SYNTHESIS OF NETWORKS

* There are three types of passive elements Viz. Inductor (L), capacitor (c) and Resistance (R).
* Two Methods are available to synthesis
  1. Foster form and
  2. Cauer form.
* By using these two Methods we can synthesize
  1. LC Network
  2. RL Network
  3. RC Network

1. LC Network:

i) Properties of LC function:

1. poles and zeros must be simple. They must also lie on the imaginary axis (i.e) $j\omega$ axis
2. poles and zeros must be interlacing
3. There must be pole or zero at the origin (i.e) $s=0$
4. there must be a pole or zero at infinity (i.e) $s=\infty$
5. The slope of $F(j\omega)$ versus ω must be positive
6. Residues of poles and zeros on the imaginary axis must be real and positive

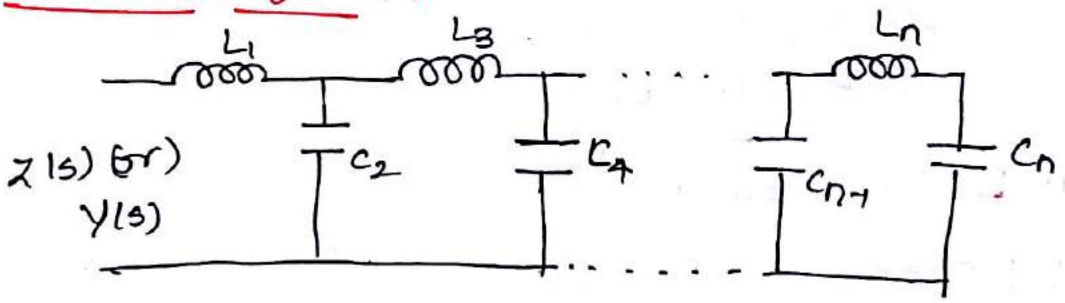
ii) Synthesize of LC Network in Cauer form:

In Cauer method, there are two types of ladder network.

1st Cauer form: Series arm - Inductor
Shunt arm - capacitor

II nd Cauer form: Series arm - Capacitor
Shunt arm - Inductor

First Cauer form:



The continued fraction expansion is

$$Z(s) = L_1 s + \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_4 s + \cdots}}}$$

* polynomial having descending powers of s

Condition for 1st Element and Last Element:

1st Element ⇒ At pole $= \infty$ $(s=\infty)$ = series Inductor
At Zero $= \infty$ $(s=\infty)$ = shunt capacitor

Last Element ⇒ At pole $= 0$ $(s=0)$ = shunt capacitor
At Zero $= 0$ $(s=0)$ = Series Inductor

1. Find the Cauer I form of the Network having

$$Z(s) = \frac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3}$$

Solution:

The given function has pole at $s = \infty$ and zero at $s = 0$ $\therefore$ Inductor is 1st & last Element.

$$s^4+4s^2+3 \;)\; 2s^5+12s^3+16s \;(\, 2s \rightarrow L_1 s$$
$$2s^5+8s^3+6s$$
$$4s^3+10s \;)\; s^4+4s^2+3 \;(\, \frac{s}{4} \rightarrow C_2 s$$
$$s^4+\frac{5}{2}s^2$$
$$\frac{3}{2}s^2+3 \;)\; 4s^3+10s \;(\, \frac{8s}{3} \rightarrow L_3 s$$
$$4s^3+8s$$
$$2s \;)\; \frac{3}{2}s^2+3 \;(\, \frac{3s}{4} \rightarrow C_4 s$$
$$\frac{3}{2}s^2$$
$$3 \;)\; 2s \;(\, \frac{2s}{3} \rightarrow L_5 s$$
$$2s$$
$$0$$

$$\therefore Z(s) = 2s + \cfrac{1}{\cfrac{s}{4} + \cfrac{1}{\cfrac{8s}{3} + \cfrac{1}{\cfrac{3s}{4} + \cfrac{1}{\cfrac{2s}{3}}}}}$$

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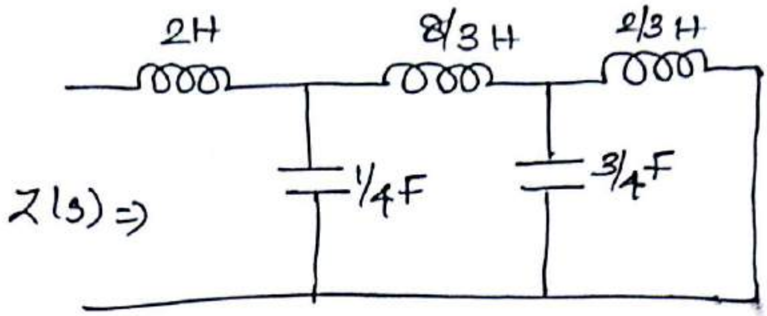
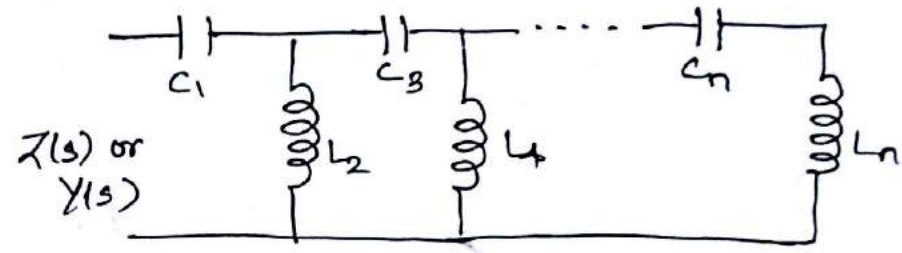


Second Cauer form :



The continued fraction expansion is

$$Z(s) = \frac{1}{C_1 s} + \cfrac{1}{\cfrac{1}{L_2 s} + \cfrac{1}{\cfrac{1}{C_3 s} + \cfrac{1}{\cfrac{1}{L_4 s} + \cdots}}}$$

* polynomial having ascending powers of s

Condition for 1st and Last Element:

1st Element => At pole = 0 (s = 0) => series capacitor
At zero = 0 (s = 0) => shunt Inductance

Last Element => At pole = ∞ (s = ∞) => Inductor
At zero = ∞ (s = ∞) => capacitor

* Numerator power should be greater than the denominator power if not inverse Z(s).

$$\therefore Y(s) = \frac{1}{Z(s)}$$

1. find the Second cauer form of the network

$$Z(s) = \frac{s^4+4s^2+3}{s^3+2s}$$

Solution:

$$Z(s) = \frac{3+4s^2+s^4}{2s+s^3}$$

$$2s+s^3\;)\;3+4s^2+s^4\;\left(\frac{3}{2s}\right. \rightarrow \frac{1}{C_1 s}$$

$$3+\frac{3}{2}s^2$$

$$\frac{5s^2}{2}+s^4\;)\;2s+s^3\;\left(\frac{4}{5s}\right. \rightarrow \frac{1}{L_2 s}$$

$$2s+\frac{4s^3}{5}$$

$$\frac{s^3}{5}\;)\;\frac{5}{2}s^2+s^4\;\left(\frac{25}{2s}\right. \rightarrow \frac{1}{C_3 s}$$

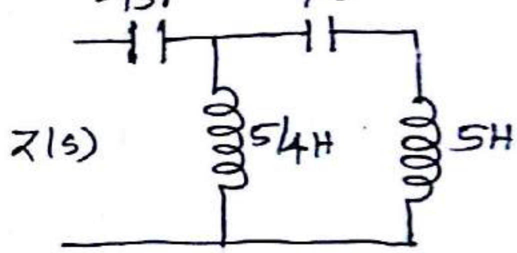
$$\frac{5}{2}s^2$$

$$s^4\;)\;\frac{s^3}{5}\;\left(\frac{1}{5s}\right. \rightarrow \frac{1}{L_4 s}$$

$$\frac{s^3}{5}$$

$$0$$

$$Z(s) = \frac{3}{2s} + \cfrac{1}{\cfrac{4}{5s} + \cfrac{1}{\cfrac{25}{2s} + \cfrac{1}{\cfrac{1}{5s}}}}$$



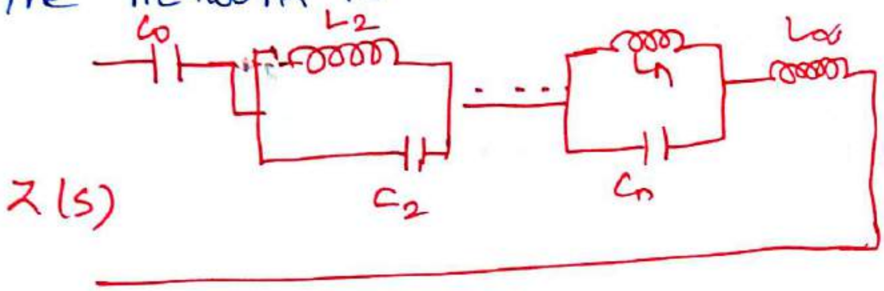
iii) Synthesize of LC Network in foster form:

Foster I Method:

The driving point impedance of a LC network is

$$Z(s) = Z_1(s) + Z_2(s) + \dots + Z_n(s)$$

The network is



$$Z_1(s) = \frac{P_0}{sC_0} = \frac{P_0}{s} \text{ where } P_0 = \frac{1}{C_0}$$

$$Z_2(s) = L_2 s \parallel \frac{1}{C_2 s} = \frac{L_2 s \times \frac{1}{sC_2}}{L_2 s + \frac{1}{sC_2}}$$

$$= \frac{L_2 s}{s^2 C_2 L_2 + 1} = \frac{(L_2 s / C_2 L_2)}{s^2 + \frac{1}{C_2 L_2}}$$

$$Z_2(s) = \frac{\left(\frac{1}{C_2}\right) s}{s^2 + \omega_2^2} = \frac{2P_n s}{s^2 + \omega_n^2}$$

$$\text{where } 2P_n = \frac{1}{C_n} \text{ \& } \omega_n^2 = \frac{1}{C_n L_n} \quad \text{— (1)}$$

$$Z_n(s) = L\infty = Hs$$

$$\therefore Z(s) = \frac{P_0}{s} + \frac{2P_n s}{s^2 + \omega_n^2} + Hs$$

from (1),

$$C_n = \frac{1}{2P_n}, \quad L_n = \frac{1}{C_n \omega_n^2}$$

$$C_0 = \frac{1}{P_0}, \quad L_\infty = H$$

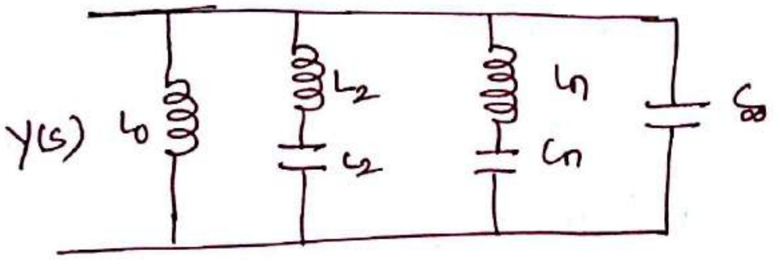
* If there is a pole at $\omega = 0$, the first element C_0 is present in the network.
* If there is a pole at $\omega = \infty$, the last element L_∞ is present in the network.

Foster II Method:

The driving point admittance $Y(s)$ of LC network is

$$Y(s) = Y_1(s) + Y_2(s) + \dots + Y_n(s)$$

$$\therefore Y(s) = \frac{H}{s} \frac{(s^2+\omega_1^2)(s^2+\omega_3^2)}{(s^2+\omega_2^2)(s^2+\omega_4^2)}$$



$$Y_1(s) = \frac{1}{L_0 s} = \frac{P_0}{s} \quad \text{where } P_0 = \frac{1}{L_0}$$

$$Y_2(s) = \frac{1}{L_2 s + \frac{1}{sC_2}} = \frac{sC_2}{s^2 L_2 C_2 + 1}$$

$$Y_2(s) = \frac{(s/L_2)}{s^2 + \frac{1}{L_2 C_2}}$$

$$Y_n(s) = C_\infty = Hs$$

$$Y(s) = \frac{P_0}{s} + \frac{2P_2 s}{s^2+\omega_2^2} + \frac{2P_4 s}{s^2+\omega_4^2} + \cdots + Hs$$

$$Y(s) = \frac{P_0}{s} + \frac{2P_n s}{s^2+\omega_n^2} + Hs$$

$$\therefore L_0 = \frac{1}{P_0}$$

$$C_\infty = H$$

$$P_n = \frac{1}{2L_n} \Rightarrow L_n = \frac{1}{2P_n}$$

$$C_n = \frac{2P_n}{\omega_n^2}$$

* If there is a pole at $\omega=0$, the first element L_0 is present in the network.

* If there is a pole at $\omega=\infty$, the last element C_∞ is present in the network.

1. The driving point impedance of a LC network is given by

$$Z(s) = \frac{5(s^2+4)(s^2+25)}{s(s^2+16)}$$

obtain first and second foster networks.

Solution :

numerator has higher power than the denominator.

First foster form:

By partial fraction,

$$Z(s) = \frac{A}{s} + \frac{B}{(s+j4)} + \frac{B^*}{s-j4}$$

Apply heaviside method,

$$A = \cancel{s}\,\frac{5(s^2+4)(s^2+25)}{\cancel{s}(s^2+16)}\Bigg|_{s=0} = \frac{500}{16} = \frac{125}{4}$$

$$B = \cancel{(s+j4)}\,\frac{5(s^2+4)(s^2+25)}{s\cancel{(s+j4)}(s-j4)}\Bigg|_{s=-j4} = \frac{-\cancel{12}\times 9\times 5}{(-j4)(-8j)}$$

$$= \frac{\cancel{20}\times\cancel{41}}{\cancel{32}} = \frac{135}{8}$$

$$B^* = \frac{135}{8}$$

$$\therefore Z(s) = \frac{(125/4)}{s} + \frac{2(135/8)s}{s^2+16} \quad -(1)$$

By inspection, $H=5$.

$$Z(s) = \frac{P_0}{s} + \frac{2P_n s}{s^2+\omega_n^2} + Hs \quad -(2)$$

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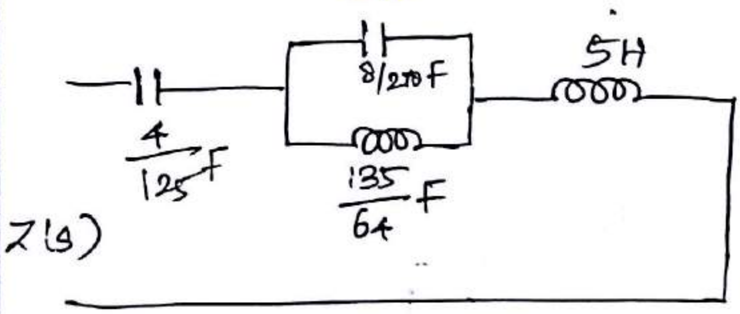
By Comparing ① & ②

$$C_0 = \frac{1}{p_0} = \frac{1}{\left(\frac{125}{4}\right)} = \frac{4}{125}\,F$$

$$L_\infty = H = 5H$$

$$C_1 = \frac{1}{2p_1} = \frac{1}{2\times\left(\frac{135}{8}\right)} = \frac{8}{2\times 135} = \frac{8}{270}\,F$$

$$L_1 = \frac{1}{C_1\omega_1^2} = \frac{1}{\frac{8}{270}\times 16} = \frac{270}{8\times 16} = \frac{270}{128} = \frac{135}{64}\,H$$



Second Foster form:

$$Y(s) = \frac{s(s^2+16)}{5(s^2+4)(s^2+25)}$$

By partial fraction,

$$Y(s) = \frac{A}{s+2j} + \frac{A^*}{s-2j} + \frac{B}{s+5j} + \frac{B^*}{s-5j}$$

$$A = \left.\frac{\cancel{(s+2j)}\, s(s^2+16)}{5\cancel{(s+2j)}(s-2j)(s^2+25)}\right|_{s=-2j}$$

$$= \frac{-\cancel{2j}\times -\cancel{12}\,6}{5\times -\cancel{4j}\times -21} = \frac{6}{105} = \frac{2}{35}$$

$$B = \left.\cancel{(s+5j)}\,\frac{s(s^2+16)}{5(s^2+4)\cancel{(s+5j)}(s-5j)}\right|_{s=-5j}$$

$$= \frac{-\cancel{5j}\times -\cancel{9}\,3}{\cancel{5}\times -\cancel{21}\,7\times -10j} = \frac{3}{70}$$

$$\therefore Y(s) = \frac{2\left(\frac{2}{35}\right)s}{s^2+4} + \frac{2\left(\frac{3}{70}\right)s}{s^2+25}$$

$$\therefore \quad Y(s) = \frac{p_0}{s} + \frac{2p_2 s}{s^2+\omega_2^2} + \cdots + Hs$$

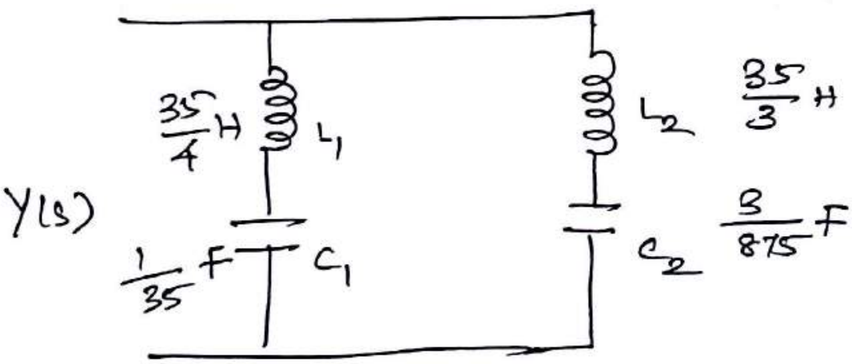
$$C_0 = 0 \quad , \quad C_\infty = 0$$

$$L_1 = \frac{1}{2p_1} \Rightarrow \frac{1}{2\times\frac{2}{35}} = \frac{35}{4}\,H$$

$$C_1 = \frac{2p_1}{\omega_1^2} = \frac{2\times 2}{35\times 4} = \frac{1}{35}\,F$$

$$L_2 = \frac{1}{2p_2} = \frac{\cancel{70}\,35}{\cancel{2}\times 3} = \frac{35}{3}\,H$$

$$C_2 = \frac{2p_2}{\omega_2^2} = \frac{\cancel{2}\times 3}{\underset{35}{\cancel{70}}\times 25} = \frac{3}{875}\,F$$



2. RC Network:

i) propeeties of RC function:

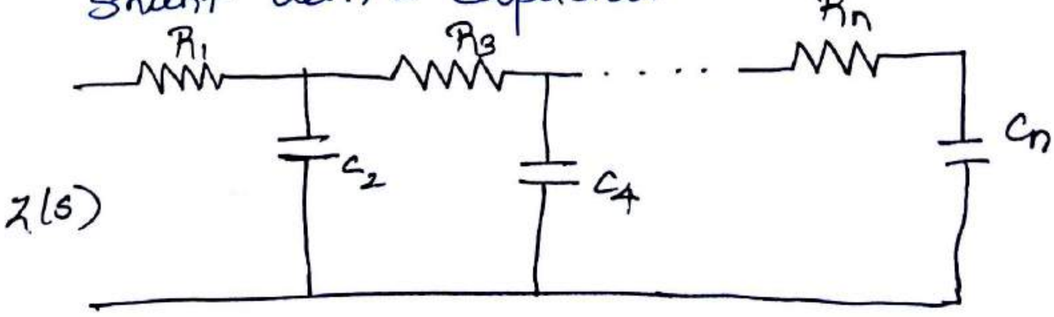
Impedance	Admittance
1. poles and zeros are simple and lie on negative real axis of 's' plane	
2. poles and zeros are interlaced	
3. There must a pole at the origin or near to the origin (s=0)	3. There must be a zero at the origin or near to the origin (s=0)
4. There must a zero at the ~~origin~~ infinity or near to the infinity (s=∞)	4. There must be a pole at the infinity or near to the infinity (s=∞)
5. The degree of the denominator can be equal to or greater than atmost by one than numerator	5. The degree of the numerator can be equal to or greater than atmost by one than the degree of the denominator.
6. Residues of the poles of impedance function must be real and positive	6. Residues of poles of admittance are real and negative. $\frac{Y_{RC}(s)}{s}$ are real and positive
7. $Z_{RC}(0) > Z_{RC}(\infty)$	7. $Y_{RC}(0) \leq Y_{RC}(\infty)$

ii) Synthesize of RC network in cauer form

First Cauer form:

series arm - Resistor

shunt arm - capacitor



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Conditions for 1st and Last Element:

* If $Z(s)$ has a zero at $s=\infty$, 1st element is C_1
* If $Z(s)$ is a constant at $s=\infty$, 1st element is R_1
* If $Z(s)$ has a pole at $s=0$, last element is C_n
* If $Z(s)$ is a constant at $s=0$, last element is R_n

* The continued fraction expansion for the first Cauer form,

$$Z(s) = R_1 + \cfrac{1}{C_2 s + \cfrac{1}{R_3 + \cfrac{1}{C_4 s + \dots}}}$$

1. find the first Cauer form

$$Z(s) = \frac{(s+2)(s+4)}{s(s+3)}$$

Solution:

$$Z(0) = \frac{8}{0} = \infty$$

$$Z(\infty) = \frac{s^2\left(1+\frac{2}{s}\right)\left(1+\frac{4}{s}\right)}{s^2\left(1+\frac{3}{s}\right)} = 1$$

$$Z(s) = \frac{s^2+6s+8}{s^2+3s}$$

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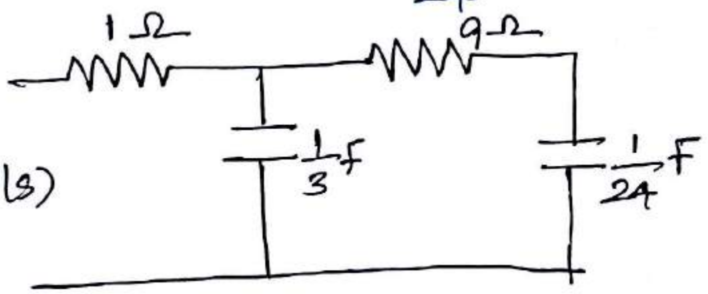
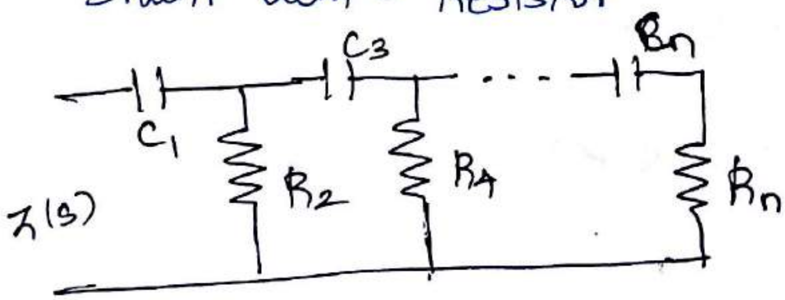
$$s^2+3s \,)\, s^2+6s+8 \,(\, 1$$
$$\underline{s^2+3s}$$
$$3s+8 \,)\, s^2+3s \,(\, \tfrac{s}{3}$$
$$\underline{s^2+\tfrac{8s}{3}}$$
$$\tfrac{s}{3} \,)\, 3s+8 \,(\, 9$$
$$\underline{3s}$$
$$8 \,)\, \tfrac{s}{3} \,(\, \tfrac{s}{24}$$
$$\underline{s/3}$$
$$\underline{0}$$

$$Z(s) = 1 + \cfrac{1}{\cfrac{s}{3} + \cfrac{1}{9 + \cfrac{1}{\cfrac{s}{24}}}}$$



Second Cauer form:

series arm - capacitor
shunt arm - Resistor



The continued fraction expansion is

$$Z(s) = \frac{1}{C_1 s} + \cfrac{1}{\frac{1}{R_2} + \cfrac{1}{\frac{1}{C_3 s} + \cfrac{1}{\frac{1}{R_4} + \cdots}}}$$

Condition for first and Last Element:

* If $Z(s)$ has a pole at $s=0$, 1^{st} element is C_1
* If $Z(s)$ has a constant at $s=0$, " " R_1
* If $Z(s)$ has a zero at $s=\infty$, last element is C_n
* If $Z(s)$ has a constant at $s=\infty$, " " R_n

1. find the Second Cauer form

$$Z(s) = \frac{s^2+6s+8}{s^2+3s}$$

Solution:

$$Z(s) = \frac{8+6s+s^2}{3s+s^2}$$

$$Z(0) = \frac{(s+2)(s+4)}{(s+0)(s+3)} = \frac{8}{0} = \infty$$

$$Z(\infty) = \frac{s^2\left(1+\frac{2}{s}\right)\left(1+\frac{4}{s}\right)}{s^2\left(1+\frac{0}{s}\right)\left(1+\frac{3}{s}\right)} = 1$$

$$Z(0) > Z(\infty)$$

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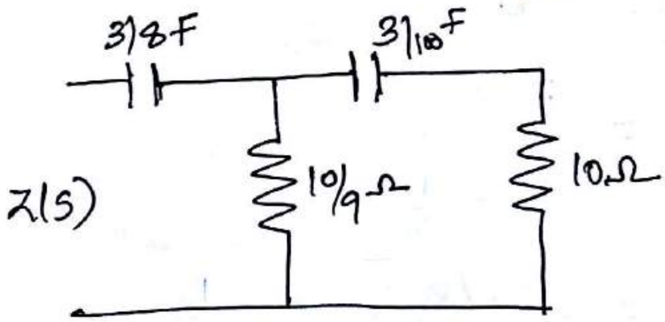
$$3s + s^2 \overline{)\, 8 + 6s + s^2 \,}\left(\frac{8}{3s}\right.$$
$$8 + \frac{8s}{3}$$

$$\frac{10s}{3} + s^2 \overline{)\, 3s + s^2 \,}\left(\frac{9}{10}\right.$$
$$3s + \frac{9s^2}{10}$$

$$\frac{s^2}{10} \overline{)\, \frac{10s}{3} + s^2 \,}\left(\frac{100}{3s}\right.$$
$$\frac{10s}{3}$$

$$s^2 \overline{)\, \frac{s^2}{10} \,}\left(\frac{1}{10}\right.$$
$$\frac{s^2}{10}$$
$$0$$

$$Z(s) = \frac{8}{3s} + \cfrac{1}{\frac{9}{10} + \cfrac{1}{\frac{100}{3s} + \frac{1}{10}}}$$



iii) Synthesize of RC Network by Foster Method:

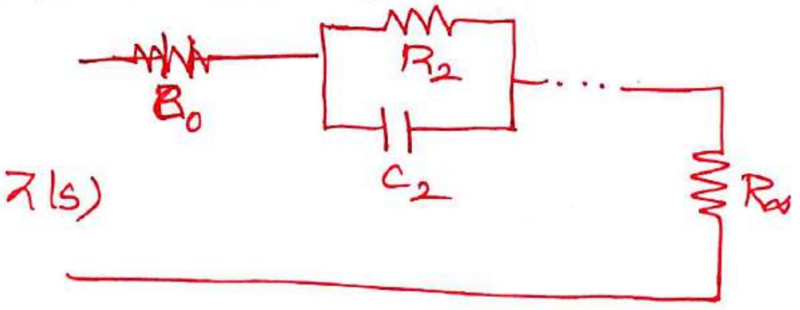
Foster -1 Form:

Properties of RL admittance is same for RC impedance also.

The driving point impedance of RC network is

$$Z(s) = \frac{H(s+\sigma_1)(s+\sigma_3)\ldots\ldots}{s(s+\sigma_2)(s+\sigma_4)\ldots\ldots}$$

The network is



$$Z(s) = Z_1(s) + Z_2(s) + \cdots + Z_n(s)$$

$$Z_1(s) = \frac{1}{sC_0} = \frac{P_0}{s} \quad \text{where } P_0 = \frac{1}{C_0}$$

$$Z_2(s) = R_2 \,||\, \frac{1}{sC_2} = \frac{R_2 \times \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{\left(R_2 \times \frac{1}{sC_2}\right)/sC_2}{R_2 sC_2 + 1}$$

$$= \frac{R_2}{1+sR_2C_2} = \frac{R_2/R_2C_2}{\frac{1}{R_2C_2}+s} = \frac{1/C_2}{s+\sigma_2}$$

$$= \frac{P_n}{s+\sigma_n} \quad \text{where } P_n = \frac{1}{C_A} \quad \sigma_A = \frac{1}{R_A C_A}$$

$$Z_n(s) = H$$

$$Z(s) = \frac{P_0}{s} + \frac{P_n}{s+\sigma_n} + \cdots + H$$

$$\therefore P_0 = \frac{1}{C_0} \Rightarrow C_0 = \frac{1}{P_0}$$

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$$P_n = \frac{1}{C_n} \Rightarrow C_n = \frac{1}{P_n}$$

$$\sigma_n = \frac{1}{R_n C_n} \Rightarrow R_n = \frac{1}{\sigma_n C_n}$$

1. find the first foster form for the deiving point function $Z(s) = \frac{(s+1)(s+3)}{s(s+2)}$

Solution:

By taking partial fraction,

$$Z(s) = \frac{(s+1)(s+3)}{s(s+2)} = \frac{s^2+4s+3}{s^2+2s}$$

$$s^2+2s \,\big)\, s^2+4s+3 \,(\, 1$$
$$\underline{s^2+2s}$$
$$\underline{2s+3}$$

$$\therefore Z(s) = 1 + \frac{2s+3}{s^2+2s}$$

$$Z(s) = 1 + \frac{A}{s} + \frac{B}{s+2}$$

By heavisides method,

$$A = s \cdot \left. \frac{(2s+3)}{s(s+2)} \right|_{s=0} = \frac{3}{2}$$

$$B = \left. (s+2) \frac{(2s+3)}{s(s+2)} \right|_{s=-2} = \frac{-1}{-2} = \frac{1}{2}$$

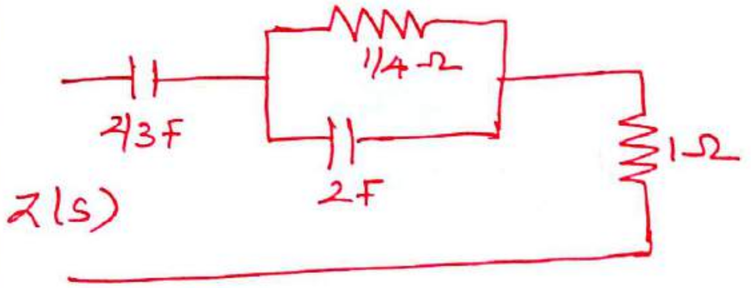
$$\therefore Z(s) = 1 + \frac{(3/2)}{s} + \frac{(1/2)}{s+2}$$

$R_\infty = H = 1\Omega$

$$C_0 = \frac{1}{P_0} = \frac{1}{(3/2)} = \frac{2}{3} F$$

$$C_1 = \frac{1}{P_1} = \frac{1}{(1/2)} = 2F$$

$$R_1 = \frac{1}{\sigma_1 C_1} \Rightarrow \frac{1}{2\times 2} = \frac{1}{4}\Omega$$



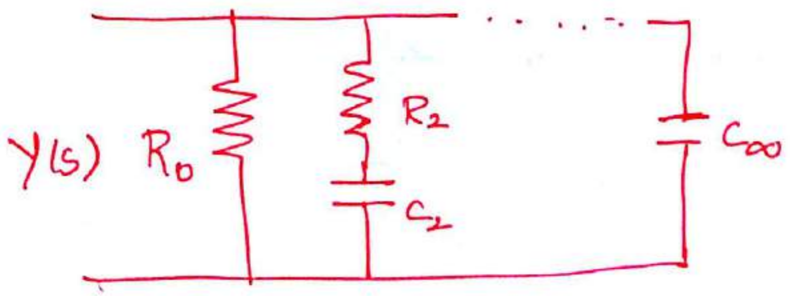
Foster II form:

* properties of RL impedance is same for RC admittance.

* The driving point function of RC network Y(s) is given by

$$Y(s) = \frac{H\,(s+\sigma_1)(s+\sigma_3)\cdots}{(s+\sigma_2)(s+\sigma_4)\cdots}$$

The network is



$$Y(s) = Y_1(s) + Y_2(s) + \cdots + Y_n(s)$$

$$Y_1(s) = P_0 = \frac{1}{R_0}$$

$$Y_2(s) = \frac{1}{R_2 + \frac{1}{sC_2}} = \frac{sC_2}{sC_2R_2+1} = \frac{sC_2/R_2C_2}{s+\frac{1}{R_2C_2}}$$

$$= \frac{(1/R_2)s}{s+\sigma_2} = \frac{P_n s}{s+\sigma_n}$$

where $P_n = \frac{1}{R_n}$

$\sigma_n = \frac{1}{R_n C_n}$

$$Y_n(s) = C_\infty = Hs$$

$$Y(s) = P_0 + \frac{P_n s}{s+\sigma_n} + Hs$$

$\therefore P_0 = \frac{1}{R_0} \Rightarrow R_0 = \frac{1}{P_0}$

$\therefore C_\infty = H$

$P_n = \frac{1}{R_n} \Rightarrow R_n = \frac{1}{P_n}$

$\sigma_n = \frac{1}{R_n C_n} \Rightarrow C_n = \frac{1}{\sigma_n R_n}$

1. find the second foster form

$$Y(s) = \frac{s(s+2)}{(s+1)(s+3)}$$

Solution:

$$Y(s) = \frac{s^2+2s}{s^2+4s+3}$$

$$\begin{array}{r} s^2+4s+3\,\big)\,s^2+2s\,(1 \\ s^2+4s+3 \\ \hline -2s-3 \\ \hline \end{array}$$

$$Y(s) = 1 + \frac{(-2s-3)}{s^2+4s+3}$$

Taking partial fraction,

$$= 1 + \frac{A}{s+1} + \frac{B}{s+3}$$

using Heavisides Method,

$$A = \cancel{(s+1)} \frac{(-2s-3)}{\cancel{(s+1)}(s+3)}\Bigg|_{s=-1} = \frac{-1}{2}$$

$$B = \cancel{(s+3)} \frac{(-2s-3)}{(s+1)\cancel{(s+3)}}\Bigg|_{s=-3} = \frac{-3}{2}$$

$$Y(s) = 1 - \frac{(1/2)}{s+1} - \frac{(3/2)}{s+3}$$

Since residues are negative, we have to expand $\frac{Y(s)}{s}$ as follows,

$$\frac{Y(s)}{s} = \frac{\cancel{s}(s+2)}{\cancel{s}(s+1)(s+3)} = \frac{s+2}{(s+1)(s+3)}$$

$$= \frac{A}{s+1} + \frac{B}{s+3}$$

using Heavisides Method,

$$A = \cancel{(s+1)} \frac{(s+2)}{\cancel{(s+1)}(s+3)}\Bigg|_{s=-1} = \frac{1}{2}$$

$$B = \cancel{(s+3)} \frac{(s+2)}{(s+1)\cancel{(s+3)}}\Bigg|_{s=-3} = \frac{1}{2}$$

$$\frac{Y(s)}{s} = \frac{(1/2)}{s+1} + \frac{(1/2)}{s+3}$$

$$Y(s) = \frac{(1/2)s}{s+1} + \frac{(1/2)s}{s+3}$$

The element values are

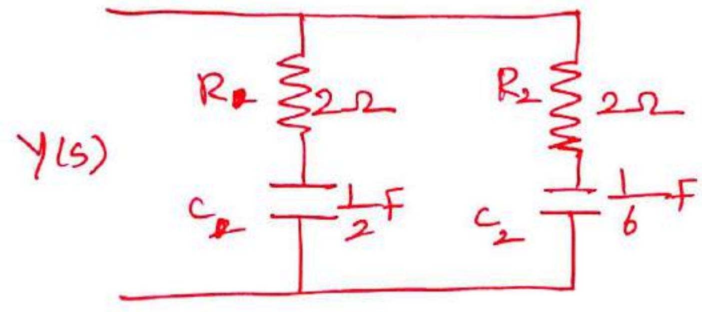
$R_0 = 0$, $C_\infty = 0$

$$\frac{p_1 s}{s+\sigma_1} = \frac{(1/2)s}{s+1} \Rightarrow p_1 = 1/2 \Rightarrow R_1 = \frac{1}{p_1} = 2\Omega$$

$$C_1 = \frac{1}{\sigma_1 R_1} = \frac{1}{1\times 2} = \frac{1}{2} F$$

$$\frac{p_2 s}{s+\sigma_2} = \frac{(1/2)s}{s+3} \Rightarrow p_2 = \frac{1}{2} \Rightarrow R_2 = \frac{1}{p_2} = 2\Omega$$

$$C_2 = \frac{1}{\sigma_2 R_2} \Rightarrow \frac{1}{3\times 2} = \frac{1}{6} F$$



3. RL Network:

i) Propeeties of RL function:

a) Propeeties of RL impedance is same as the propeeties of RC Admittance

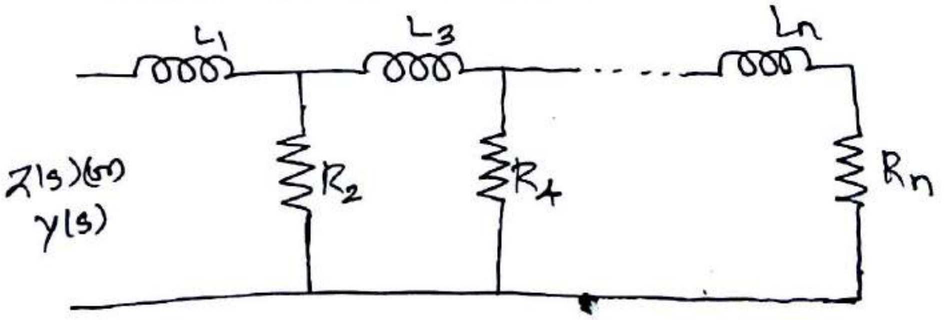
b) propeeties of RL Admittance is same as the propeeties of RC impedance.

ii) Synthesize of RL network in cauer form:

First Cauer form:

Selies aem - Inductor

Shunt aem - Resistor



The continued fraction expansion is

$$Z(s) = sL_1 + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{sL_3 + \cfrac{1}{\frac{1}{R_4} + \cdots}}}$$

Condition for 1st and Last Element:

* If $Z(s)$ has a pole at $s=\infty$, 1st element is L_1
* If $Z(s)$ is a constant at $s=\infty$, " " R_1
* If $Z(s)$ has a zero at $s=0$, Last element is L_n
* If $Z(s)$ is a constant at $s=0$, " " R_n

1) find first cauer form

$$Z(s) = \frac{(s+4)(s+8)}{(s+2)(s+6)}$$

Solution:

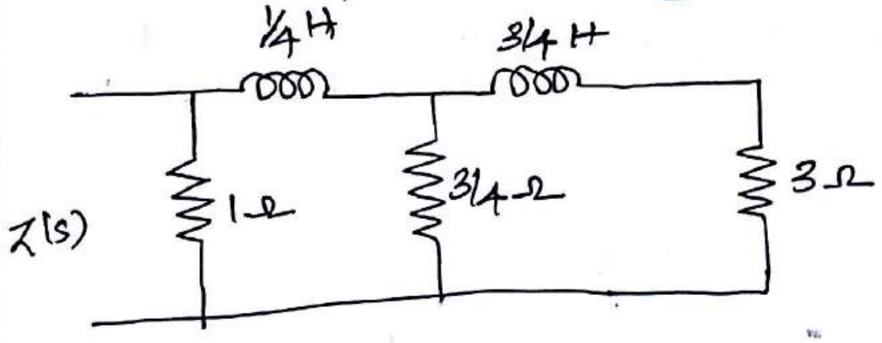
$$Z(0) = \frac{32}{12} = 2.67$$

$$Z(\infty) = \frac{s^2(1+\frac{4}{s})(1+\frac{8}{s})}{s^2(1+\frac{2}{s})(1+6/s)} = 1$$

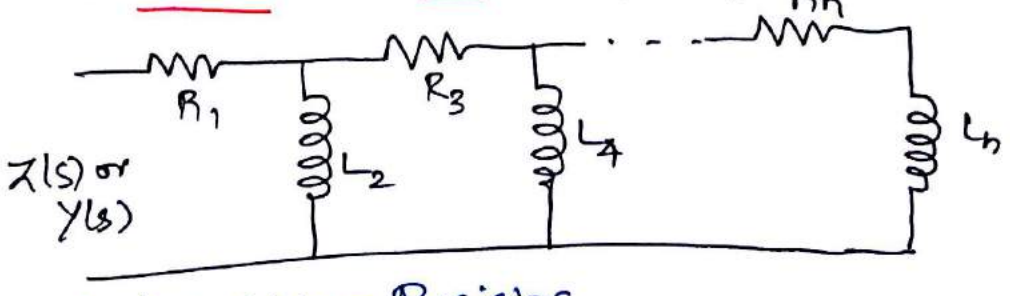
$$Z(0) > Z(\infty)$$

$$s^2+8s+12\,\big)\; s^2+12s+32\;\big(\,1$$
$$\underline{s^2+8s+12}$$
$$4s+20\,\big)\; s^2+8s+12\;\big(\,\tfrac{s}{4}$$
$$\underline{s^2+5s}$$
$$3s+12\,\big)\; 4s+20\;\big(\,\tfrac{4}{3}$$
$$\underline{4s+16}$$
$$4\,\big)\; 3s+12\;\big(\,\tfrac{3s}{4}$$
$$\underline{3s}$$
$$12\,\big)\; 4\;\big(\,\tfrac{1}{3}$$
$$\underline{4}$$
$$\underline{0}$$

$$Z(s) = \cfrac{1}{1+\cfrac{s}{4}+\cfrac{1}{\cfrac{4}{3}+\cfrac{1}{\cfrac{3}{4}s+\cfrac{1}{\cfrac{1}{3}}}}}$$



Second Cauer form:



Series arm - Resistor
Shunt arm - Inductor

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The continued fraction expansion is

$$Z(s) = R_1 + \cfrac{1}{\cfrac{1}{sL_2} + \cfrac{1}{R_3 + \cfrac{1}{sL_4} + \cfrac{1}{R_5 + \cdots}}}$$

Condition for 1st and Last Element:

* If $Z(s)$ has a zero at $s=0$, 1st element is L_1
* If $Z(s)$ is a constant at $s=0$, " " R_1
* If $Z(s)$ has a pole at $s=\infty$, last element is L_n
* If $Z(s)$ is a constant at $s=\infty$, " " R_n

1. find the second cauer form

$$Z(s) = \frac{2s^2+8s+6}{s^2+8s+12}$$

Solution:

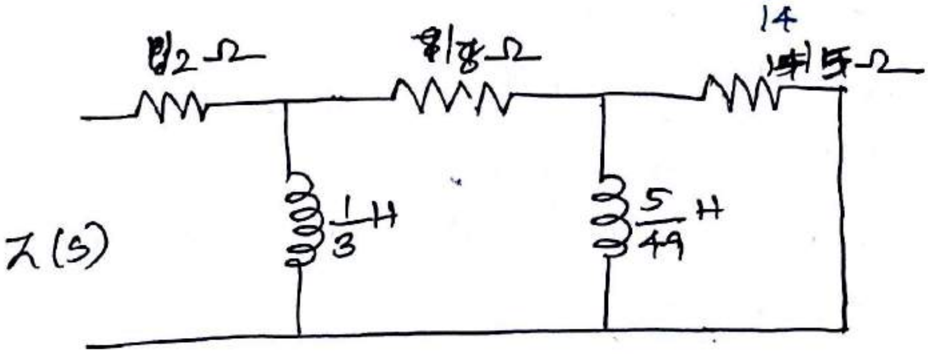
$$Z(0) = \frac{6}{12} = 0.5$$

$$Z(\infty) = \frac{(s+1)(s+3)}{(s+2)(s+6)} = \frac{s^2\left(1+\frac{1}{s}\right)\left(1+\frac{3}{s}\right)}{s^2\left(1+\frac{2}{s}\right)\left(1+\frac{6}{s}\right)} = 1$$

$$Z(0) < Z(\infty)$$

$$12+8s+s^2 \overline{\big)\, 6+8s+2s^2 \,}\big(\tfrac{1}{2}$$
$$6+4s+\frac{s^2}{2}$$
$$\overline{}$$
$$4s+\frac{3}{2}s^2 \big)\, 12+8s+s^2 \,\big(\tfrac{3}{s}$$
$$12+\frac{9}{2}s$$
$$\overline{}$$
$$\frac{7}{2}s+s^2 \big)\, 4s+\frac{3}{2}s^2 \,\big(\tfrac{8}{7}$$
$$4s+\frac{8s^2}{7}$$
$$\overline{}$$
$$\frac{5}{14}s^2 \big)\, \frac{7}{2}s+s^2 \,\big(\tfrac{49}{5s}$$
$$\frac{7}{2}s$$
$$\overline{}$$
$$s^2 \big)\, \frac{5}{14}s^2 \,\big(\tfrac{5}{14}$$
$$\frac{5}{14}s^2$$
$$\overline{}$$
$$0$$

$$Z(s)=\frac{1}{2}+\cfrac{1}{\frac{3}{s}+\cfrac{1}{\frac{8}{7}+\cfrac{1}{\frac{49}{5s}+\cfrac{1}{\frac{5}{14}}}}}$$

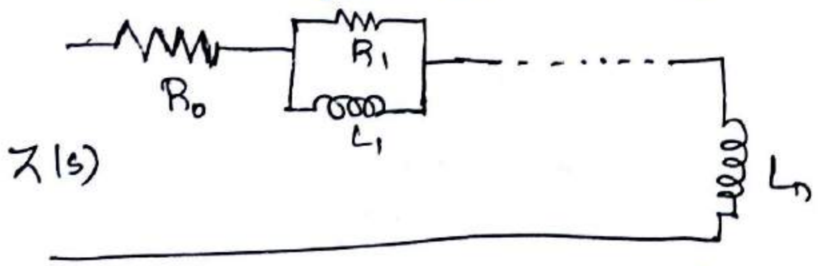


iii) Synthesize of RL network by the Foster form.

Foster -1 form:

The driving point impedance function of RL network is given by

$$Z(s) = \frac{H(s+\sigma_1)(s+\sigma_3)\cdots}{(s+\sigma_2)(s+\sigma_4)\cdots}$$



Expression for parallel combination of R_1 & L_1,

$$Z_1(s) = R_1 \| L_1 s = \frac{R_1 \times sL_1}{R_1 + sL_1}$$

$\div$ by $L_1 \Rightarrow Z_1(s) = \dfrac{R_1 s}{\frac{R_1}{L_1} + s} = \dfrac{R_1 s}{s+\sigma_1}$ — (1)

where $\sigma_1 = \dfrac{R_1}{L_1}$

$$\frac{Z_1(s)}{s} = \frac{R_1}{s+\sigma_1}$$

$Z(s)$ can be expressed as

$$Z(s) = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}{b_0 s^m + b_1 s^{m-1} + \cdots + b_m} \quad \text{where } n > m \quad \text{— (2)}$$

The degree of numerator is greater than that of the denominator by one.

By separating the constant term and linear terms in eq (2), the RL impedance function can be written as

$$Z(s) = P_0 + \frac{P_i s}{s+\sigma_i} + \cdots + Hs \quad \text{— (3)}$$

If we divide the total impedance into a series of impedance $Z_1(s), Z_2(s), \ldots, Z_n(s)$

$$Z(s) = Z_1(s) + Z_2(s) + \cdots + Z_n(s) \quad \text{— (4)}$$

comparing eq (3) & (4)

$Z_1(s) = P_0$, a constant represents resistor

$Z_n(s) = Hs$ represents inductor L_∞

The remaining terms represent parallel combination of an inductor and a resistor

$$\boxed{\begin{aligned} P_n &= R_n \\ \sigma_n &= \frac{R_n}{L_n} \\ R_0 &= P_0 \\ L_\alpha &= Hs \end{aligned}}$$

1. find first foster form for the function

$$Z(s) = \frac{5(s+1)(s+4)}{(s+3)(s+5)}$$

Solution:

$Z(s)$ represents RL impedance because it satisfies all the properties.

By partial fraction expansion,

$$\frac{5(s+1)(s+4)}{(s+3)(s+5)} = \frac{5s^2+25s+20}{s^2+8s+15}$$

$$s^2+8s+15 \overline{)\,5s^2+25s+20\,}\,(5$$
$$\underline{5s^2+40s+75}$$
$$\underline{-15s-55}$$

$$= 5 + \frac{(-15s-55)}{s^2+8s+15}$$

$$\frac{-15s-55}{s^2+8s+15} = \frac{A}{s+3} + \frac{B}{s+5}$$

$$A = \left.\frac{\cancel{(s+3)}(-15s-55)}{\cancel{(s+3)}(s+5)}\right|_{s=-3} = \frac{45-55}{2}$$

$$\boxed{A = -5}$$

$$B = \left.\frac{\cancel{(s+5)}(-15s-55)}{(s+3)\cancel{(s+5)}}\right|_{s=-5} = \frac{75-55}{-2}$$

$$\boxed{B = -10}$$

$$\therefore Z(s) = 5 - \frac{5}{s+3} - \frac{10}{s+5}$$

$Z(s)$ has negative residues, so $\frac{Z(s)}{s}$

$$\frac{Z(s)}{s} = \frac{5(s+1)(s+4)}{s(s+3)(s+5)}$$

$$= \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+5}$$

Apply heavisides method,

$$A = \left.\frac{\cancel{s}\,5(s+1)(s+4)}{\cancel{s}(s+3)(s+5)}\right|_{s=0} = \frac{20}{15} = \frac{4}{3}$$

$$B = \left.\frac{\cancel{(s+3)}\,5(s+1)(s+4)}{s\cancel{(s+3)}(s+5)}\right|_{s=-3} = \frac{-10}{-6} = \frac{5}{3}$$

$$C = \left.\frac{\cancel{(s+5)}\,5(s+1)(s+4)}{s(s+3)\cancel{(s+5)}}\right|_{s=-5} = \frac{-20}{-10} = 2$$

$$\therefore \frac{Z(s)}{s} = \frac{4}{3s} + \frac{5}{3(s+3)} + \frac{2}{(s+5)}$$

$$Z(s) = \frac{4}{3} + \frac{5s}{3(s+3)} + \frac{2s}{s+5}$$

$$Z(s) = p_0 + \frac{p_1 s}{(s+\sigma_1)} + \frac{p_2 s}{s+\sigma_2}$$

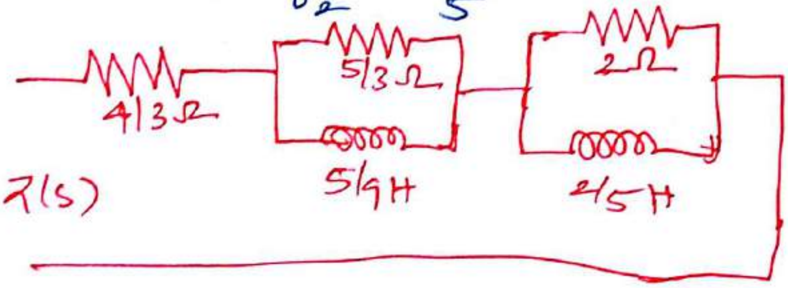
$$P_0 = \frac{4}{3}\Omega = R_0$$

$$P_1 = \frac{5}{3}\Omega = R_1$$

$$\sigma_1 = 3 \Rightarrow L_1 = \frac{R_1}{\sigma_1} = \frac{5}{3\times 3} = \frac{5}{9} H$$

$$P_2 = 2\Omega = R_2$$

$$\sigma_2 = 5 \Rightarrow L_1 = \frac{R_2}{\sigma_2} = \frac{2}{5} H$$

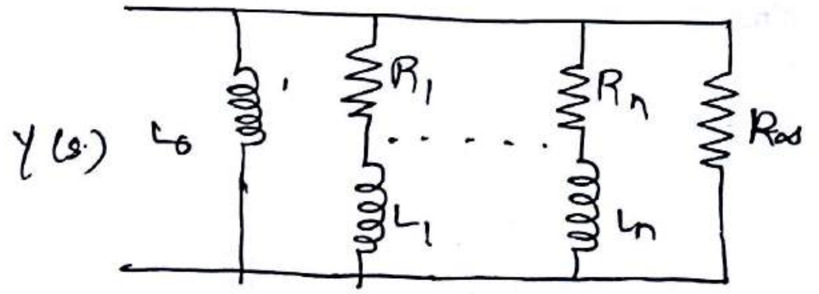


Foster II Form:

The driving point admittance function of the RL network $Y(s)$ is given by

$$Y(s) = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_n}$$

$$= \frac{H(s+\sigma_1)(s+\sigma_3)\dots}{s(s+\sigma_2)(s+\sigma_4)\dots} \quad \text{— (1)}$$



The RL admittance function can be written as,

$$Y(s) = \frac{P_0}{s} + \frac{P_i}{s+\sigma_i} + \dots + H \quad \text{— (2)}$$

Series combination of RL Network is

$$Y_1(s) = \cancel{R_1 + \frac{1}{sL_1}} \quad \frac{1}{R_1 + sL_1} \quad -③$$

Compare ② & ③

$$R_n = \frac{\sigma_n}{p_n}$$

$$L_n = \frac{1}{p_n}$$

$$Y(s) = Y_1(s) + Y_2(s) + \cdots + Y_n(s)$$

$Y_1(s) = \frac{p_0}{s}$ represents inductor L_0, $L_0 = \frac{1}{p_0}$

$Y_n(s) = H$ represents resistor R_∞, $R_\infty = H$

$$\boxed{\begin{aligned} L_0 &= \frac{1}{p_0} \\ R_\infty &= H \\ R_n &= \frac{\sigma_n}{p_n} \\ L_n &= \frac{1}{p_n} \end{aligned}}$$

1. Find the second foster form of admittance function. $Y(s) = \frac{2s^2 + 16s + 30}{s^2 + 6s + 8}$

Solution:

$Y(s)$ satisfies all the properties

Apply partial fraction,

$$Y(s) = \frac{2s^2 + 16s + 30}{s^2 + 6s + 8}$$

$$= 2 + \frac{4s + 14}{s^2 + 6s + 8}$$

$$s^2 + 6s + 8 \overline{)\, 2s^2 + 16s + 30 \,(2}$$
$$2s^2 + 12s + 16$$
$$\overline{\quad 4s + 14 \quad}$$

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$$Y(s) = 2 + \frac{A}{s+2} + \frac{B}{s+4}$$

Using Heaviside Method,

$$A = (s+2)\left.\frac{4s+14}{(s+2)(s+4)}\right|_{s=-2} = 3$$

$$B = (s+4)\left.\frac{4s+14}{(s+2)(s+4)}\right|_{s=-4} = 1$$

The residues are positive. Hence

$$Y(s) = 2 + \frac{3}{s+2} + \frac{1}{s+4}$$

$$\therefore Y(s) = \cancel{\frac{P_0}{s}} H + \frac{P_1}{s+\sigma_1} + \frac{P_2}{s+\sigma_2}$$

$R_\infty = H = 2\Omega$

$P_1 = 3,\ \sigma_1 = 2 \Rightarrow R_1 = \frac{\sigma_1}{P_1} = \frac{2}{3}\Omega$

$L_1 = \frac{1}{P_1} = \frac{1}{3} H$

$P_2 = 1,\ \sigma_2 = 4 \Rightarrow R_2 = \frac{\sigma_2}{P_2} = 4\Omega$

$L_2 = \frac{1}{P_2} = 1H$

The RL network's second foster form is