Chapter 10

CLARKE'S AND PARK'S TRANSFORMATIONS

10.1 Introduction

The performance of three-phase ac machines are described by their voltage equations and inductances. It is well known that some machine inductances are functions of rotor speed. The coefficients of the differential equations, which describe the behavior of these machines, are time varying except when the rotor is stalled. A change of variables is often used to reduce the complexity of these differential equations. There are several different methods to transform variables. In this chapter, the well-known Clarke and Park transformations are introduced, modeled, and implemented on the LF2407 DSP. Using these transformations, many properties of electric machines can be studied without complexities in the voltage equations. These transformations make it possible for control algorithms to be implemented on the DSP. By this approach, many of the basic concepts and interpretations of this general transformation are concisely established.

10.2 Clarke's Transformation

The transformation of stationary circuits to a stationary reference frame was developed by E. Clarke [2]. The stationary two-phase variables of Clarke's transformation are denoted as α and β. As shown in Fig. 10.1, α-axis and β-axis are orthogonal.

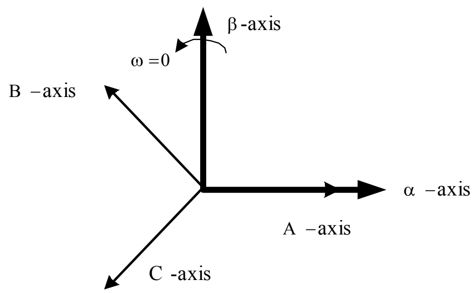

Figure 10.1 Clarke's transformation.

In order for the transformation to be invertible, a third variable, known as the zero-sequence component, is added. The resulting transformation is

$$\left[f_{\alpha\beta 0}\right] = T_{\alpha\beta 0}\left[f_{abc}\right] \tag{10.1}$$

where

$$\left[f_{\alpha\beta 0}\right] = \left[f_\alpha \quad f_\beta \quad f_0\right]^T$$

and

$$\left[f_{abc}\right] = \left[f_a \quad f_b \quad f_c\right]^T$$

where f represents voltage, current, flux linkages, or electric charge and the transformation matrix, $T_{\alpha\beta0}$, is given by

$$T_{\alpha\beta0} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{10.2}$$

The inverse transformation is given by

$$\left[f_{abc}\right] = T_{\alpha\beta0}{}^{-1}\left[f_{\alpha\beta0}\right] \tag{10.3}$$

where the inverse transformation matrix is presented by

$$T_{\alpha\beta0}{}^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix} \tag{10.4}$$

10.3 Park's Transformation

In the late 1920s, R.H. Park [1] introduced a new approach to electric machine analysis. He formulated a change of variables which replaced variables such as voltages, currents, and flux linkages associated with fictitious windings rotating with the rotor. He referred the stator and rotor variables to a reference frame fixed on the rotor. From the rotor point of view, all the variables can be observed as constant values. Park's transformation, a revolution in machine analysis, has the unique property of eliminating all time varying inductances from the voltage equations of three-phase ac machines due to the rotor spinning.

Although changes of variables are used in the analysis of ac machines to eliminate time-varying inductances, changes of variables are also employed in the analysis of various static and constant parameters in power system components. Fortunately, all known real transformations for these components are also contained in the transformation to the arbitrary reference frame. The same general transformation used for the stator variables of ac machines serves the rotor variables of induction machines. Park's transformation is a well-known three-phase to two-phase transformation in synchronous machine analysis. Park's transformation is presented in Fig. 10.2.

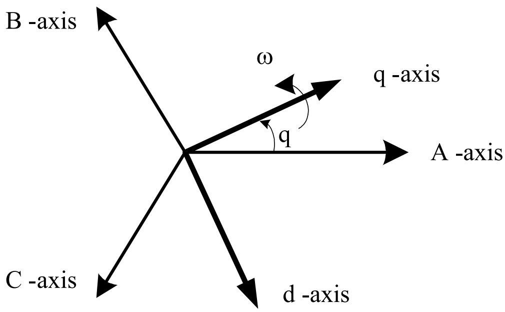

Figure 10.2 Park's transformation.

The transformation equation is of the form

$$\left[f_{qd0s}\right] = T_{qd0}(\theta)\left[f_{abcs}\right] \tag{10.5}$$

where

$$\left[f_{qd0s}\right] = \left[f_{qs} \quad f_{ds} \quad f_{0s}\right]^T$$

and $\left[f_{abcs}\right] = \left[f_{as} \quad f_{bs} \quad f_{cs}\right]^T$

and the dq0 transformation matrix is defined as

$$T_{qd0s}(\theta) = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{10.6}$$

θ is the angular displacement of Park's reference frame and can be calculated by

$$\theta = \int_0^t \omega(\zeta)d\zeta + \theta(0) \tag{10.7}$$

where ζ is the dummy variable of integration. It can be shown that for the inverse transformation we can write

$$\left[f_{abcs}\right] = T_{qd0}(\theta)^{-1} \cdot \left[f_{qd0s}\right] \tag{10.8}$$

where the inverse of Park's transformation matrix is given by

$$T_{qd0}(\theta)^{-1} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix} \tag{10.9}$$

In the previous equations, the angular displacement θ must be continuous, but the angular velocity associated with the change of variables is unspecified. The frame of reference may rotate at any constant, varying angular velocity, or it may remain stationary. The angular velocity of the transformation can be chosen arbitrarily to best fit the system equation solution or to satisfy the system constraints. The change of variables may be applied to variables of any waveform and time sequence; however, we will find that the transformation given above is particularly appropriate for an a-b-c sequence.

10.4 Transformations Between Reference Frames

In order to reduce the complexity of some derivations, it is necessary to transform the variables from one reference frame to another one. To establish this transformation between any two reference frames, we can denote *y* as the new reference frame and *x* as the old reference frame. Both new and old reference frames are shown in Fig. 10.3.

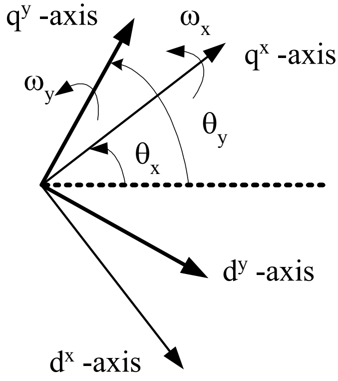

Figure 10.3 Transformation between two reference frames.

It is assumed that the reference frame *x* is rotating with angular velocity ω_x and the reference frame *y* is spinning with the angular velocity ω_y. θ_x and θ_y are angular displacements of reference frames *x* and *y*, respectively. In this regard, we can rewrite the transformation equation as

$$\left[f_{qd0s}^{y}\right] = T_{qd0s}^{x \to y} \cdot \left[f_{qd0s}\right] \tag{10.10}$$

But we have

$$\left[f^{x}_{qd0s}\right] = T^{x}_{qd0s} \cdot \left[f_{abcs}\right] \tag{10.11}$$

If we substitute (10.11) in (10.10) we get

$$\left[f^{y}_{qd0s}\right] = T^{x\to y}_{qd0s} \cdot T^{x}_{qd0s} \cdot \left[f_{abcs}\right] \tag{10.12}$$

In another way, we can find out that

$$\left[f^{y}_{qd0s}\right] = T^{y}_{qd0s} \cdot \left[f_{abcs}\right] \tag{10.13}$$

From (10.12) we obtain

$$T^{x\to y}_{qd0s} = T^{y}_{qd0s} \cdot {T^{x}_{qd0s}}^{-1} \tag{10.14}$$

Then, the desired transformation can be expressed by the following matrix:

$$T^{x\to y}_{qd0s} = \begin{bmatrix} \cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) & 0 \\ \sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) & 0 \\ 1 & 1 & 1 \end{bmatrix} \tag{10.15}$$

10.5 Field Oriented Control (FOC) Transformations

In the case of FOC of electric machines, control methods are performed in a two-phase reference frame fixed to the rotor *(q^r-d^r)* or fixed to the excitation reference frame *(q^e-d^e)*. We want to transform all the variables from the three-phase a-b-c system to the two-phase stationary reference frame and then retransform these variables from the stationary reference frame to a rotary reference frame with arbitrary angular velocity of ω. These transformations are usually cascaded. The block diagram of this procedure is shown in Fig. 10.4.

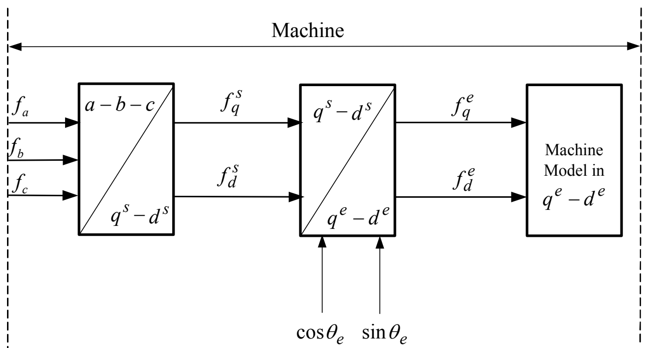

Figure 10.4 Machine side transformation in field oriented control.

In this figure, f denotes the currents or voltages and q^e-d^e represents the arbitrary rotating reference frame with angular velocity ω_e and q^s-d^s represents the stationary reference frame. In the vector control method, after applying field-oriented control it is necessary to transform variables to stationary a-b-c system. This can be achieved by taking the inverse transformation of variables from the arbitrary rotating reference frame to the stationary reference frame and then to the a-b-c system. The block diagram of this procedure is shown in Fig. 10.5. In this block diagram, * is a representation of commanded or desired values of variables.

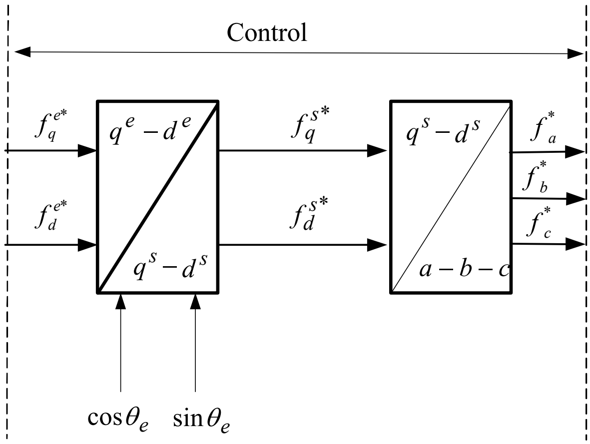

Figure 10.5 Variable transformation in the field oriented control.

10.6 Implementing Clarke's and Park's Transformations on the LF240X

10.6.1 Implementing Clarke's Transformation

It is desired to transfer the three-phase stationary parameters f_a, f_b, and f_c from the a-b-c system to the two-phase stationary reference frame. It is assumed that the system is balanced and we have

$$f_a + f_b + f_c = 0 \tag{10.16}$$

We can rewrite (10.1) as follows:

$$f_\alpha = \frac{2}{3} f_a - \frac{1}{3} f_b - \frac{1}{3} f_c \tag{10.17}$$

$$f_\beta = \frac{1}{\sqrt{3}} (f_b - f_c) \tag{10.18}$$

Substituting f_c from (10.16) into (10.17) and (10.18) results in

$$f_\alpha = f_a \tag{10.19}$$

$$f_\beta = \frac{1}{\sqrt{3}}(f_a + 2f_b) \qquad (10.20)$$

10.6.1.1 Inputs and Outputs of Clarke's Transformation Block

The inputs and outputs of Clarke's transformation are shown in Fig. 10.6. As it is shown in this figure, f_a and f_b are inputs and f_α and f_β are outputs of this transformation.

Figure 10.6 Clarke transformation.

To enjoy better resolution of the variables in fixed point DSP, we transfer all variables to the Q15-based format. With this consideration, the maximum value of inputs and outputs can be (2^{15}-1) or in hexadecimal, the format shall be $7FFF_h$. In this base, the variables can vary in the range 8000_h-$7FFF_h$. This transformation converts balanced three-phase quantities into balanced two-phase quadrature quantities as shown in Fig. 10.7.

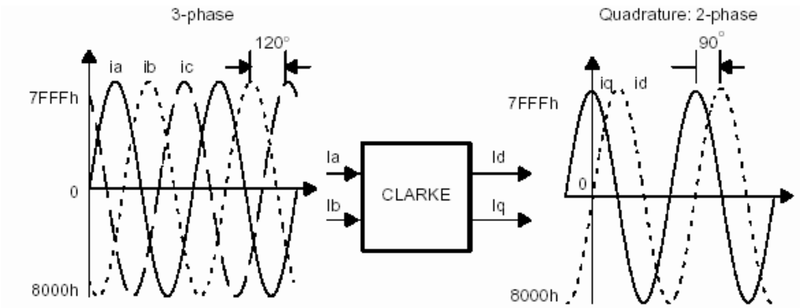

Figure 10.7 Quantities in Clarke's transformation. (Courtesy of Texas Instruments)

As we previously noted, our calculations are based on the Q15 format . So all the coeficients are present in this representation. Then 1/√3 is represented by

```
LDP     #sqrt3inv           ;sqrt3inv=(1/sqrt(3))
                            ;=0.577350269
SPLK    #018830,sqrt3inv    ;1/sqrt(3) (Q15)
```

Clarke's transformation is implemented as follows:

```
SETC    SXM              ;Sign extension mode on
LDP     #clark_a         ;clark_alfa = clark_a
LACC    clark_a          ;ACC = clark_a
SACL    clark_alfa       ;clark_d = clark_a
                         ;clark_beta=(2*clark_b+clark_a)/
                         ;sqrt(3)
SFR                      ;ACC = clark_a/2
ADD     clark_b          ;ACC = clark_a/2 + clark_b
SACL    clk_temp         ;clk_temp = clark_a/2 + clark_b
LT      clk_temp         ;TREG = clark_a/2 + clark_b
MPY     sqrt3inv         ;PREG=(clark_a/2+clark_b)*
                         ;(1/sqrt(3))
PAC                      ;ACC=(clark_a/2+clark_b)*
                         ;(1/sqrt(3))
SFL                      ;ACC=(clark_a+clark_b*2)*
                         ;(1/sqrt(3))
SACH    clark_beta       ;clark_beta=(clark_a+clark_b*2
                         ;(1/sqrt(3))
SPM     0                ;SPM reset
RET
```

10.6.2 Inverse Clarke's Transformation

From (10.3), the inverse Clarke functions for a balanced system can be obtained as

$$f_a = f_\alpha$$
$$f_b = \frac{-f_\alpha + \sqrt{3} * f_\beta}{2} \tag{10.21}$$
$$f_c = \frac{-f_\alpha - \sqrt{3} * f_\beta}{2}$$

This transformation converts balanced two-phase quadrature quantities into balanced three-phase quantities. The block diagram of the inverse Clarke transformation is shown in Fig. 10.8.

Figure 10.8 Inverse Clark transformation block.

In this block diagram, f_α and f_β are inputs and f_a, f_b, and f_c are outputs. Inputs and outputs are represented in Q15 format. Variation of quantities in the inverse Clark transformation is shown in Fig. 10.9.

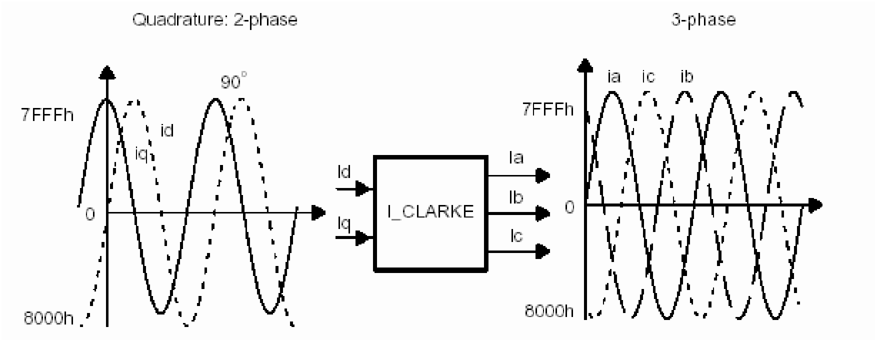

Figure 10.9 Quantities in inverse Clarke's transformation. (Courtesy of Texas Instruments)

Implementation of the inverse Clarke transformation via assembly code is as follows

```
I_CLARKE_INIT:
        LDP     #half_sqrt3         ;Variables data page
        SPLK    #28377,half_sqrt3   ;Set constant sqrt(3)*0.5 in Q15
                                    ;format
RET
I_CLARKE:
        LDP     #f_clark_alpha      ;Variables data page
        SPM     1                   ;SPM set for Q15 multiplication
        SETC    SXM                 ;Sign extension mode on
        LACC    f_clark_alpha       ;ACC = f_alpha
        SACL    f_clark_a           ;f_a = f_alpha
        LT      f_clark_beta        ;TREG = f_clark_beta
        MPY     half_sqrt3          ;PREG=f_clark_beta * half_sqrt3
        PAC                         ;ACC= f_clark_beta * half_sqrt3
SUB     f_clark_alpha,15            ;ACC=f_beta*half_sqrt3-f_alpha/2
        SACH    f_clark_b
        PAC                         ;ACC high = f_beta*half_sqrt3
        NEG                         ;ACC high = - f_beta*half_sqrt3
        SUB     f_clark_alpha,15    ;ACC high=-f_beta*half_sqrt3-
                                    ;f_alpha/2
        SACH    f_clark_c           ;f_c = - f_beta * half_sqrt3 -
                                    ;f_alpha/2
        SPM     0                   ;SPM reset
        CLRC    SXM                 ;Sign extension mode off
RET
```

10.6.3 Calculation of Sine/Cosine with Fast Table Direct Look-Up and Linear Interpolation

To implement the Park and the inverse Park transforms, the sine and cosine functions need to be implemented. This method realizes the sine/cosine functions with a look-up table of 256 values for 360° of sine and cosine functions. The method includes linear interpolation with a fixed step table to provide a minimum harmonic distortion. This table is loaded in program memory. The sine value is presented in Q15 format with the range of -1<value<1. The first few rows of the look-up sine table are presented as follows:

```
       ;SINVALUE                ;       Index     Angle      Sin(Angle)
        ----------------                --------  --------   ------------
SINTAB_360
        .word          0        ;         0       0          0.0000
        .word          804      ;         1       1.41       0.0245
       .word           1608     ;         2       2.81       0.0491
        .word          2410     ;         3       4.22       0.0736
        .word          3212     ;         4       5.63       0.0980
```

The following assembly code is written to read values of sine from the sine Table in Q15 format:

```
LACC    theta_p, 9              ;Input angle in Q15 format and
                                ;left shifted by 15
SACH    t_ptr                   ;Save high ACC to t_ptr (table
                                ;pointer)
LACC    #SINTAB_360
ADD     t_ptr
TBLR    sin_theta               ;sin_theta = Sin(theta_p) in Q15
```

Note that 0 < theta_p < 7FFFh (i.e., equivalent to 0 < theta_p < 360 deg). The TBLR instruction transfers a word from a location in program memory to a data-memory location specified by the instruction. The program-memory address is defined by the low-order 16 bits of the accumulator. For this operation, a read from program memory is performed, followed by a write to data memory.

To calculate the cosine values from the sine Table in Q15 format, we write the following code:

```
LACC    theta_p
ADD     #8192                   ;add 90 deg, cos(A)=sin(A+90⁰)
AND     #07FFFh                 ;Force positive wrap-around
SACL    GPR0_park               ;here 90 deg = 7FFFh/4
LACC    GPR0_park,9
SACH    t_ptr
LACC    #SINTAB_360
```

10.6.4 Implementation of Park's Transformation on LF2407

As discussed in Section 10.5, with field-oriented control of motors, it is necessary to transform variables, i.e., currents and voltages, from a-b-c system to two-phase stationary reference frame, q^s-d^s, and from two-phase stationary reference frame q^s-d^s to arbitrary rotating reference frame with angular velocity of ω (*q*-*d* reference frame). The first transformation is dual to Clarke's transformation but the q^s axis is in the direction of α–axis, and d^s axis is in negative direction of β –axis. These two transformations are explained in the following sections.

10.6.4.1 Transformation from 3-phase to 2-phase Stationary Reference Frame $(a-b-c) \rightarrow (q^S - d^S)$

This transformation transfers the three-phase stationary parameters, f_a, f_b, and f_c from an a-b-c system to a two-phase orthogonal stationary reference frame. If we substitute θ=0 in (10.6) and assuming that the system is balanced, we get:

$$f_q^S = f_a \tag{10.23}$$

$$f_d^S = -\frac{1}{\sqrt{3}}(2f_b + f_a) \tag{10.24}$$

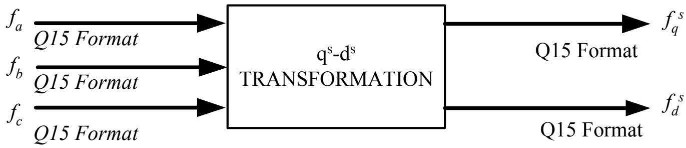

Figure 10.11 Two-phase stationary transformation.

Both input and output are represented in Q15 format with a block diagram of the transformation being shown in Fig. 10.11. The developed code is similar to what was mentioned in Section 10.6.1.1.

10.6.4.2 Transformation from the Stationary Reference Frame to the Arbitrary Rotary Reference Frame $(q^S - d^S) \rightarrow (q-d)$

This transformation converts vectors in a balanced two-phase orthogonal stationary system into an orthogonal rotary reference frame. The inputs are f_q^S, f_d^S, and θ, and the outputs are f_q and f_d. This is the transformation between the stationary reference frame and the arbitrary reference frame rotating with the angular velocity of ω. If we substitute $\theta_x = 0$ and $\theta_y = \theta$ we obtain:

$$\begin{aligned} f_q &= \cos\theta . f_q^S - \sin\theta \cdot f_d^S \\ f_d &= \sin\theta . f_q^S + \cos\theta \cdot f_d^S \end{aligned} \tag{10.25}$$

where θ is the angular displacement.

In this transformation, it is necessary to calculate $\sin\theta$ and $\cos\theta$, where the method to calculate them was presented in a previous section. In Fig. 10.12, the input and output of the Park transformation block has been shown. All the input and outputs are in the Q15 format and in the range of 8000_h-$7FFF_h$.

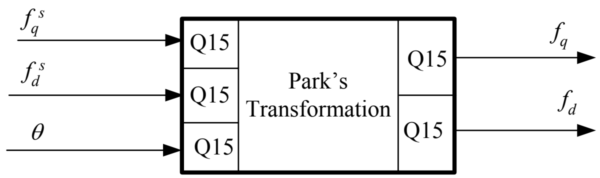

Figure 10.12 Park transformation block.

The following code is written to implement Park's transformation:

```
        SPM     1                   ;SPM set for Q15 multiplication
        ZAC                         ;Reset accumulator
        LT      f_q_s               ;TREG = f_q_s
        MPY     sin_theta           ;PREG = f_q_s * sin(theta)
        LTA     f_d                 ;ACC = f_q_s * sin(theta) and
                                    ;TREG =f_q_s
        MPY     cos_theta           ;PREG = f_d_s* cos_teta
        MPYA    sin_theta           ;ACC=f_q_s*sin_teta+f_d_s*
                                    ;cos_teta andPREG=f_q_s*sin_teta
        SACH    park_D              ;f_d =f_q_s * cos_teta + f_d_s*
                                    ;sin(theta)
        LACC    #0                  ;Clear ACC
        LT      f_d_s               ;TREG = f_d_s
        MPYS    cos_theta           ;ACC=- f_d_s* *sin(theta) and
                                    ;PREG = f_q_s * cos(theta)
        APAC                        ;ACC=- f_d_s*sin(theta) +f_q_s*
                                    ;cos(theta)
        SACH    f_q                 ;fq = -f_d_s*sin(theta) +f_q_s*
                                    ;cos(theta)
        SPM     0                   ;SPM reset
RET
```

10.6.5 Transformation of the Arbitrary Rotating Reference Frame to the Stationary Reference Frame $(q-d)\rightarrow(q^s-d^s)$

This transformation projects vectors in an orthogonal rotating reference frame into a two-phase orthogonal stationary frame. From (10.15) we get:

$$\begin{aligned} f_q^s &= \cos\theta\cdot f_q + \sin\theta\cdot f_d \\ f_d^s &= -\sin\theta\cdot f_q + \cos\theta\cdot d_d \end{aligned} \tag{10.26}$$

In this transformation, θ is the angular displacement. To transform variables to Park's reference frame, it is necessary to calculate $\sin\theta$ and $\cos\theta$. Use the method presented in the previous section. In Fig. 10.13, inputs and outputs of the inverse

Park transformation block are shown. The inputs are f_d, f_q, and θ, and the outputs are f_α and f_β. All the inputs and outputs are in the Q15 format and in the range of 8000_h-$7FFF_h$.

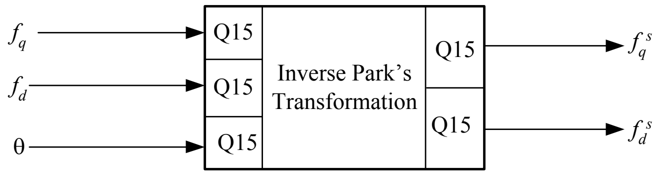

Figure 10.13 Inverse Park's transformation block.

The following code is written to implement this transformation:

```
SPM     1                       ;SPM set for Q15 multiplication
        ZAC                     ;Reset accumulator
        LT      f_q             ;TREG = fq
        MPY     cos_theta       ;PREG = fq * cos(theta)
        LTA     f_d             ;ACC=fq*cos(theta) and TREG =fd
        MPY     sin_theta       ;PREG = fd * sin(theta)
        MPYA    sin_theta       ;ACC=fq*cos(theta)+fd*sin(theta)
                                ;and PREG=fd*sin(theta)
        SACH    f_q_s           ;fd=fq*cos(theta)+fd*sin(theta)
        LACC    #0              ;Clear ACC
        LT      f_d             ;TREG = fd
        MPYS    cos_theta       ;ACC = -fd*sin_theta and
                                ;PREG = fd*cos_theta
        APAC
        SACH    f_d_s
        SPM     0               ;SPM reset
RET
```

10.6.6 The 2-Phase to 3-Phase Transformation $(q^s - d^s) \rightarrow (a - b - c)$

This transformation transforms the variables from the stationary two-phase q^s-d^s frame to the stationary a-b-c system. This system is also dual to the inverse Clarke transformation where the q^s-axis is in the direction of the α axis and the d^s-axis is in the negative direction of β–axis.

If we substitute θ=0 in (10.9) and assume a balanced system we get:

$$
\begin{aligned}
f_a &= f_q^s \\
f_b &= \frac{-f_q^s - \sqrt{3} f_d^s}{2} \\
f_c &= \frac{-f_q^s - \sqrt{3} f_d^s}{2}
\end{aligned}
\tag{10.27}
$$

The implemented code is similar to the inverse Clarke transformation which will not be repeated in here.

10.7 Conclusion

With FOC of synchronous and induction machines, it is desirable to reduce the complexity of the electric machine voltage equations. The transformation of machine variables to an orthogonal reference frame is beneficial for this purpose. Park's and Clarke's transformations, two revolutions in the field of electrical machines, were studied in depth in this chapter. These transformations and their inverses were implemented on the fixed point LF2407 DSP.

References

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