

Chapter 12

DSP-BASED CONTROL OF PERMANENT MAGNET SYNCHRONOUS MACHINES

12.1 Introduction

As described in [Chapter 9](#), the permanent magnet synchronous motor (PMSM) is a PM motor with a sinusoidal back-EMF. Compared to the BLDC motor, it has less torque ripple because the torque pulsations associated with current commutation do not exist. A carefully designed machine in combination with a good control technique can yield a very low level of torque ripple (<2% rated), which is attractive for high-performance motor control applications such as machine tool and servo applications.

In this chapter, following the same procedures used in [Chapter 9](#), the principles of the PMSM drive system will be introduced. Later, the control implementation using the LF2407 DSP will be described in detail.

12.2 The Principle of the PMSM

12.2.1 Mathematical Model of PMSM in the abc Stationary Reference Frame

[Figure 12.1](#) depicts a cross-section of the simplified three-phase surface mounted PMSM motor for our discussion. The stator windings, as-as', bs-bs', and cs-cs', are shown as lumped windings for simplicity, but are actually distributed about the stator. The rotor has two poles. Mechanical rotor speed and position are denoted as ω_{rm} and θ_{rm} , respectively. Electrical rotor speed and position, ω_r and θ_r , are defined as P/2 times the corresponding mechanical quantities, where P is the number of poles.

Based on the above motor definition, the voltage equation in the abc stationary reference frame is given by

$$V_{abc} = R_s i_{abc} + \frac{d}{dt} \lambda_{abc} \quad (12.1)$$

where

$$f_{abc} = [f_{as} \quad f_{bs} \quad f_{cs}]^T \quad (12.2)$$

and the stator resistance matrix is given by

$$R_s = \text{diag}[r_s \quad r_s \quad r_s] \quad (12.3)$$

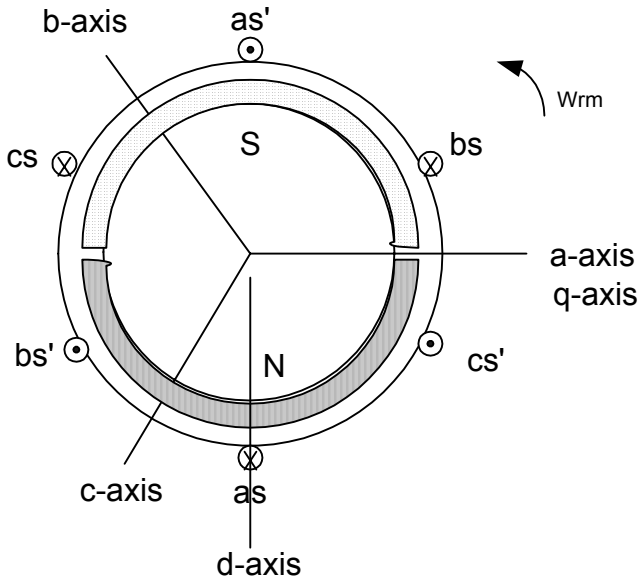


Figure 12.1 The cross-section of PMSM.

The flux linkages equation can be expressed by

$$\lambda_{abcS} = L_S i_{abcS} + \lambda'_m \begin{bmatrix} \sin \theta_r \\ \sin(\theta_r - \frac{2\pi}{3}) \\ \sin(\theta_r - \frac{4\pi}{3}) \end{bmatrix} \tag{12.4}$$

where λ'_m denotes the amplitude of the flux linkages established by the permanent magnet as viewed from the stator phase windings. Note that in (12.4) the back-EMFs are sinusoidal waveforms that are 120° apart from each other. The stator self inductance matrix, L_S , is given as

$$L_S = \begin{bmatrix} L_{ls} + L_A - L_B \cos 2\theta_r & -\frac{1}{2}L_A - L_B \cos 2(\theta_r - \pi/3) & -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \pi/3) \\ -\frac{1}{2}L_A - L_B \cos 2(\theta_r - \pi/3) & L_{ls} + L_A - L_B \cos 2(\theta_r - 2\pi/3) & -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \pi) \\ -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \pi/3) & -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \pi) & L_{ls} + L_A - L_B \cos 2(\theta_r + 2\pi/3) \end{bmatrix} \tag{12.5}$$

The electromagnetic torque may be written as

$$T_e = \frac{P}{2} \left\{ \lambda'_m \left[(i_{as} - \frac{1}{2}i_{bs} - \frac{1}{2}i_{cs}) \cos \theta_r - \frac{\sqrt{3}}{2} (i_{bs} - i_{cs}) \sin \theta_r \right] + \frac{L_{md} - L_{mq}}{3} \left[(i_{as}^2 - \frac{1}{2}i_{bs}^2 - \frac{1}{2}i_{cs}^2 - i_{as}i_{bs} - i_{as}i_{cs} + 2i_{bs}i_{cs}) \sin 2\theta_r + \frac{\sqrt{3}}{2} (i_{bs}^2i_{cs}^2 - 2i_{as}i_{bs} + 2i_{as}i_{cs}) \cos 2\theta_r \right] \right\} + T_{cog}(\theta_r) \tag{12.6}$$

In (12.6), $T_{cog}(\theta_r)$ represents the cogging torque and the d- and q-axes magnetizing inductances are defined by

$$L_{md} = \frac{3}{2} (L_A - L_B)$$

and

$$L_{mq} = \frac{3}{2} (L_A + L_B) \tag{12.7}$$

The torque and speed are related by the electromechanical motion equation

$$J \frac{d}{dt} \omega_{rm} = \frac{P}{2} (T_e - T_L) - B_m \omega_{rm} \tag{12.8}$$

where J is the rotational inertia, B_m is the approximated mechanical damping due to friction, and T_L is the load torque.

12.2.2 Mathematical Model of PMSM in Rotor Reference Frame

The voltage and torque equations can be expressed in the rotor reference frame in order to transform the time-varying variables into steady state constants. Since the stator has two poles and the rotor has four poles, the transformation of the three-phase variables in the stationary frame to the rotor reference frame is defined as

$$f_{qd0r} = K_r f_{abcs} \tag{12.9}$$

where

$$K_r = \frac{2}{3} \begin{bmatrix} \cos \theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin \theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

If the applied stator voltages are given by

$$\begin{cases} V_{as} = \sqrt{2}V_s \cos \theta_{ev} \\ V_{bs} = \sqrt{2}V_s \cos(\theta_{ev} - \frac{2\pi}{3}) \\ V_{cs} = \sqrt{2}V_s \cos(\theta_{ev} + \frac{2\pi}{3}) \end{cases} \quad (12.10)$$

Then, applying (12.9) to (12.1), (12.4) and (12.10) yields

$$v_{qs}^r = r_s i_{qs}^r + \omega_r \lambda_{ds}^r + \frac{d}{dt} \lambda_{qs}^r \quad (12.11)$$

$$v_{ds}^r = r_s i_{ds}^r - \omega_r \lambda_{qs}^r + \frac{d}{dt} \lambda_{ds}^r \quad (12.12)$$

$$\lambda_{qs}^r = L_{qs} i_{qs}^r \quad (12.13)$$

$$\lambda_{ds}^r = L_{ds} i_{ds}^r + \lambda_m^r \quad (12.14)$$

where the q- and d-axes self inductances are given by $L_{qs} = L_{ls} + L_{mq}$ and $L_{ds} = L_{ls} + L_{md}$, respectively.

The electromagnetic torque can be written as

$$T_e = \frac{3}{2} \frac{P}{2} [\lambda_m^r i_{qs}^r + (L_{ds} - L_{qs}) i_{qs}^r i_{ds}^r] \quad (12.15)$$

From (12.15), it can be seen that torque is related only to the d- and q-axes currents. Since $L_q \geq L_d$ (for surface mount PMSM, both of inductances are equal), the second item contributes a negative torque if the flux weakening control has been used. In order to achieve the maximum torque/current ratio, the d-axis current is set to zero during the constant torque control so that the torque is proportional only to q-axis current. Hence, this results in the control of q-axis current for regulating the torque in rotor reference frame.

12.3 PMSM Control System

Based on the above analysis, a PMSM drive system is developed as shown in Fig. 12.2. The total drive system looks similar to that of the BLDC motor and consists of a PMSM, power electronics converter, sensors, and controller. These components are discussed in detail in the following sections.

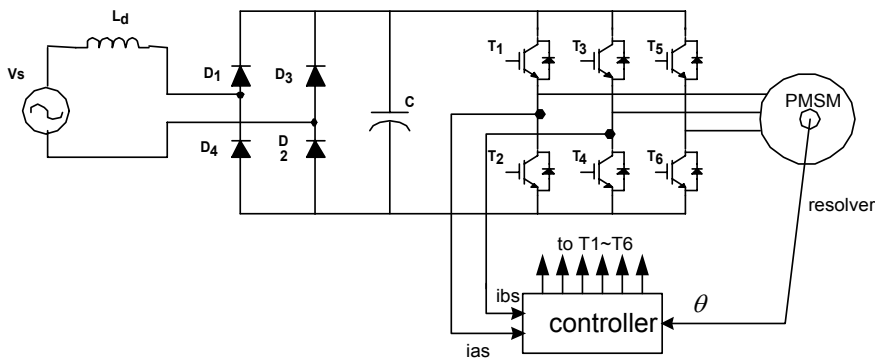


Figure 12.2 The PMSM speed control system.

12.3.1 PMSM Machine

The design consideration of the PMSM is to first generate the sinusoidal back-EMF. Unlike the BLDC, which needs concentrated windings to produce the trapezoidal back-EMF, the stator windings of PMSM are distributed in as many slots per pole as deemed practical to approximate a sinusoidal distribution. To reduce the torque ripple, standard techniques such as skewing and chorded windings are applied to the PMSM. With the sinusoidally excited stator, the rotor design of the PMSM becomes more flexible than the BLDC motor where the surface mount permanent magnet is a favorite choice. Besides the common surface mount non-salient pole PM rotor, the salient pole rotor, like inset and buried magnet rotors, are often used because they offer appealing performance characteristics during the flux weakening region. A typical PMSM with 36 stator slots in stator and four poles on the rotor is shown in Fig. 12.3.

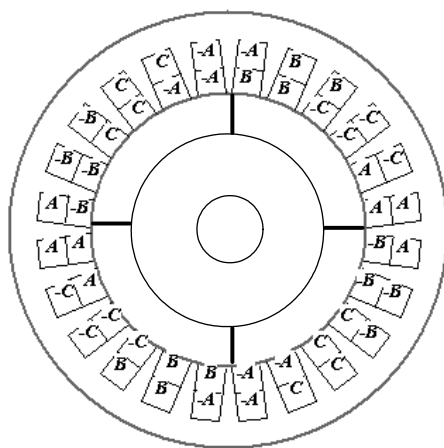


Figure 12.3 A four-pole 24-slot PMSM.

12.3.2 Power Electronic Converter

The PMSM shares the same topology of the power electronics converter as the BLDC motor drive system. The converter is the standard two-stage configuration with a dc link capacitor between a front-end rectifier and a three-phase full-bridge inverter as the output. The rectifier is either a full-bridge diode or power switch rectifier.

Due to the sinusoidal nature of the PMSM, control algorithms such as V/f and vector control, developed for other AC motors, can be directly applied to the PMSM control system. If the motor windings are Y-connected without a neutral connection, three phase currents can flow through the inverter at any moment. With respect to the inverter switches, three switches, one upper and two lower in three different legs conduct at any moment as shown in Fig. 12.4. PWM current control is still used to regulate the actual machine current. Either a hysteresis current controller, a PI controller with sine-triangle, or a SVPWM strategy is employed for this purpose. Unlike the BLDC motor, the three switches are switched at any time.

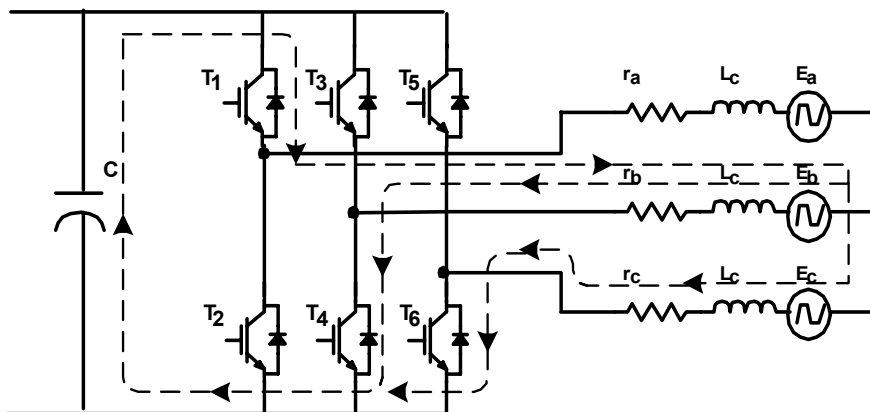


Figure 12.4 The current path when the three phases are chopped.

12.3.3 Sensors

There are two types of sensors used in the PMSM drive system: the current sensor, which measures the phase currents, and the position sensor which is used to sense the rotor position and speed. The resistances in series with the power switches as shown in Fig. 12.2 are usually used as shunt resistor phase current sensors. Either an encoder or resolver serves as the position sensor. Rotor position is needed in order to synchronize the stator excitation of the PMSM with the rotor speed and position.

Figure 12.5 shows the structure of an optical encoder. It consists of a light source, slotted disk, and photo sensors. The disk rotates with the rotor. The two photo sensors output a logic “1” when they detect light. When the light is blocked, a logic “0” is generated by the sensors. When the light passes through the slots of

the disk and strikes the sensor, a logic “1” is produced. These logic signals are shown in Fig. 12.5. By counting the number of pulses, the motor speed can be calculated. The direction of rotation can be determined by detecting the leading edge between signal A and signal B.

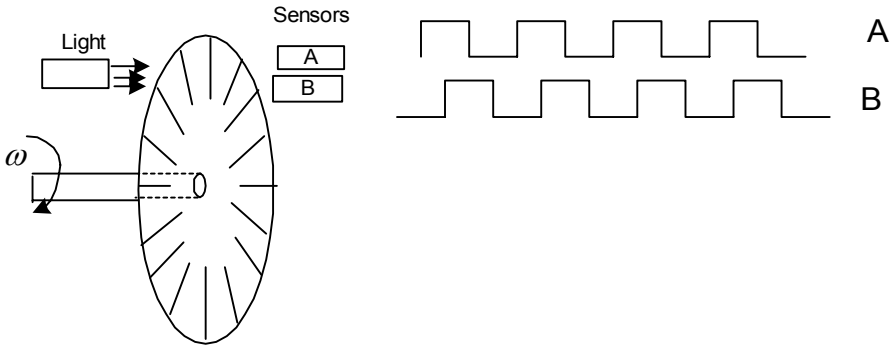


Figure 12.5 The structure of encoder.

A resolver is a rotary electromechanical transformer. It outputs two sinusoidal signals such that one wave is a sinusoidal function of the rotor angle θ , while the other signal is a cosinusoidal function of θ . The difference between these two waveforms reveals the position of the rotor. Integrated circuits such as the AD2S80 can be used to decode the signals. The resolver output waveform and the corresponding rotor position are given in Fig. 12.6.

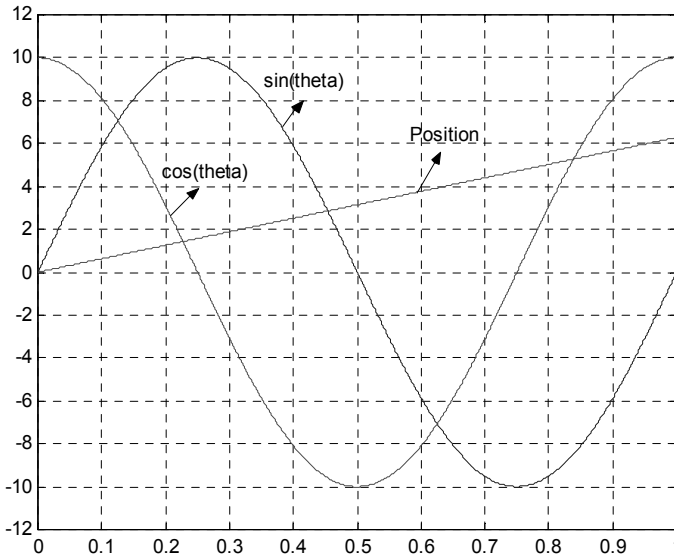


Figure 12.6 The resolver output and the corresponding rotor position.

12.3.4 Controller

The LF2407 is used as the controller to implement speed control of the PMSM system. The interface of the LF2407 is illustrated in Fig. 12.7. Similar to the BLDC motor control system, three input channels are selected to read the two phase currents and resolver signal. Because a resolver is used in one case, the QEP inputs are not used. QEP inputs work only with a QEP signal that a rotary encoder supplies. The DSP output pins PWM1-PWM6 used to supply the gating signals to the switches and form the output of the control part of the system.

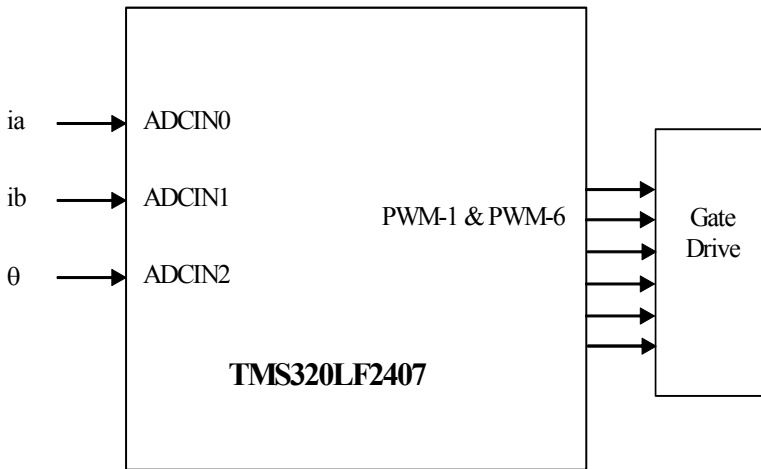


Figure 12.7 The interface of LF2407.

12.4 Implementation of the PMSM System Using the LF2407

A block diagram of the PMSM drive system is displayed in Fig. 12.8. An assembly code algorithm was written for the LF2407 to implement the control system shown inside the dashed line in Fig. 12.8.

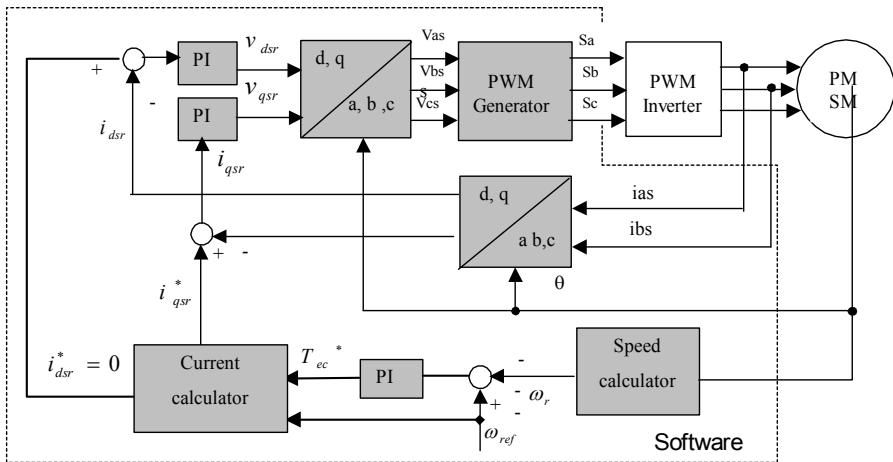


Figure 12.8 Block diagram of PMSM speed control system.

The flowchart of the developed software is shown in Fig. 12.9. The control program of the PMSM has one main routine and includes four modules:

1. Initialization procedure
2. DAC module
3. ADC module
4. Speed control module

The first three items introduced in Chapter 9. Hence, in the following section, only the speed control module is discussed in detail, with the corresponding assembly code given.

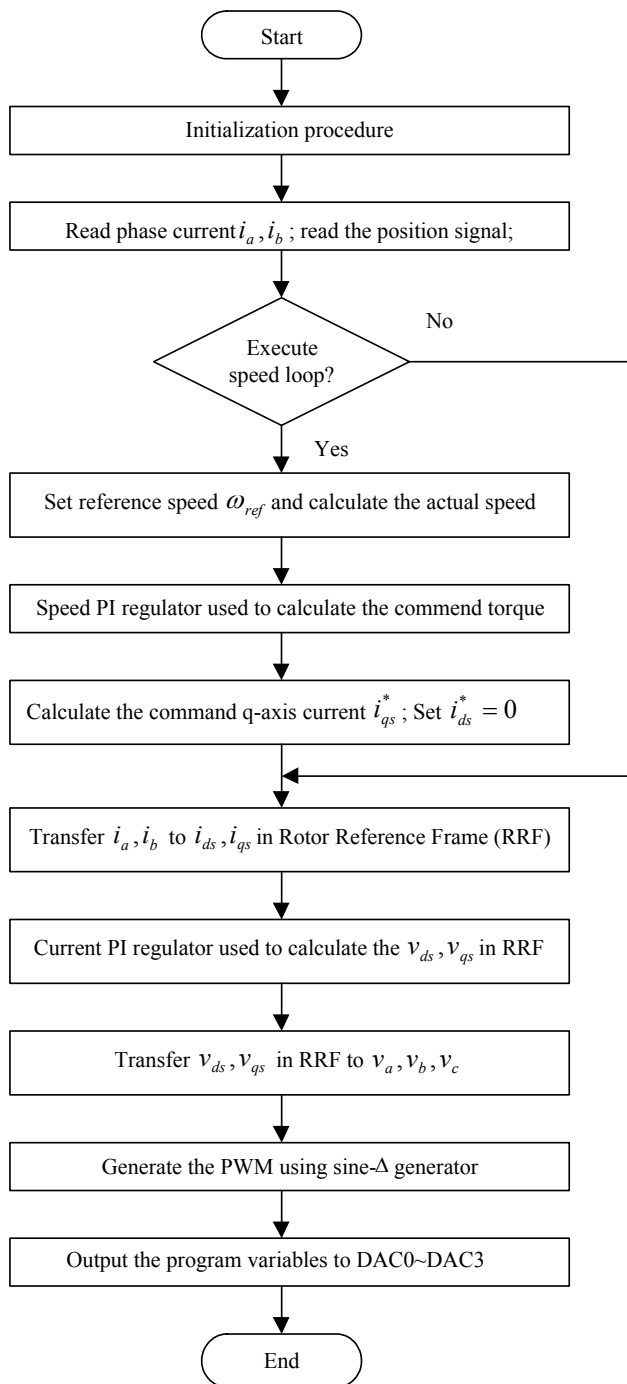


Figure 12.9 The flow chart of PMSM control system.

12.4.1 The Speed Control Algorithm

In the BLDC motor control system the Timer 1 underflow interrupt is used for the subroutine of speed control. This routine performs the tasks of:

- Reading the current and position signal, then generating the commanded speed profile.
- Calculating the actual motor speed, transferring the variables in the *abc* model to the *d-q* model and reverse.
- Regulating the motor speed and currents using the vector control strategy.
- Generating the PWM signal based on the calculated motor phase voltages.

The PWM frequency is determined by the time interval of the interrupt, with the controlled phase voltages being recalculated every interrupt. The modules of this routine are detailed in the following section. The code below shows this routine.

```
T1_PERIOD_ISR:
;Context save regs
    MAR    *,AR1                ;AR1 is stack pointer
    MAR    *+                  ;skip one position
    SST    #1, *+              ;save ST1
    SST    #0, *+              ;save ST0
    SACH   *+                  ;save acc high
    SACL   *                   ;save acc low
    POINT_EV
    SPLK   #0FFFFh,EVIFRA      ;Clear all Group A interrupt
                                           ;flags (T1 ISR)

READ_SIG
    CALL   ADC_CONV
    CALL   CAL_TRIANGLE
    CALL   ADC_DQ
    POINT_B0
    LACC   CL_SPD_FLG
    BCND   CURRENT_CNTRL,GT     ;speed-loop?
; speed control
SPEED_CNTRL:
    POINT_B0
    CALL   SPEED_PROFILE
    CALL   VTIMER_SEC
    CALL   SPEED_CAL
    CALL   D_PID_spd
    BLDD   #D_PID_out          ;iqsr
    SPLK   #0, idsr_ref

; current control
CURRENT_CNTRL
    CALL   D_PID_cur
    BLDD   #D_out_iq, Vqr
    BLDD   #D_out_id, Vdr
    CALL   DQ_ABC
    BLDD   #a_out, Va
    BLDD   #b_out, Vb
    BLDD   #c_out, Vc
PWM_GEN     CALL   PWM_DRV
DA_CONV     CALL   DAC_VIEW_Q15I
;Restore Context
END_ISR:
    MAR    *, AR1                ;make stack pointer active
```

```

LACL    *-                ;Restore Acc low
ADDH    *-                ;Restore Acc high
LST     #0, *-           ;load ST0
LST     #1, *-           ;load ST1
CLRC    INTM
RET

```

12.4.1.1 The Calculation of $\sin\theta$ and $\cos\theta$

A lookup table is used to calculate the sine and cosine values of the rotor position θ . The rotor electrical angle depends only on its sine value in lookup table. The cosine value is calculated by shifting the sine value 90 degrees. The sine and cosine values, which are used in the transformation, can be obtained by simply knowing the rotor angle. The code below shows how to read the 1:1 look-up table with the LF2407.

```

TRI_CAL
...
LACC    TRI_INT          ;load accumulator
AND     #0ffh           ;get lower bits
ADD     #SINTAB         ;table read
TBLR    sine_a
...
RET

```

The block of code below shows a portion of the sine value lookup table.

SINTAB	;SINVAL		Index	Angle	Sin(Angle)
....					
.word	12539	;	16	22.50	0.3827
.word	13279	;	17	23.91	0.4052
.word	14010	;	18	25.31	0.4276
.word	14732	;	19	26.72	0.4496
.word	15446	;	20	28.13	0.4714
.word	16151	;	21	29.53	0.4929
.word	16846	;	22	30.94	0.5141
.word	17530	;	23	32.34	0.5350
.word	18204	;	24	33.75	0.5556
....					
RET					

12.4.1.2 The abc-to-dq Transformation

The abc-to-dq transformation is defined in (12.9). It transfers the three-phase stationary motor model to a two-phase rotational motor model. In other words, under the restriction of the same motor performance, three phase stationary stator windings with 120° separation can be replaced by a two-phase rotational winding with the q-phase 90° ahead of d-phase. The two-phase currents are related to the three-phase currents as defined by the transformation in (12.9). After this transformation, a significant simplification is achieved. The d and q-axis variables are decoupled and independent with time and rotor position, which implies that these variables become constant in steady state. It is possible to control the d and q

variables independently. Since the d-axis variables are associated with the field variable and q-axis variables are related to the torque, this feature enables us to control the ac motor similar to a dc motor. For more detailed information on this topic we can refer to vector control theory. A portion of the abc-to-dq transformation using the assembly code is given in the code below:

```

ABC_DQ:
    ...
    ...
    LACC    #0
    LT      ABC_ain
    MPY     sone_a
    LTA     ABC_bin
    MPY     sone_b
    LTA     ABC_cin
    MPY     sone_c
    LTA     ABC_ain
    SACH    ABC_D_out
    ...
    ...
    RET
    
```

12.4.1.3 The d-q to a-b-c Transformation

After the commanded d and q-axes variables are calculated, these two variables are transferred to the a-b-c stationary frame to drive the motor. This reverse transform is defined as follows:

$$f_{abc} = K_r' f_{qd0r} \tag{12.16}$$

where

$$K_r' = \begin{bmatrix} \cos\theta_r & \sin\theta & \frac{1}{2} \\ \cos(\theta_r - \frac{2\pi}{3})_r & \sin(\theta_r - \frac{2\pi}{3}) & \frac{1}{2} \\ \cos(\theta_r + \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) & \frac{1}{2} \end{bmatrix} \tag{12.17}$$

An example of the assembly code to implement the above equation is given in the code below:

```

DQ_ABC
    ...
    ...
    LACC    #0
    LT      DQ_D_ref
    MPY     sone_a
    LTA     DQ_Q_ref
    MPY     cosone_a
    MPYA    cosone_b
    SACH    DQ_aout
    ...
    ...
    RET
    
```

12.4.1.4 PWM Generation

The PWM circuits of the 2407 Event Manager are used to generate the gating signals. Figure 12.10 displays the principle of this method. The control signal with frequency f_1 is constantly compared with a triangle signal which has a high-frequency f_2 (usually $f_2/f_1 > 21$). If the controlled signal is larger than the triangle signal, a PWM output signal becomes a logic “1”. Otherwise, a “0” is given.

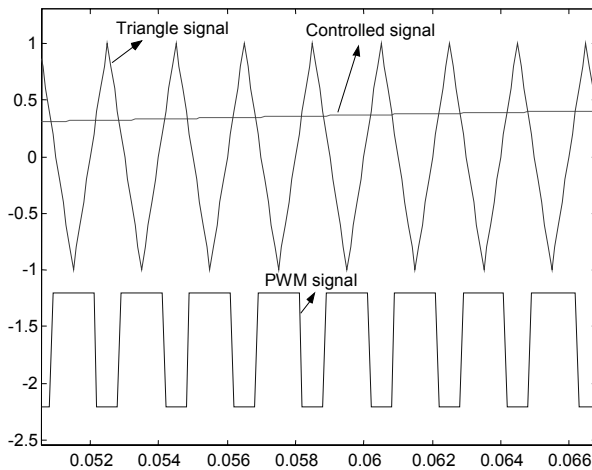


Figure 12.10 The principle of sine-triangle PWM generation.

The full-compare units have been used to generate the PWM outputs. The PWM signal is high when the output of current PI regulation matches the value of T1CNT and set low when the Timer underflow occurs. The switch states are controlled by the ACTR register. As discussed in Section 3.2, the lower switches should always be on and the upper switches should be chopped. From the point of implementation on the LF2407, this requires that the ACTR register is reset for each interval. Therefore, PWM1, PWM3, and PWM5, which trigger the upper switches, are set as *active low/high* and PWM2, PWM4, and PWM6, which trigger the lower switches are set as *force high*. The code below illustrates this implementation.

```
SINE_PWM:
    ....
    ....
POINT_B0
    MPY    Ub
    PAC
    ADD    PERIOD, 15
    POINT_EV
    SACH   CMR2
    ....
    RET
```