

and $$\frac{d\theta}{dt} = \omega_r$$

Together with the specified initial conditions (the state of the system at time zero in terms of the state variables):

$$i_1\big|_{t=0} = i_{10}\,,\ \ i_2\big|_{t=0} = i_{20}\,,\ \ \omega_r\big|_{t=0} = \omega_{r0}\,,\ \text{and}\ \ \theta\big|_{t=0} = \theta_0\,,$$

the above state equations can be used to simulate the dynamic performance of the doubly excited rotating actuator. Following the same rule, we can derive the state equation model of any electromechanical systems.