

**Transformations to Stationary Reference Frames:**

The applied approach eliminates the redundancy of poly phase winding substituting these by their two phase equivalent. Such redundancy is given for both line voltages and line currents of three phase ac machines. In a three phase machine with isolated neutral point (as shown in figure 1) the sum of line voltages and line currents has to be zero at all times according to kirchoff's laws

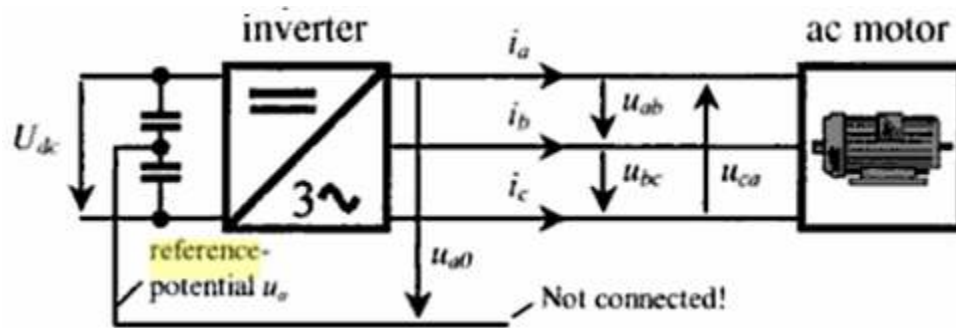


Figure 1 Three phase machine with isolated neutral point.

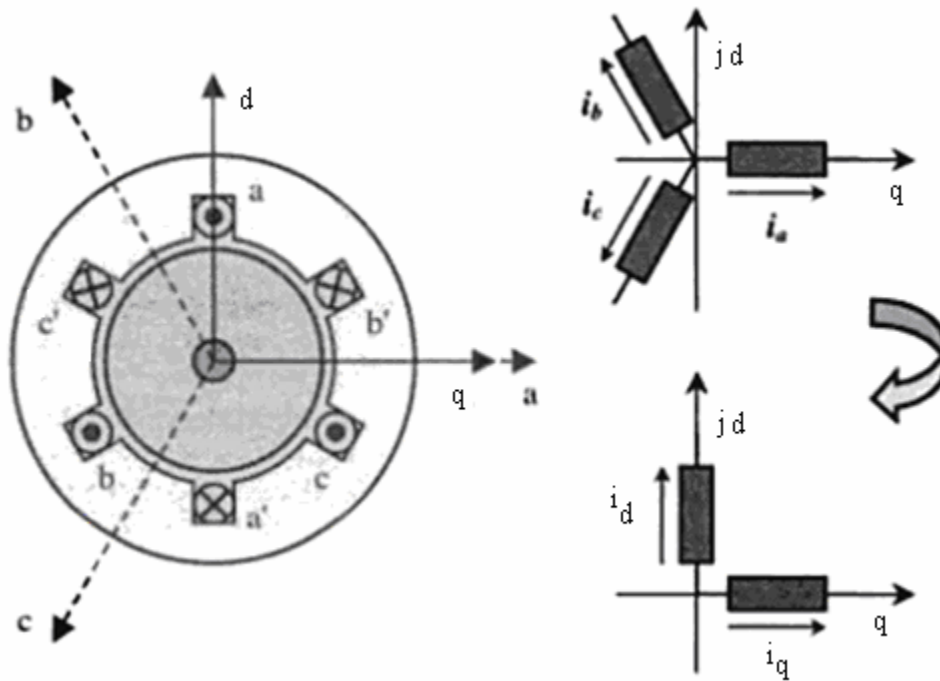
$$i_a + i_b + i_c = 0 \dots\dots\dots(1)$$

$$U_{ab} + U_{bc} + U_{ca} = 0 \dots\dots\dots(2)$$

According to (1) and (2) the input of a three phase ac motor is completely described by two line voltages and two currents since the third value equals the negative sum of the two others. Thus when describing the dynamic behavior of ac machines, the polyphase motor windings can be reduced to a set of two windings. The most suitable choice of describing the behavior of by a two axes equivalent is a set of field coils, having their magnetic axes arranged in quadrature. When saturation is neglected this approach is eliminates the mutual magnetic coupling of the three phase windings. In two axes equivalent, the flux of one winding does not interact with the perpendicular winding and vice versa.

The simplified diagram of a three phase ac motor (Fig 2) shows only the stator windings for each phase displaced by 120 degrees in space. The same is valid for rotor also. The transformation of stator and rotor variables to a two phase reference frame (indicated by subscripts q and d), where the coils are perpendicular, guarantees that there

is no interaction between perpendicular windings as there is no saturation, referred to as cross saturation.



**Fig 2. Three Phase Winding and Two phase equivalent**

Due to redundancy in (1)-(2), it is obvious that voltage and current equations of any ac machine can be reduced to a set of each two appropriate variables of q and d reference frame. One straight forward, but unusual option of transferring variables to the qd reference frame is a simple vector addition of the three phase variables. According to figure 2, a geometrical calculation yields.

$$[i_a + i_b \cos(120) + i_c \cos(240)] = i_q \text{ (Vector Addition).....(3)}$$

Solving the trigonometric functions and replacing  $i_c$  according to (1) results in

$$\Rightarrow i_a - \frac{i_b}{2} - \frac{(-i_a - i_b)}{2} = i_q \text{.....(4)}$$

$$\Rightarrow i_q = \frac{3}{2} i_a \text{.....(5)}$$

And similarly for d axis

$$[i_b \cos 30^\circ + i_c \cos 150^\circ] = i_d \dots\dots\dots(6)$$

$$\Rightarrow \frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} (-i_a - i_b) = i_d \dots\dots\dots(7)$$

$$\Rightarrow i_d = \frac{\sqrt{3}}{2} i_a + \frac{\sqrt{3}}{2} i_b \dots\dots\dots(8)$$

However when the above transformation is equally applied to both phase current and voltages, the system is not power invariant.

$$p(t) = u_a i_a + u_b i_b + u_c i_c \neq u_q i_q + u_d i_d \dots\dots\dots(9)$$

$$\begin{aligned} u_q i_q + u_d i_d &= \frac{3}{2} u_a \frac{3}{2} i_a + \left( \frac{\sqrt{3}}{2} u_a + \sqrt{3} u_b \right) \left( \frac{\sqrt{3}}{2} i_a + \sqrt{3} i_b \right) \\ &= 3u_a i_a + 3u_b i_b + \frac{3}{2} u_a i_b + \frac{3}{2} u_b i_a \dots\dots\dots(10) \\ &= \frac{3}{2} [u_a i_a + u_b i_b + (-u_a - u_b)(-i_a - i_b)] \\ &= \frac{3}{2} [u_a i_a + u_b i_b + u_c i_c] = \frac{3}{2} P(t) \end{aligned}$$

Note, that the variables in the qd reference frame are no real values. They are fictitious and well chosen to describe the real electrical and dynamic behavior of the machine. Obviously, the original current phasor can be equally described by two perpendicular vector components multiplied by a constant scaling factor. According to (10) the power in the qd reference frame reduced by a scaling factor  $\frac{2}{3}$  guarantees power invariance.

Thus this scaling factor can be distributed amongst qd current and voltages in different ways. Eg qd current remains and qd voltages are reduced by a factor of  $\frac{2}{3}$  or vice versa.

Therefore the final expression after power invariance is given by, from (1) and (6)

$$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \dots\dots\dots(11)$$

$$i_{qd} = K_s i_{abc}$$

$K_s$  is known as transformation matrix

The above expression is valid for voltages, flux or flux linkages etc.

Unfortunately, while the sum of line voltages is zero, the phase voltages or phase to neutral voltages may contain a homopolar component dependent on the PWM method used. The effect of homopolar component on the machine with isolated neutral point is equal to the change of reference potential (eg changing the potential  $u_0$  in the figure (1)). In other words the potential of all the three phases is varied uniformly and simultaneously. Fortunately, the sum of line voltages is always zero. Thus this homopolar component is unseen by the motor terminals and therefore not reflected in the motor behavior. However, since the motor voltages are often calculated by means of reference voltages containing such homopolar component, a more elaborate transformation must be used for the qd voltage calculation. The transformation matrix is obtained similarly to above method by geometrical calculations with out replacing the phase voltages  $u_c$ . For completeness the zero components are also given. According to fig (3) the zero components indicates a displacement of reference potential  $u_0$  with respect to the centre  $m$  of the line voltages. Obviously, the phase voltages ( $u_a, u_b, u_c$ ) are equal to the affiliated line to neutral voltages ( $u_{am}, u_{bm}, u_{cm}$ ) plus the zero component  $u_{m0}$ . Since the sum of line to neutral voltages is zero, following relation is valid.

$$U_{am} + U_{bm} + U_{cm} = (U_a - U_{m0})(U_b - U_{m0})(U_c - U_{m0}) = 0 \dots \dots (12)$$

$$\Rightarrow U_{m0} = \frac{1}{3}(u_a + u_b + u_c)$$

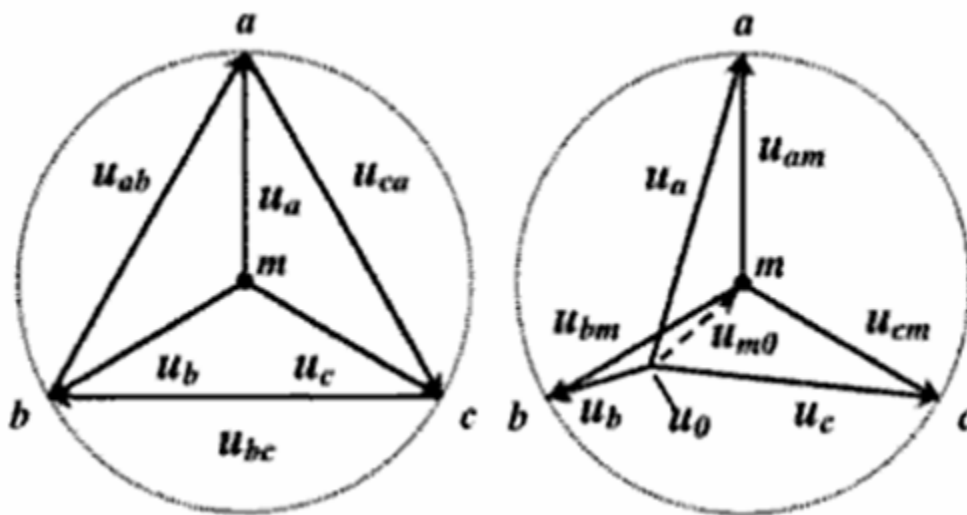


Fig (3) Zero component of Phase Voltages; **Left:** Phase and Line Voltages without zero component **Right:** Phase voltages and displacement of the neutral point

Scaling of power in variance factor yields the zero the zero component in the qd reference frame.

$$\begin{bmatrix} i_q \\ i_d \\ i_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \dots\dots\dots(12)$$

$$i_{qd} = K_s i_{abc}$$

$K_s$  is known as transformation matrix

The inverse transformation is

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix} \begin{bmatrix} i_q \\ i_d \\ i_o \end{bmatrix} \dots\dots\dots(13)$$

$$i_{abc} = K_s^{-1} i_{qd}$$

$K_s^{-1}$  is known as inverse transformation matrix

The above transformation matrix can be written in trigonometric functions as

$$\mathbf{K}_s = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin \theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix},$$