UNIT-3

1. Introduction to filter
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    - classification
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2. Filter networks
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        - T & π Network - Z_0, γ
    - classification of pass and stop band
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7. Band pass filter
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I. FILTERS:

Definition:

* A filter is a reactive network that freely passes the desired band while almost totally suppressing all other bands.

(or)

* A filter is a frequency selective circuit in which signals having certain frequencies are passed, whereas signals having other frequencies are blocked or attenuated.

* Most filter sections are two port networks with one pair of input terminals and another pair of output terminals.

* The band of frequencies that are passed through the filter which produces no attenuation in the desired band are called as pass band (or) transmission band.

* The band of frequencies that are rejected or blocked through the filter are called as stop band or attenuation band which should provide total or infinite attenuation at other than desired band of frequencies.

* The frequency band between the pass band and stop band is called transition band.

* The frequency which separates the pass band and stop band is defined as cutoff frequency of wave filter.

II. CLASSIFICATION OF FILTER:

1. Based on Components:

Based on components filters are classified into Active and passive filters

a) Active filters:

Active filters contains op-amp or any other electronic circuits.

b) passive filters:

* passive filters use L & C which is arranged in the form of series and parallel
* T and π section arrangement is most widely used
* uses ladder network.

Drawbacks of passive filter:

1. Inductors are bulky and expensive at lower frequencies.
2. Difficult to tune the passive filter.

2. Based on Applications:

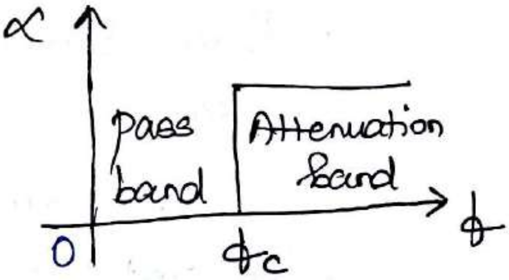
Depending upon the applications, the filters are classified as

1. Low pass filter
2. High pass filter
3. Band pass filter
4. Band Elimination filter.

a) Low pass filter:

A low pass filter is one which passes all frequencies upto the cutoff frequency f_c without attenuation and attenuates all other frequencies greater than f_c.

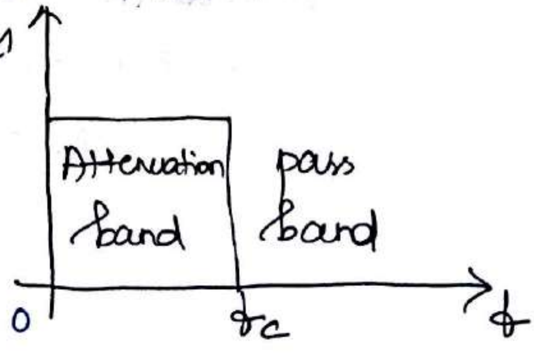
* pass band = 0 to f_c
* Attenuation band = $> f_c$



b) High pass filter:

A high pass filter attenuates all frequencies below desired cutoff frequency f_c and passes all the frequencies above f_c.

* pass band = $> f_c$
* Attenuation band = 0 to f_c



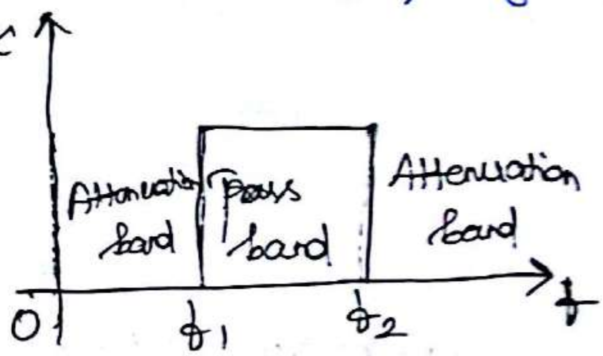
c. Band pass filter:

A band pass filter passes frequencies between two designated cut-off frequencies and attenuates all other frequencies.

This filter has two cut off frequencies and will have the pass band between f_2 (upper cut off frequency) and f_1 (lower cut off frequency)

* pass band = between f_1 & f_2 $(f_2 - f_1)$

* Attenuation band ⇒ $< f_1$ & $> f_2$

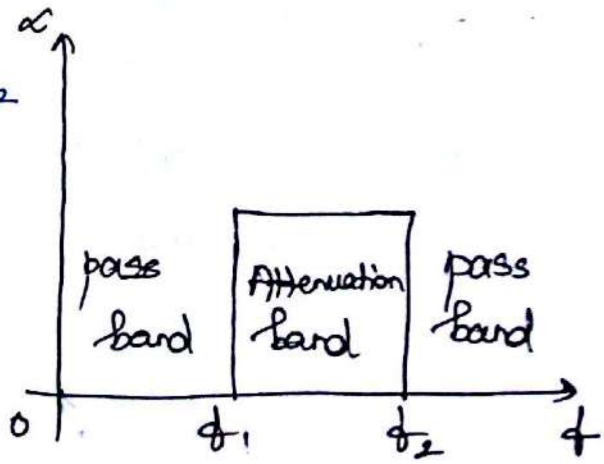


d. Band Elimination filter (or) Band stop filter:

A band stop filter passes all frequencies below and above cut off frequencies (f_1 & f_2), while it attenuates all the frequencies between the two cut off frequencies.

* pass band = $< f_1$ & $> f_2$

* Attenuation band ⇒ b/w f_1 & f_2 $(f_2 - f_1)$



iii. **Unit for a filter:**

The attenuation of a wave filter can be expressed in terms of decibel or neper.

Decibel:

A decibel is defined as ten times the common logarithmic ratio of input power to the output power.

$$D = 10 \log_{10}\left(\frac{P_1}{P_2}\right) \qquad (1 \text{ bel} = 10 \text{ dB})$$

The decibel can also be expressed in terms of the ratio of input voltage (or current) and the output voltage (or current).

$$D = 20 \log_{10}\left(\frac{V_1}{V_2}\right) \quad \text{(or)} \qquad (P = V^2)$$

$$D = 20 \log_{10}\left(\frac{I_1}{I_2}\right)$$

Neper:

Neper is defined as the natural logarithmic ratio of input current to the output current or input voltage to the output voltage.

$$N = \log_e\left(\frac{V_1}{V_2}\right) \quad \text{(or)} \quad N = \log_e\left(\frac{I_1}{I_2}\right)$$

$$N = \frac{1}{2}\log_e\left(\frac{P_1}{P_2}\right)$$

$$\boxed{1 \text{ decibel} = 0.115 \text{ neper}}$$

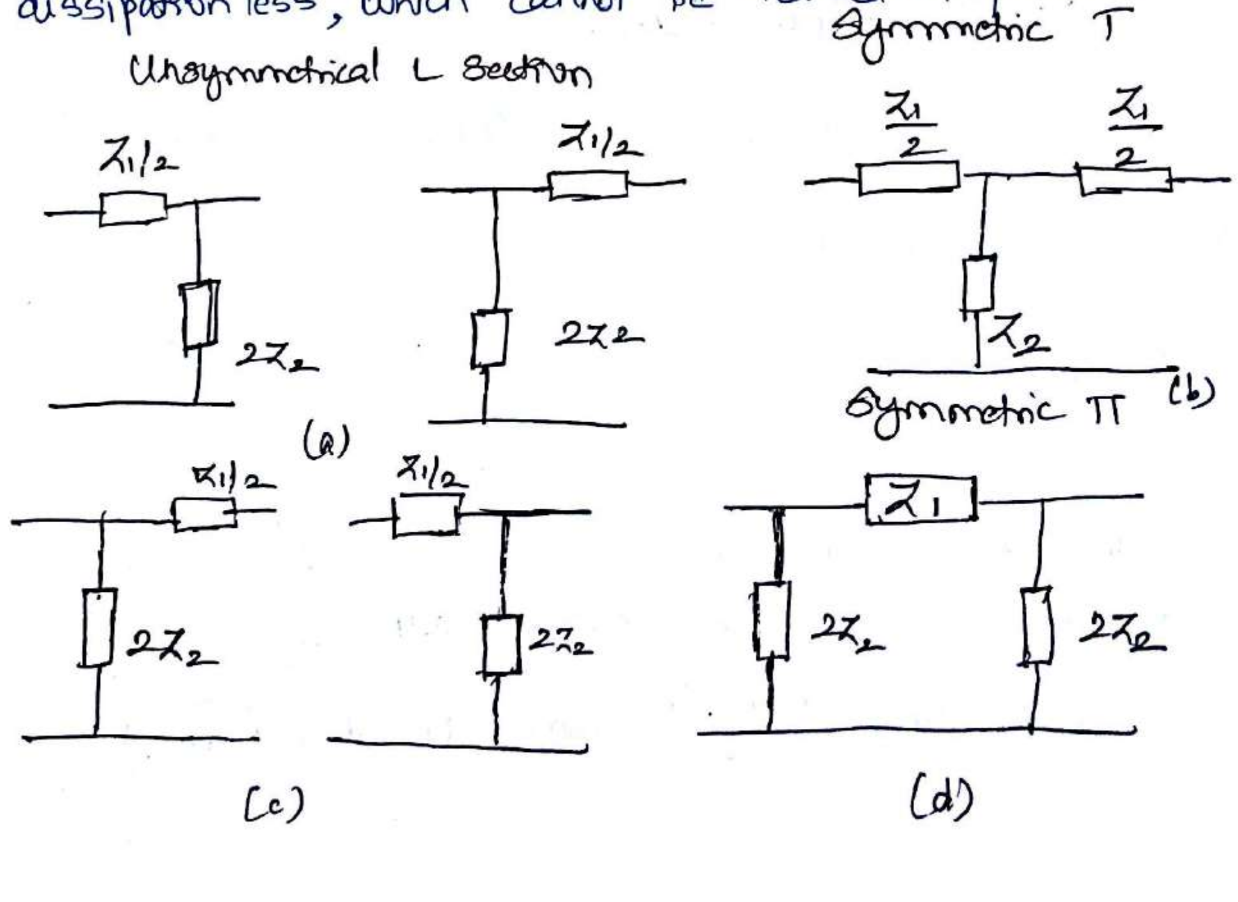
iv. **APPLICATIONS OF FILTER:**

Filter networks are used in

* Communication systems to separate various voice channels in telephone circuits.
* Instrumentation, telemetering equipments etc. where it is necessary to transmit or attenuate a limited range of frequencies.

v. **FILTER NETWORKS :**

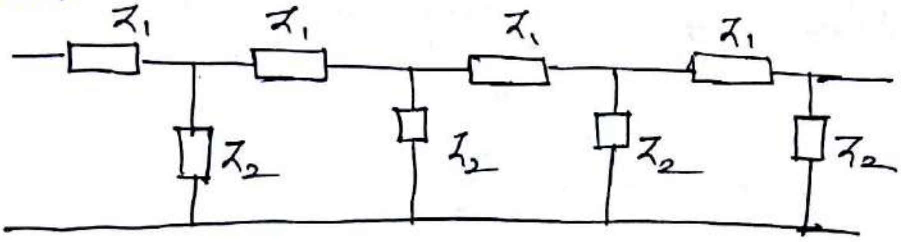
* Ideally a filter should have zero attenuation in the pass band. This condition can only be satisfied if the elements of the filter are dissipation less, which cannot be realized in practice.



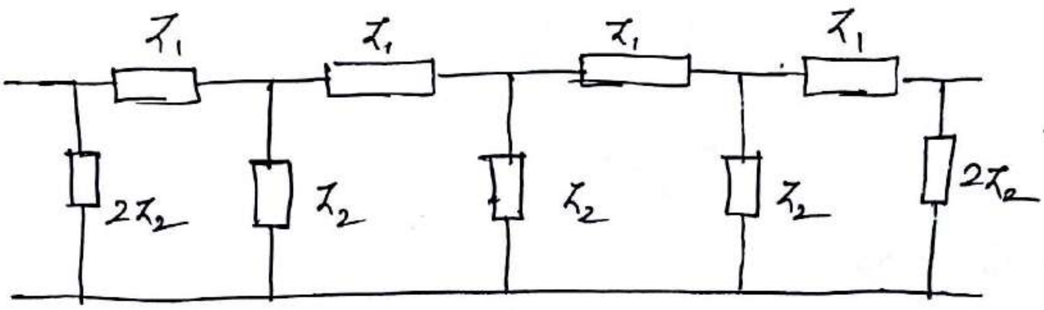
* Filters are designed with an assumption that the elements of the filters are purely reactive.

* Filters are made of symmetrical T or π sections, which can be considered as combinations of unsymmetrical L sections.

* A ladder structure is common form of filter network.



T network



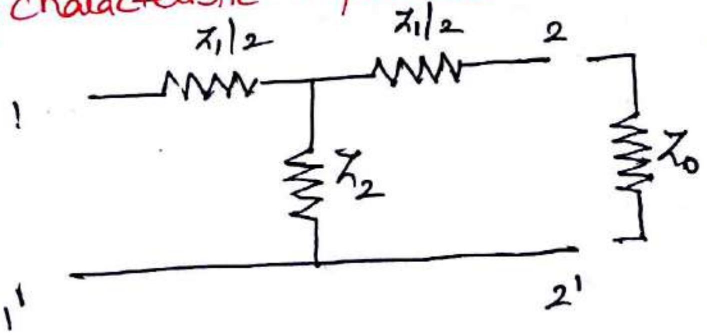
π Network

Equations of filter networks:

To study the behaviour of any filter requires the calculation of its propagation constant γ, attenuation α, phase shift β and its characteristic impedance Z_0.

1. T Network:

a) Characteristic Impedance



* Consider a symmetrical T-network as shown in figure

* If the image impedances at port 1-1' and 2-2' are equal to each other, the image impedance is then called the <u>characteristic or the iterative impedance Z_0.</u>

* Thus if the network is terminated in Z_0, its input impedance will also be Z_0.

* The value of input impedance for the T-network when it is terminated in Z_0 is given by

$$Z_{in} = \frac{Z_1}{2} + Z_2 \parallel \left(\frac{Z_1}{2} + Z_0\right)$$

$$= \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_0\right)}{Z_2 + \frac{Z_1}{2} + Z_0}$$

also $Z_{in} = Z_0$

$$\therefore Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left(\frac{Z_1}{2} + Z_0\right)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1}{2} + \frac{(Z_1Z_2 + 2Z_0Z_2)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1^2 + 2Z_1Z_2 + 2Z_0Z_1 + 2Z_1Z_2 + 4Z_0Z_2}{2(Z_1 + 2Z_2 + 2Z_0)}$$

$$\cancel{= 4Z_1Z_2 +}$$

$$\cancel{2Z_0Z_1} - \cancel{4Z_0Z_2} + 4Z_0^2 = Z_1^2 + 4Z_1Z_2 + \cancel{2Z_0Z_1} + \cancel{4Z_0Z_2}$$

$$4Z_0^2 = Z_1^2 + 4Z_1Z_2$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

The characteristic impedance of a symmetrical T section is

$$\boxed{Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}} \quad - (1)$$

* Z_{OT} can also be expressed in terms of open circuit impedance Z_{OC} and short circuit impedance Z_{SC} of T-network.

The open circuit impedance is

$$Z_{OC} = \frac{Z_1}{2} + Z_2 \quad - (2)$$

The short circuit impedance is

$$Z_{SC} = \frac{Z_1}{2} + \frac{\frac{Z_1}{2} \times Z_2}{\frac{Z_1}{2} + Z_2}$$

$$Z_{SC} = \frac{Z_1^2 + 4Z_1Z_2}{2Z_1 + 4Z_2} \quad - (3)$$

$$Z_{OC} \times Z_{SC} = \frac{(Z_1 + \cancel{2}Z_2)}{2} \times \frac{(Z_1^2 + 4Z_1Z_2)}{2(Z_1 + \cancel{2}Z_2)}$$

$$= Z_1 Z_2 + \frac{Z_1^2}{4}$$

$$= Z_{OT}^2$$

$$\boxed{Z_{OT} = \sqrt{Z_{OC} Z_{SC}}} \quad - (4)$$

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b) Propagation Constant (γ):

For a network to be terminated in Z_0, the ratio V_1/V_2 is the same as I_1/I_2. It is convenient to express this ratio in exponential form i-e

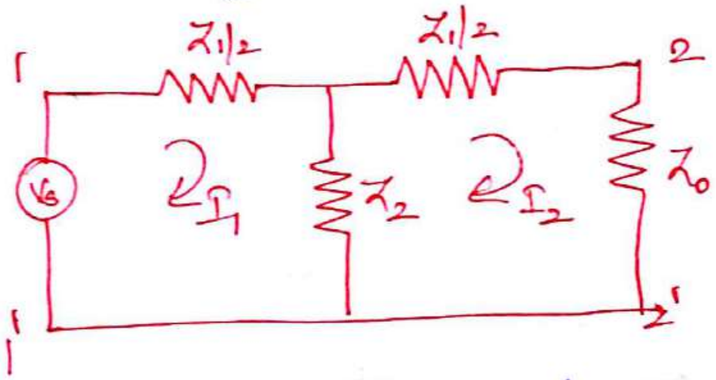
$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = e^{\gamma} = e^{\alpha + j\beta}$$

γ = propagation constant

α = Attenuation constant

β = phase shift

$$e^{\gamma} = \frac{I_1}{I_2} \Rightarrow \gamma = \log_e \left(I_1 / I_2\right)$$



* writing the mesh equation for the 2nd loop,

$$I_1 Z_2 = I_2\left(Z_2 + \frac{Z_1}{2} + Z_0\right)$$

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_2 + Z_0}{Z_2} = e^{\gamma}$$

$$Z_2 e^{\gamma} = \frac{Z_1}{2} + Z_2 + Z_0$$

$$Z_0 = Z_2\left(e^{\gamma} - 1\right) - \frac{Z_1}{2} \quad \text{—— (1)}$$

The characteristic impedance of a T network is given by

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4}+Z_1Z_2} \quad — (2)$$

Squaring eq (1) & (2)

$$Z_0^2 = \left(Z_2^2\,(e^{\gamma}-1)^2 - \left(\frac{Z_1}{2}\right)\right)^2 \quad — (3)$$

$$Z_{OT}^2 = \frac{Z_1^2}{4}+Z_1Z_2 \quad — (4)$$

Subtract eq (4) from (3)

$$Z_2^2\,(e^{\gamma}-1)^2 + \cancel{\frac{Z_1^2}{4}} - \frac{\cancel{2}Z_1Z_2\,(e^{\gamma}-1)}{\cancel{2}} - \cancel{\frac{Z_1^2}{4}} - Z_1Z_2 = 0$$

$$Z_2^2\,(e^{\gamma}-1)^2 - Z_1Z_2\,(\cancel{1}+e^{\gamma}-\cancel{1}) = 0$$

$$Z_2^2\,(e^{\gamma}-1)^2 - Z_1Z_2\,e^{\gamma} = 0$$

$$Z_2\,(e^{\gamma}-1)^2 - Z_1\,e^{\gamma} = 0$$

$$(e^{\gamma}-1)^2 = \frac{Z_1\,e^{\gamma}}{Z_2}$$

$$\left(e^{2\gamma}+1-2e^{\gamma}\right) = \frac{Z_1}{Z_2\,e^{-\gamma}}$$

Rearranging the above equation,

$$e^{-\gamma}\left(e^{2\gamma}+1-2e^{\gamma}\right) = \frac{Z_1}{Z_2}$$

$$\left(e^{\gamma}+e^{-\gamma}-2\right) = \frac{Z_1}{Z_2}$$

Divide both sides by 2,

$$\frac{(e^{\gamma}+e^{-\gamma}-2)}{2} = \frac{Z_1}{2Z_2}$$

$$\frac{e^{\gamma}+e^{-\gamma}}{2}-1 = \frac{Z_1}{2Z_2}$$

$$\frac{e^{\gamma}+e^{-\gamma}}{2} = 1+\frac{Z_1}{2Z_2}$$

$$\cosh\gamma = 1+\frac{Z_1}{2Z_2} \quad — (5)$$

To obtain hyperbolic tangent expression for γ,

$$\sinh\gamma = \sqrt{\cosh^2\gamma - 1}$$

$$= \sqrt{\left(1+\frac{Z_1}{2Z_2}\right)^2 - 1}$$

$$= \sqrt{\cancel{1}+\left(\frac{Z_1}{2Z_2}\right)^2 + \frac{Z_1}{Z_2} - \cancel{1}}$$

$$= \sqrt{\frac{Z_1}{Z_2}+\left(\frac{Z_1}{2Z_2}\right)^2}$$

$$\sinh\gamma = \frac{1}{Z_2}\sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$= \frac{Z_{OT}}{Z_2} \quad — (6)$$

Divide eq (6) by (5)

$$\frac{\sinh\gamma}{\cosh\gamma} = \frac{Z_{OT}/Z_2}{1+\frac{Z_1}{2Z_2}}$$

$$= \frac{Z_{OT} \times 2 \cancel{Z_2}}{\cancel{Z_2}(2Z_2 + Z_1)}$$

$$= \frac{Z_{OT}}{Z_2 + \frac{Z_1}{2}}$$

from eq ② in Z_0 derivation,

$$Z_{oc} = Z_2 + \frac{Z_1}{2}$$

from eq ④ in Z_0 derivation,

$$Z_{OT} = \sqrt{Z_{oc} Z_{sc}}$$

$$\therefore \tan h\gamma = \frac{\sqrt{Z_{oc} Z_{sc}}}{Z_{oc}}$$

$$= \frac{\cancel{Z_{oc}}\sqrt{Z_{sc}}}{\cancel{Z_{oc}}}$$

$$= \frac{\sqrt{\cancel{Z_{oc}} Z_{sc}}}{\sqrt{\cancel{Z_{oc}} Z_{oc}}}$$

$$\tan h\gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

Also $\sin h\gamma = \sqrt{\frac{1}{2}(\cos h\gamma - 1)}$

$$\cos h\gamma = 1 + \frac{Z_1}{2Z_2} \quad \text{(from ⑤)}$$

$$\therefore \sin h\gamma = \sqrt{\frac{1}{2}\left(\cancel{1} + \frac{Z_1}{2Z_2} - \cancel{1}\right)} =$$

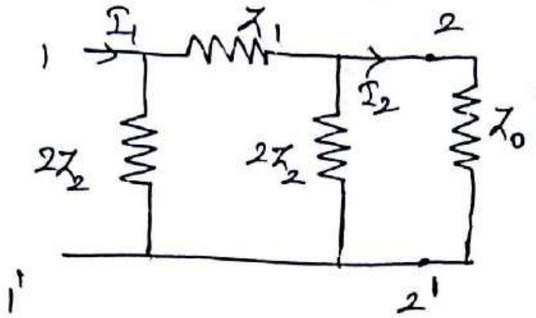
$$\sin h\gamma = \sqrt{\frac{Z_1}{4Z_2}}$$

2. π- Network:

characteristic Impedance (Z_0):

(same as like T network)

* when the network is terminated in Z_0 at port 2-2', its input impedance is given by



$$Z_{in} = 2Z_2 \,||\, (Z_1 + (2Z_2 \,||\, Z_0))$$

$$= 2Z_2 \,||\, \left[Z_1 + \frac{2Z_2 \times Z_0}{2Z_2 + Z_0}\right]$$

$$Z_{in} = \frac{2Z_2 \times \left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{2Z_2 + Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}}$$

By definition of characteristic impedance $Z_{in} = Z_0$.

$$\therefore Z_0 = \frac{2Z_2 \times \left[Z_1 + \frac{2Z_0 Z_2}{2Z_2 + Z_0}\right]}{2Z_2 + Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}}$$

$$2Z_2 Z_0 + Z_1 Z_0 + \frac{2Z_2 Z_0^2}{2Z_2 + Z_0} = \frac{2Z_2 \left(2Z_1 Z_2 + Z_0 Z_1 + 2Z_0 Z_2\right)}{\left(2Z_2 + Z_0\right)}$$

$$\frac{4Z_2^2 Z_0 + 2Z_2 Z_0^2 + 2Z_1 Z_2 Z_0 + Z_1 Z_0^2 + 2Z_2 Z_0^2}{2Z_2 + Z_0} = RHS$$

$$RHS = \frac{4Z_1 Z_2^2 + 2Z_0 Z_1 Z_2 + 4Z_0 Z_2^2}{(2Z_2 + Z_0)}$$

$$(2Z_2 + Z_0)(2Z_2 Z_0^2 + Z_1 Z_0^2 + 2Z_2 Z_0^2) = (2Z_2 + Z_0)(4Z_1 Z_2^2)$$

$$4Z_2^2 Z_0^2 + 2Z_1 Z_0^2 Z_2 + 4Z_2^2 Z_0^2 + 2Z_2 Z_0^3 + Z_1 Z_0^3 + 2Z_2 Z_0^3 =$$

$$8Z_1 Z_2^2 + 4Z_0 Z_1 Z_2^2$$

$$8Z_2^2 Z_0^2$$

$$2Z_2 Z_0^2 + Z_1 Z_0^2 + 2Z_2 Z_0^2 = 4Z_1 Z_2^2$$

$$4Z_2 Z_0^2 + Z_1 Z_0^2 = 4Z_1 Z_2^2$$

$$Z_0^2 (4Z_2 + Z_1) = 4Z_1 Z_2^2$$

$$Z_0^2 = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2} = \frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}$$

$$Z_0 = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} \quad \text{— (1)}$$

~~Divide~~ Nr Multiply Nr & Dr by $(Z_1 Z_2)$

$$Z_{0\pi} = \sqrt{\frac{(Z_1 Z_2)(Z_1 Z_2)}{(Z_1 Z_2) + \frac{Z_1 (Z_1 Z_2)}{4Z_2}}}$$

$$= \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}} \quad \text{— (2)}$$

from eq (1) in characteristic impedance of T network.

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

Sub this on Eq (2)

$$\therefore Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

Also $Z_{0\pi} = Z_{oc} \times Z_{sc}$ (π network)

The input impedance at port 1-1' when port 2-2' is open is

$$Z_{oc} = 2Z_2 \| (Z_1 + 2Z_2)$$

$$= \frac{2Z_2(Z_1 + 2Z_2)}{2Z_2 + Z_1 + 2Z_2}$$

$$= \frac{2Z_2(Z_1 + 2Z_2)}{Z_1 + 4Z_2}$$

The input impedance at port 1-1' when port 2-2' is short circuit is

$$Z_{sc} = \frac{2Z_1 Z_2}{2Z_2 + Z_1} = Z_1 \| 2Z_2$$

Hence $Z_{oc} \times Z_{sc} = \dfrac{2Z_2(Z_1 + 2Z_2)}{Z_1 + 4Z_2} \times \dfrac{2Z_1 Z_2}{(2Z_2 + Z_1)}$

$$= \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

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$\div$ nr & dr by $4Z_2$,

$$Z_{oc} \times Z_{sc} = \frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}$$

$$= Z_{o\pi}^2 \quad \text{(from eq 1)}$$

$$\therefore Z_{o\pi} = \sqrt{Z_{oc} \times Z_{sc}}$$

Propagation constant of π-network:

The propagation constant of a symmetrical π-section is same as that for symmetrical T-section.

$$\cos h\, \gamma = 1 + \frac{Z_1}{2Z_2}$$

iii. Classification of pass and stop band:

* By using propagation constant, the characteristics of filters can be verified.

* By using propagation constant γ, being a function of frequency, the pass band, stop band and the cut-off point (ie) the point of separation between two bands can be identified.

* for symmetrical T or π sections, the expression for propagation constant γ in terms of hyperbolic functions,

$$\sin h \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

* If Z_1 and Z_2 are both pure imaginary values, their ratio and hence $\frac{Z_1}{4Z_2}$ will be a pure real number.

$\because$ Z_1 and Z_2 may lie in the range of $-j\infty$ to $+j\infty$

$\frac{Z_1}{4Z_2}$ will have real values in $-j\infty$ to $+j\infty$.

$\sinh\frac{\gamma}{2}$ will also have infinite limit.

* Propagation constant is a complex function,

$$\gamma = \alpha + j\beta$$

α = Measure of the change in magnitude of the current or voltage in the network called attenuation constant

β = Measure of the difference in phase between the input and output currents or voltages, known as phase shift constant.

* α & β values depends on $\frac{Z_1}{4Z_2}$

$$\sinh\frac{\gamma}{2} = \sinh\left(\frac{\alpha}{2} + \frac{j\beta}{2}\right)$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2}$$

$$= \sqrt{\frac{Z_1}{4Z_2}} \quad \text{--- (1)}$$

Case A:

If Z_1 and Z_2 are the same type of reactances, then $\frac{Z_1}{4Z_2}$ is real and equal to say $\alpha + x$

The imaginary part of Eq (1) is equal to zero.

$$\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = 0 \quad \text{— (2)}$$

$$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = x \quad \text{— (3)}$$

Eq (2) can be satisfied if $\frac{\beta}{2} = 0$ or $n\pi$, where $n = 0, 1, 2, \ldots$ then

$$\cos\frac{\beta}{2} = 1 \text{ \& } \sinh\frac{\alpha}{2} = x = \sqrt{\frac{Z_1}{4Z_2}}$$

x should be always positive

$\left|\frac{Z_1}{4Z_2}\right| > 0$ and

$$\alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

Since $\alpha \neq 0$, it indicates that the attenuation exists.

Case B:

If Z_1 and Z_2 are opposite reactances then $\sqrt{\frac{Z_1}{4Z_2}}$ is imaginary and equal to Jx.

The real part of Eq ① is zero.

$$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2}=0 \quad — ⑤$$

$$\cosh\frac{\alpha}{2}\sin\frac{\beta}{2}=x \quad — ⑥$$

Eq ⑤ must be satisfied when $\alpha=0$ or $\beta=\pi$.

i) when $\alpha=0$, from eq ⑤

$$\sinh\frac{\alpha}{2}=0$$

from ⑥, $\sin\frac{\beta}{2}=x=\sqrt{\frac{Z_1}{4Z_2}}$

* Sine can have a Maximum value of 1

<u>Note</u>:

$\sin \theta = 1$

$\sin 90^\circ = -1$

* The eq ⑥ is valid only for negative $\frac{Z_1}{4Z_2}$ and having maximum value of unity.

* It indicates the condition of pass band with zero attenuation and the condition as

$$-1 \le \frac{Z_1}{4Z_2} \le 0$$

$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}} \quad — ⑦$$

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ii) when $\beta = \pi$ from eq (5)

$\cos \frac{\beta}{2} = 0$

from 6, $\sin \frac{\beta}{2} = \pm 1$

$\cosh \frac{\alpha}{2} = x = \sqrt{\frac{Z_1}{4Z_2}}$

* $\cos h\frac{\alpha}{2} \geq 1$, this solution is valid for negative $Z_1/4Z_2$ and having Magnitude greater than or equal to unity.

* This is the condition for stop band since $\alpha \neq 0$

$$-\alpha \leq \frac{Z_1}{4Z_2} \leq -1$$

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \quad \text{---} \quad (8)$$

* Three limits for case A and B.

* Knowing the values of Z_1 and Z_2, it is possible to determine the case to the applied to the filter.

* Z_1 & Z_2 are made of different types of reactances or combination of reactances so that as frequency changes, a filter may pass from one case to another.

* Case A and (ii) in case B are attenuation band

* (i) in case B is the pass band.

* The frequency which separates the attenuation band from pass band or vice versa is called cut off frequencies.

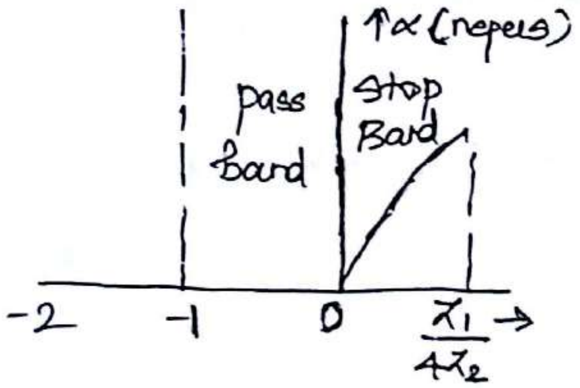
* It is defined by f_c & termed as nominal frequency.

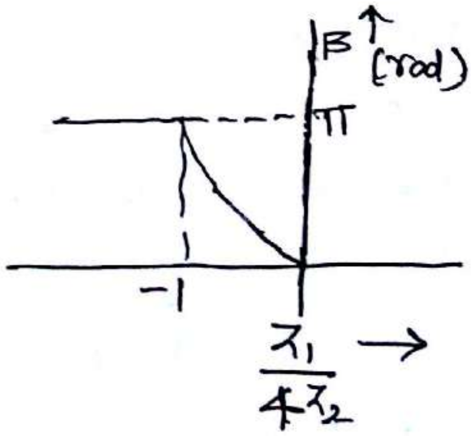
* Since Z_0 is real in the pass band and imaginary in an attenuation band, f_c is the frequency at which Z_0 changes from being real to being imaginary.

* These frequencies occur at,

$$\left.\begin{aligned} \frac{Z_1}{4Z_2} &= 0 \quad \text{or} \quad Z_1 = 0 \\ \frac{Z_1}{4Z_2} &= -1 \quad \text{or} \quad Z_1 + 4Z_2 = 0 \end{aligned}\right\} \quad — (9)$$

* The above conditions can be represented graphically,





Characteristic Impedance in pass band and stop band:

The characteristic impedance of T network is given by

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} \quad - \quad (1)$$

If Z_1 & Z_2 are purely reactive,

Let $Z_1 = jx_1$ & $Z_2 = jx_2$ then Eq (1) becomes

$$Z_{0T} = \sqrt{-x_1 x_2 \left(1 + \frac{x_1}{4x_2}\right)} \quad - \quad (2)$$

* A pass band exists when x_1 & x_2 are of opposite reactances,

$$-1 < \frac{x_1}{4x_2} < 0 \quad - \quad (3)$$

Sub (3) in (2),

Z_{0T} = positive and real.

* A stop band exists when x_1 & x_2 are of same reactance,

$$\frac{x_1}{4x_2} > 0$$

Z_{0T} = Imaginary

* Another stop band exist when x_1 & x_2 are of same reactance,

$$\frac{x_1}{4x_2} < -1$$

$$Z_{OT} = \text{Imaginary}$$

* In a pass band if a network is terminated in a pure Resistance R_0 ($Z_{OT} = R_0$), the input impedance is R_0 and the network transmits the power received from the source to the R_0 without any attenuation.

* If a network is terminated in a pure reactance (Z_0 = pure reactance), the input impedance is reactive and cannot receive or transmit power. However the network transmits voltage and current with 90° phase difference and with attenuation.

* The characteristic impedance of a symmetrical π section can be expressed in terms of T.

$$Z_{O\pi} = \frac{Z_1 Z_2}{Z_{OT}}$$

* Since Z_1 and Z_2 are purely reactive, $Z_{O\pi}$ is real if Z_{OT} is real and $Z_{O\pi}$ is imaginary if Z_{OT} is imaginary.

* The conditions developed for T sections are valid for π sections.

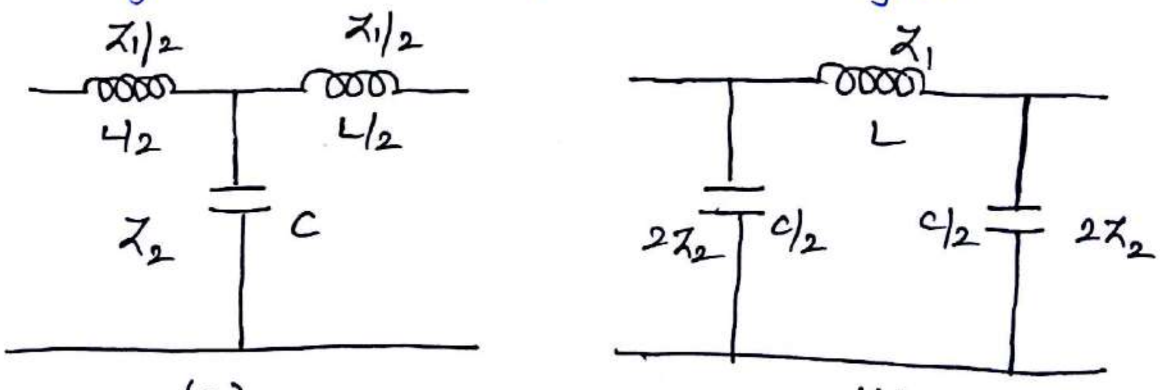
CONSTANT-K LOW PASS FILTER:

* A network either T or π is said to be of the constant-K type if Z_1 & Z_2 of the network satisfy the condition

$$Z_1 Z_2 = K^2 \quad \text{— (1)}$$

where Z_1 & Z_2 are impedances in the T and π sections.

* Eq (1) states that Z_1 & Z_2 are inverse if their product is a constant, independent of frequency.
* K is real constant, that is the resistance.
* K is often termed as design impedance or nominal impedance of the constant K filter.
* The constant K, T or π type filter is also known as the prototype because other more complex networks can be derived from it.
* A prototype T and π sections are shown in the figure where $Z_1 = j\omega L$ and $Z_2 = \frac{1}{j\omega C}$.



(a) (b)

* $Z_1 Z_2 = K^2$

$\Rightarrow j\omega L \times \frac{1}{j\omega C} = K^2 \Rightarrow K^2 = \frac{L}{C}$

$\therefore K = \sqrt{\frac{L}{C}}$ which is independent of frequency — (2)

* Since the product Z_1 & Z_2 is constant, the filter is a constant K type.

* from eq (4) in classification of pass & stop band, the cut off frequencies are

$\frac{Z_1}{4Z_2} = 0$

$\Rightarrow \frac{j\omega L}{4\frac{1}{j\omega C}} = 0$

$\Rightarrow \frac{j\omega L \times j\omega C}{4} = 0 \Rightarrow \frac{-\omega^2 LC}{4} = 0$

And also $\frac{Z_1}{4Z_2} = -1$

$\frac{-\omega^2 LC}{4} = -1$

$\frac{\omega^2 LC}{4} = 1$

$\therefore \frac{(2\pi f_c)^2 LC}{4} = 1$

$f_c = \frac{1}{\pi\sqrt{LC}}$ — (3)

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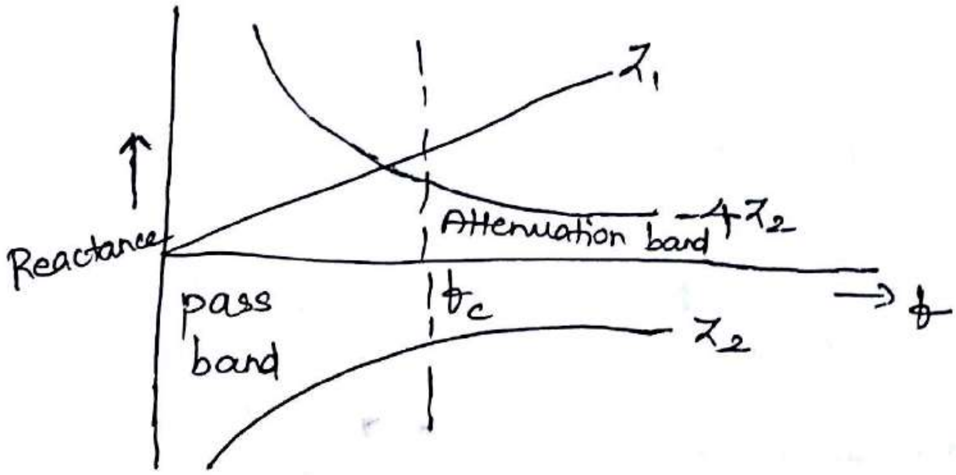
* The pass band can be determined graphically The reactances of Z_1 and $4Z_2$ will vary with frequency as in fig.

* The cut off frequencies at the intersection of the curves Z_1 and $-4Z_2$ is indicated as f_c.

* On the x-axis as $Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$ and $Z_1 = -4Z_2$.



* All the frequencies above f_c lie in a stop or attenuation band. Thus the network is called low pass filter.

* from equation of propagation constant γ,

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{j\omega L}{4\times\frac{1}{j\omega C}}} = \sqrt{\frac{-\omega^2 LC}{4}}$$

$$\sinh\frac{\gamma}{2} = \frac{j\omega\sqrt{LC}}{2} \quad — (4)$$

from eq (3), $f_c = \frac{1}{\pi\sqrt{LC}} \Rightarrow \sqrt{LC} = \frac{1}{f_c\pi}$ —(5)

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Sub (5) in (4)

$$\therefore \sinh\frac{\gamma}{2} = \frac{j2\pi f \times (1/f_c\pi)}{2}$$

$$= \frac{2\pi f}{2\pi f_c}$$

$$\therefore \sinh\frac{\gamma}{2} = j\frac{f}{f_c} \quad \text{— (6)}$$

In pass band, $-1 < \frac{Z_1}{4Z_2} < 0$

$$-1 < -\frac{\omega^2 LC}{4} < 0$$

$$-1 < -\left(\frac{f}{f_c}\right)^2 < 0$$

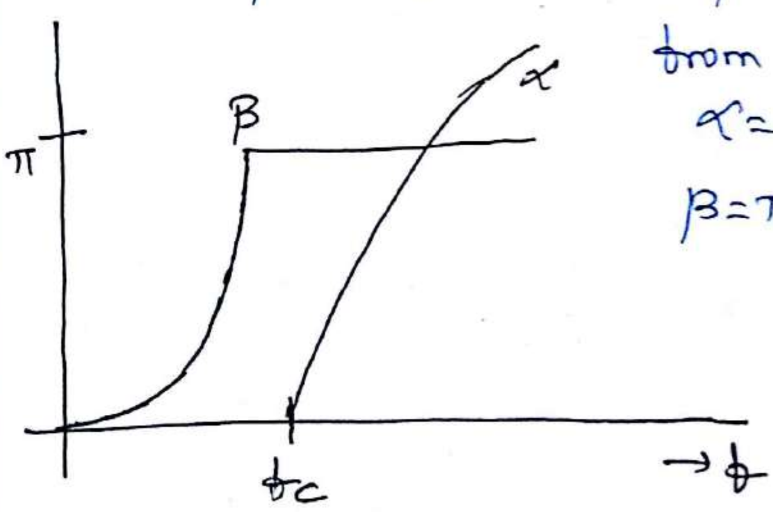
$$\Rightarrow \frac{f}{f_c} < 1 \text{ and } \beta = 2\sin^{-1}\left(\frac{f}{f_c}\right); \alpha = 0 \quad \text{— (7)}$$

In attenuation band, $\frac{Z_1}{4Z_2} < -1$

$$\Rightarrow \frac{f}{f_c} < 1$$

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right); \beta = \pi \quad \text{— (8)}$$

* The plots for α and β is shown in fig.



from fig,

$\alpha = 0, \beta = 2\sinh^{-1}\left(\frac{f}{f_c}\right)$ $f < f_c$

$\beta = \pi, \alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$ $f > f_c$

Calculation of characteristic impedance:

The characteristic impedance can be calculated as,

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{j\omega L \times \frac{1}{j\omega C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

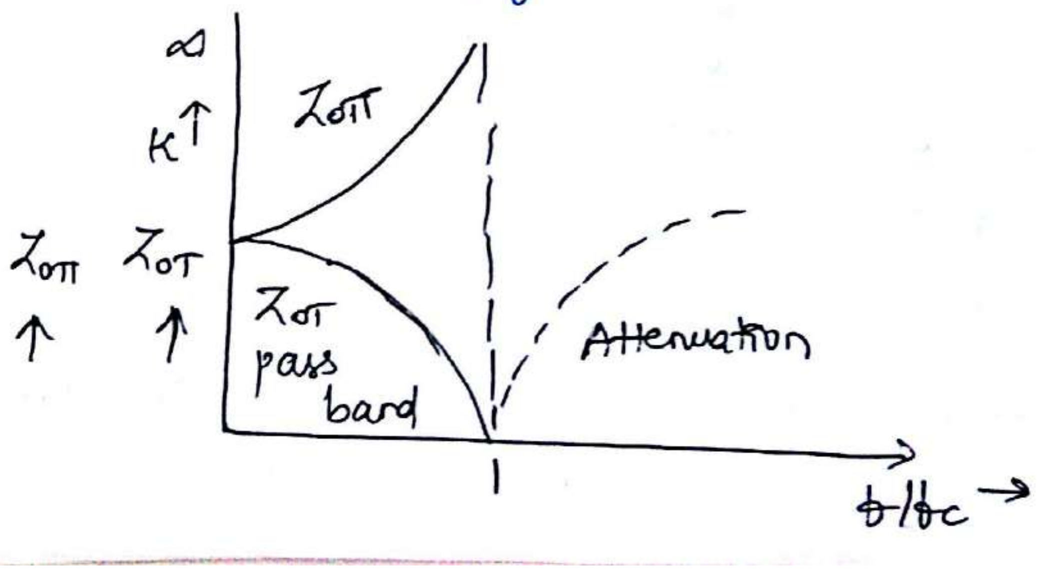
$$= \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

$$Z_{0T} = K\sqrt{1 - \left(\frac{f}{f_c}\right)^2} \quad \text{—— (9)}$$

from eq (9), * Z_{0T} is real when $f < f_c$ (ie) in pass band

* $Z_{0T} = 0$ for $f > f_c$
* Z_{0T} = Imaginary in the attenuation band rising to infinite reactance at infinite frequency.

* The variation of Z_{0T} with frequency is shown in the figure.



for π - network:

The characteristic impedance of a π-network is given by

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{K^2}{K\sqrt{1-(f/f_c)^2}}$$

$$Z_{0\pi} = \frac{K}{\sqrt{1-(f/f_c)^2}} \quad \text{— (10)}$$

* The variation of $Z_{0\pi}$ with f is shown in previous fig.

* $Z_{0\pi}$ = real for $f < f_c$

$Z_{0\pi}$ = infinite at $f = f_c$

$Z_{0\pi}$ = imaginary at $f > f_c$

Design of Low pass filter:

A low pass filter can be designed from the specifications of cut-off frequency and load resistance.

At cut off frequency, $Z_1 = -4Z_2$

$$j\omega_c L = \frac{-4}{j\omega_c C}$$

$$j^2 \omega_c^2 LC = -4$$

$$-4\pi^2 f_c^2 LC = -4$$

$$\pi^2 f_c^2 LC = 1 \quad \text{— (1)}$$

we know that, $K = \sqrt{\frac{L}{C}}$ is design impedance or load resistance

Sub ⑤ in ① $K^2 = \frac{L}{C}$ — ②

$\pi^2 f_c^2 K^2 C^2 = 1$ — ③

from ③, $\boxed{C = \frac{1}{\pi f_c K}}$ shunt capacitance

from ②, $L = K^2 C = \frac{K^2}{\pi f_c K}$

$\boxed{L = \frac{K}{f_c \pi}}$ series inductance

Problem:

1. Design a low pass filter (both T & π sections) having cut off frequency of 2 KHz to operate with a terminated load resistance of 500 Ω.

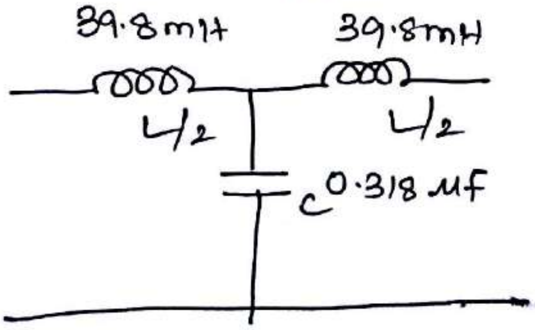
Solution:

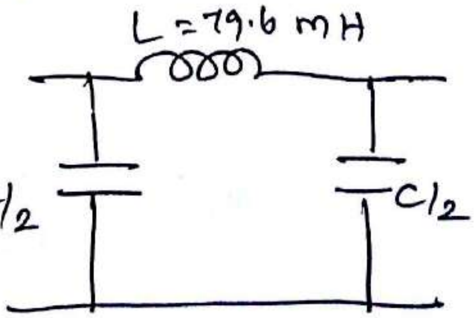
Given: $f_c = 2\,KHz$

$K = \sqrt{\frac{L}{C}} = 500\,\Omega$

$L = \frac{K}{f_c \pi} = \frac{500}{2000 \times \pi} = 79.6\,mH$

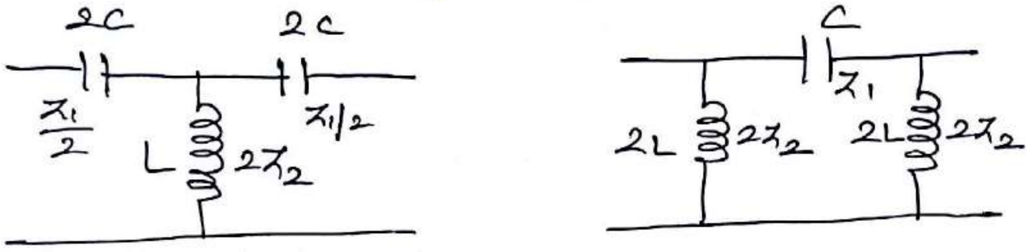
$C = \frac{1}{\pi f_c K} = \frac{1}{\pi \times 2000 \times 500} = 0.318\,\mu F$





2) CONSTANT K HIGH PASS FILTER:

* Constant K high pass filter can be obtained by changing the positions of series and shunt arms of the network as shown in fig



* Here $Z_1 = \frac{-j}{\omega C}$ & $Z_2 = j\omega L$

* product of Z_1 & Z_2 is constant, which is independent of frequency. Thus $Z_1 Z_2$ is

$$Z_1 Z_2 = \frac{-j}{\omega C} \times j\omega L = \frac{L}{C} = K^2$$

$$K = \sqrt{\frac{L}{C}} \quad — ①$$

* The cut off frequencies are given by

$Z_1 = 0$ & $Z_1 = -4Z_2$

$$Z_1 = 0 \Rightarrow \frac{-j}{\omega C} = 0 \text{ or } \omega - \infty \quad — ②$$

$$Z_1 = -4Z_2 \Rightarrow \frac{-j}{\omega C} = -4 j\omega L$$

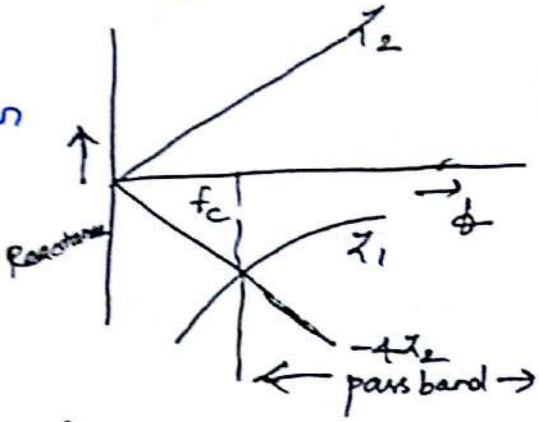
$$\omega^2 LC = \frac{1}{4} \Rightarrow (2\pi f_c)^2 LC = \frac{1}{4}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}} \quad — ③$$

* The reactances of Z_1 and Z_2 are sketched as functions of frequency as shown in fig.

* As seen from fig, the filter transmits all frequencies between $f = f_c$ to $f = \alpha$.

* The point f_c from the graph is a point at which $Z_1 = -4Z_2$



* $\sin h \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{1/j\omega C}{4j\omega L}} = \sqrt{\frac{-1}{4\omega^2 LC}}$

from eq ③, $f_c = \frac{1}{4\pi\sqrt{LC}}$

$$\sqrt{LC} = \frac{1}{4\pi f_c} \Rightarrow LC = \frac{1}{(4\pi f_c)^2}$$

$$\sin h \frac{\gamma}{2} = \sqrt{\frac{-1 \times (4\pi f_c)^2}{4\omega^2}} = \sqrt{\frac{-(4\pi)^2 (f_c)^2}{4\omega^2}}$$

$$= \sqrt{\frac{-16\pi^2 f_c^2}{4 \times (2\pi f)^2}} = \sqrt{\frac{-16\pi^2 f_c^2}{4 \times 4 \times \pi^2 \times f^2}}$$

$$\sin h \frac{\gamma}{2} = \sqrt{\frac{-f_c^2}{f^2}}$$

$$\sin h \frac{\gamma}{2} = j \frac{f_c}{f} \quad - ④$$

In the pass band $-1 < \frac{Z_1}{4Z_2} < 0 \Rightarrow$

$\alpha = 0$ or the region in which $\frac{f_c}{f} < 1$ is a pass band.

$$\therefore \beta = 2 \sin^{-1}\left(\frac{f_c}{f}\right)$$

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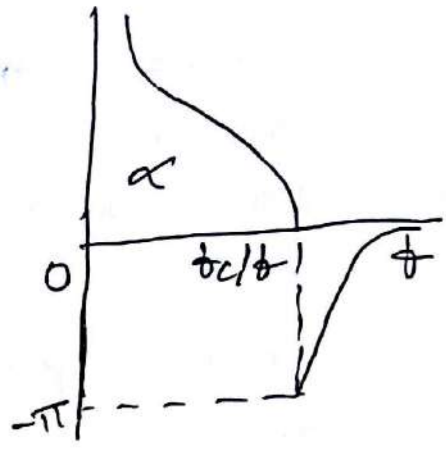
In the attenuation band $\frac{Z_1}{4Z_2} < -1$ (ie) $\frac{f_c}{f} > 1$

$$\alpha = 2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right]$$

$$\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right); \beta = \pi$$



* The plots of α and β for pass and stop bands of a high pass filter network are shown in figure.

The characteristic impedance can be calculated as

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

Sub $Z_1 Z_2 = \frac{L}{C}$ & $\frac{Z_1}{4Z_2} = \frac{-1}{4\omega^2 LC}$ (from previous section)

$$= \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

$$Z_{0T} = K\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad -⑤$$

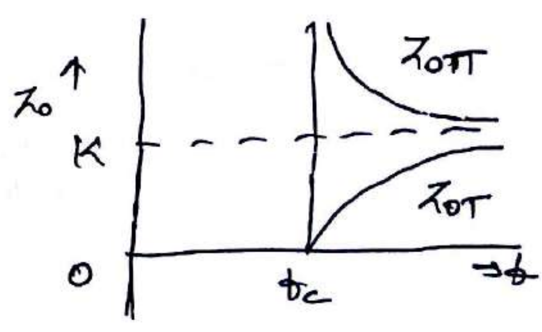
for π network:

The characteristic impedance of a π-network is given by, $Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{K^2}{Z_{0T}}$

from ⑤,

$$= \frac{K^2}{K\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\therefore Z_{0\pi} = \frac{K}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad -⑥$$



Design of high pass filter:

At cut off frequency, $Z_1 = -4Z_2$

$$\frac{1}{j\omega_c C} = -4j\omega_c L$$

$$\frac{1}{j(2\pi f_c)C} = -4j(2\pi f_c)L$$

$$4 \times 4\pi^2 f_c^2 LC = 1$$

$$16\pi^2 f_c^2 LC = 1 \quad -(7)$$

$$K = \sqrt{\frac{L}{C}} \Rightarrow K^2 = \frac{L}{C} \quad -8$$

sub (8) in (7)

$$\therefore 16\pi^2 f_c^2 \cancel{L}\, C\,(K^2 C) = 1$$

$$16\pi^2 f_c^2 K^2 C^2 = 1$$

$$\therefore \boxed{C = \frac{1}{4\pi f_c K}} \Rightarrow \text{Capacitance}$$

$$\& \; L = K^2 C = \frac{K^2}{\frac{1}{4\pi f_c K}}$$

$$= K^2 \times \frac{1}{4\pi f_c K}$$

$$\boxed{L = \frac{K}{4\pi f_c}} \Rightarrow \text{Inductance}$$

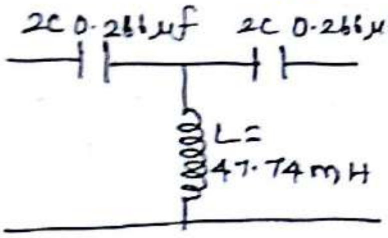
Problem:

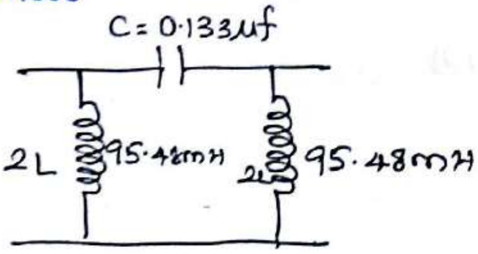
1. Design a high pass filter having a cut-off frequency of 1KHz with a load resistance of $600\,\Omega$.

Solution: $R_L = K = 600\,\Omega$ & $f_c = 1KHz$

$$L = \frac{K}{4\pi f_c} = \frac{600}{4 \times \pi \times 1 \times 10^3} = 47.74\ mH$$

$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi \times 600 \times 1000} = 0.133\mu f$$





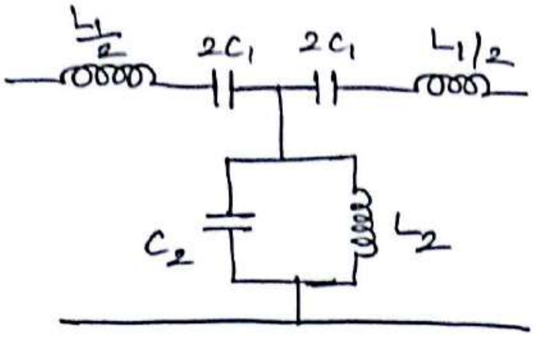
BAND PASS FILTER:

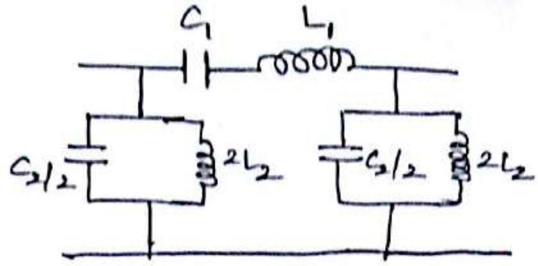
* A band pass filter is one which attenuates all frequencies below a lower cut-off frequency f_1 and above an upper cut off frequency f_2. frequencies lying between f_1 to f_2 comprise the pass band and are transmitted with zero attenuation.

* A band pass filter may be obtained by using a low pass filter followed by a high pass filter in which the cut-off frequency of the low pass filter is above the cut-off frequency of the high pass filter, the overlap thus allowing only a band of signal frequencies to pass.

* This is not economical in practice, it is more economical to combine the low and high pass functions into a single filter section.

* Consider the circuit each arm has a resonant circuit with same resonant frequency (ie) the resonant frequency of the series arm and the resonant frequency of the shunt arm are made equal to obtain the band pass characteristic.





By definition, $Z_1 Z_2 = k^2 = \frac{L_2}{C_1}$ — (1)

The impedance of the series arm Z_1 is

$$Z_1 = j\omega L_1 + \frac{1}{j\omega C_1} = j\omega L_1 - \frac{j}{\omega C_1}$$

$$= j\left[\frac{\omega^2 C_1 L_1 - 1}{\omega C_1}\right]$$

The impedance of the shunt arm Z_2 is

$$Z_2 = j\omega L_2 \,||\, \frac{1}{j\omega C_2} = \frac{j\omega L_2 \cdot \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}}$$

$$= \frac{j\omega L_2 C_2 / C_2}{j^2 \omega^2 L_2 C_2 + 1}$$

$$= \frac{j\omega L_2}{1 - \omega^2 L_2 C_2}$$

$$Z_1 Z_2 = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right]\left[\frac{j\omega L_2}{1 - \omega^2 L_2 C_2}\right]$$

$$Z_1 Z_2 = -\frac{L_2}{C_1}\left[\frac{\omega^2 L_1 C_1 - 1}{1 - \omega^2 L_2 C_2}\right] \quad \text{— (2)}$$

for the condition of equal resonant frequencies,

$$\omega_0 \frac{L_1}{2} = \frac{1}{2\omega_0 C_1} \text{ for the series arm} \quad \text{— (3)}$$

$$\frac{1}{\omega_0 C_2} = \omega_0 L_2 \text{ for the shunt arm} \quad (4)$$

from ③ & ④

$$\omega_0^2 L_1 C_1 = 1 \quad — ⑤$$

$$\omega_0^2 L_2 C_2 = 1 \quad — ⑥$$

$$⑤ = ⑥ \Rightarrow \omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$

$$\boxed{L_1 C_1 = L_2 C_2} \quad — ⑦$$

Sub ⑦ in ②

$$Z_1 Z_2 = \frac{-L_2}{C_1}\left[\frac{-(1-\omega^2 L_1 C_1)}{1-\omega^2 L_1 C_1}\right]$$

$$\boxed{Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2} \quad — ⑧$$

To determine resonant frequency f_0:

for pass band, $-1 < \frac{Z_1}{4Z_2} < 0$

At cut off frequency, $\frac{Z_1}{4Z_2} = -1$

$$Z_1 = -4Z_2$$

Multiply by $Z_1 \Rightarrow Z_1^2 = -4Z_1 Z_2$

$$Z_1^2 = -4k^2 \quad (\because Z_1 Z_2 = k^2)$$

$$Z_1 = \pm j2k$$

(ie) the value of Z_1 at lower cut off frequency is equal to the negative of the value of Z_1 at the upper cutoff frequency.

$$\therefore Z_1 = -j2k \quad — ⑨$$

$$\left(\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right) = -\left(\frac{1}{j\omega_2 C_1} + j\omega_2 L_1\right)$$

$$j\left(\omega_1 L_1 - \frac{1}{\omega_1 C_1}\right) = -j\left(\omega_2 L_1 - \frac{1}{\omega_2 C_1}\right)$$

$$\omega_1 L_1 - \frac{1}{\omega_1 C_1} = \frac{1}{\omega_2 C_1} - \omega_2 L_1$$

$$\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 C_1} = \frac{1 - \omega_2^2 L_1 C_1}{\omega_2 C_1}$$

$$(\omega_1^2 L_1 C_1 - 1)\omega_2 C_1 = (1 - \omega_2^2 L_1 C_1)\omega_1 C_1$$

$$(\omega_1^2 L_1 C_1 - 1) = \frac{\omega_1}{\omega_2}(1 - \omega_2^2 L_1 C_1) \quad — (10)$$

from eq (5), $L_1 C_1 = \frac{1}{\omega_0^2}$ - (11)

Sub (11) in (10)

$$\left(\frac{\omega_1^2}{\omega_0^2} - 1\right) = \frac{\omega_1}{\omega_2}\left(1 - \frac{\omega_2^2}{\omega_0^2}\right)$$

$$(\omega_1^2 - \omega_0^2)\omega_2 = \omega_1(\omega_0^2 - \omega_2^2)$$

$$\omega_1^2\omega_2 - \omega_0^2\omega_2 = \omega_1\omega_0^2 - \omega_2^2\omega_1$$

$$-\omega_0^2\omega_2 - \omega_1\omega_0^2 = -\omega_2^2\omega_1 - \omega_1^2\omega_2$$

$$\omega_0^2(\omega_2 + \omega_1) = \omega_1\omega_2(\omega_2 + \omega_1)$$

$$\omega_0^2 = \frac{\omega_1\omega_2\cancel{(\omega_2 + \omega_1)}}{\cancel{(\omega_2 + \omega_1)}}$$

$$\omega_0^2 = \omega_1\omega_2$$

Sub $\omega_0 = 2\pi f_0$, $\omega_1 = 2\pi f_1$ & $\omega_2 = 2\pi f_2$

$$(2\pi f_0)^2 = (2\pi f_1)(2\pi f_2)$$

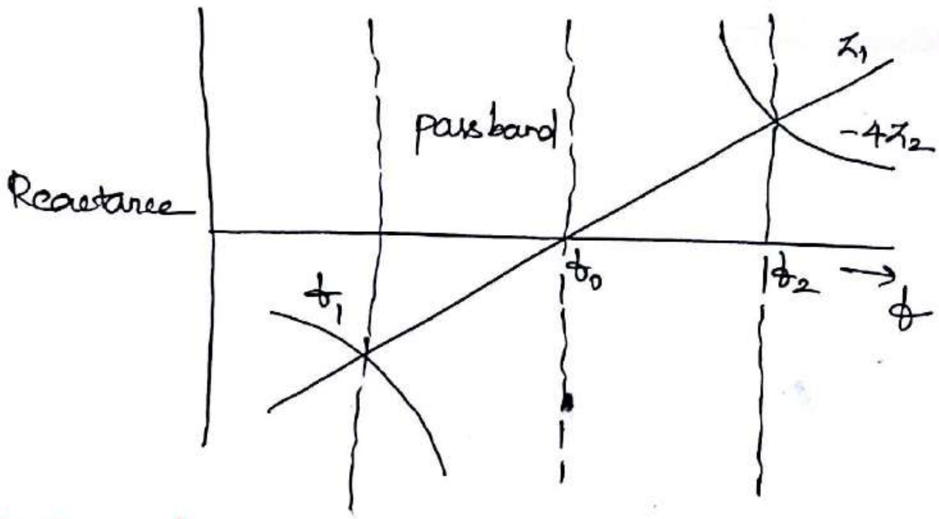
$$f_0^2 = \frac{\cancel{4\pi^2} f_1 f_2}{\cancel{4\pi^2}}$$

$$f_0^2 = f_1 f_2$$

$$\boxed{f_0 = \sqrt{f_1 f_2}} \quad — (12)$$

* The resonant frequency is the geometric mean of the cut-off frequencies.

* The variation of the reactances with respect to frequency is shown in the figure.



Design of Band pass filter:

$$Z_1 = -j2K$$

$$\left[\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right] = -2jK$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2K$$

$$1 - \omega_1^2 L_1 C_1 = 2K\omega_1 C_1$$

from ⑤, $L_1 C_1 = \frac{1}{\omega_0^2}$

$$1 - \frac{\omega_1^2}{\omega_0^2} = 2K\omega_1 C_1$$

Sub $\omega_1 = 2\pi f_1$ & $\omega_0 = 2\pi f_0$

$$1 - \frac{(2\pi f_1)^2}{(2\pi f_0)^2} = 2K \times 2\pi f_1 \times C_1$$

$$1 - \left(\frac{f_1}{f_0}\right)^2 = 4\pi K f_1 C_1$$

from eq (12), $f_0 = \sqrt{f_1 f_2}$

$$\therefore 1 - \frac{f_1^2}{f_1 f_2} = 4\pi K f_1 C_1$$

$$\frac{f_2 - f_1}{f_2} = 4\pi K f_1 C_1$$

$$f_2 - f_1 = 4\pi K f_1 f_2 C_1$$

$$\therefore \boxed{C_1 = \frac{f_2 - f_1}{4\pi K f_1 f_2}} \quad \text{— (13)}$$

from (5), $L_1 C_1 = \frac{1}{\omega_0^2}$

$$\therefore L_1 = \frac{1}{\omega_0^2 C_1}$$

sub eq (13) in L_1, $L_1 = \frac{1 \times 4\pi K f_1 f_2}{\omega_0^2 (f_2 - f_1)}$

$$L_1 = \frac{4\pi K f_0^2}{4\pi^2 f_0^2 (f_2 - f_1)} \qquad (f_1 f_2 = f_0^2)$$

$$\boxed{L_1 = \frac{K}{\pi (f_2 - f_1)}} \quad \text{— (14)}$$

from eq (8),

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = K^2$$

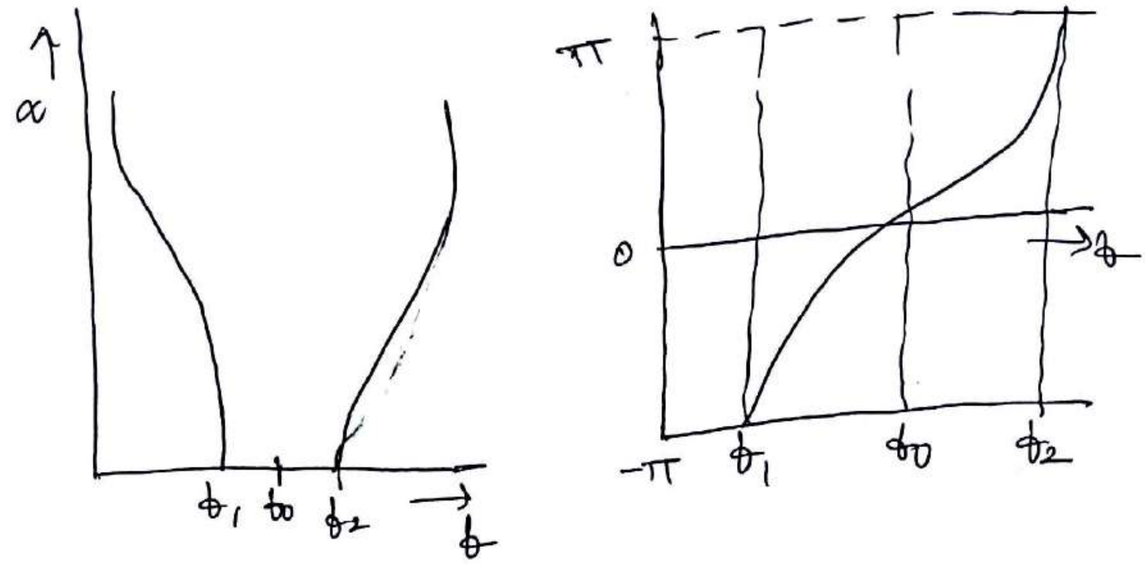
$$\therefore L_2 = C_1 K^2 = \frac{(f_2 - f_1) \times K^2}{4\pi K f_1 f_2}$$

$$\boxed{L_2 = \frac{(f_2 - f_1) K}{4\pi f_1 f_2}} \quad \text{— (15)}$$

$$C_2 = \frac{L_1}{K^2} = \frac{K}{\pi (f_2 - f_1) K^2}$$

$$\boxed{C_2 = \frac{1}{K\pi (f_2 - f_1)}} \quad \text{— (16)}$$

The Variation of α, β with respect to frequency as shown in the figure.



Problem:

1\. Design K-type band pass filter having a design impedance of 500Ω and cut off frequencies 1 KHz and 10KHz.

Solution:

Given, $K = 500\Omega$, $f_1 = 1\,KHz$ & $f_2 = 10\,KHz$

$$L_1 = \frac{K}{\pi(f_2 - f_1)} = \frac{500}{\pi(10000 - 1000)}$$

$$\boxed{L_1 = 16.68\ mH}$$

$$C_1 = \frac{f_2 - f_1}{4\pi K f_1 f_2} = \frac{(10000 - 1000)}{4 \times \pi \times 500 \times 1000 \times 10000}$$

$$\boxed{C_1 = 0.143\ \mu F}$$

$$L_2 = C_1 K^2 = 0.143 \times 10^{-6} \times (500)^2$$

$$\boxed{L_2 = 3.57\ mH}$$

$$C_2 = \frac{L_1}{K^2} = \frac{16.68 \times 10^{-3}}{(500)^2}$$

$$\boxed{C_2 = 0.0707\ \mu F}$$

* Each of the two series arms of the constant K, T section filter is given by

$$\frac{L_1}{2} = \frac{17.68}{2} = 8.84\,mH$$

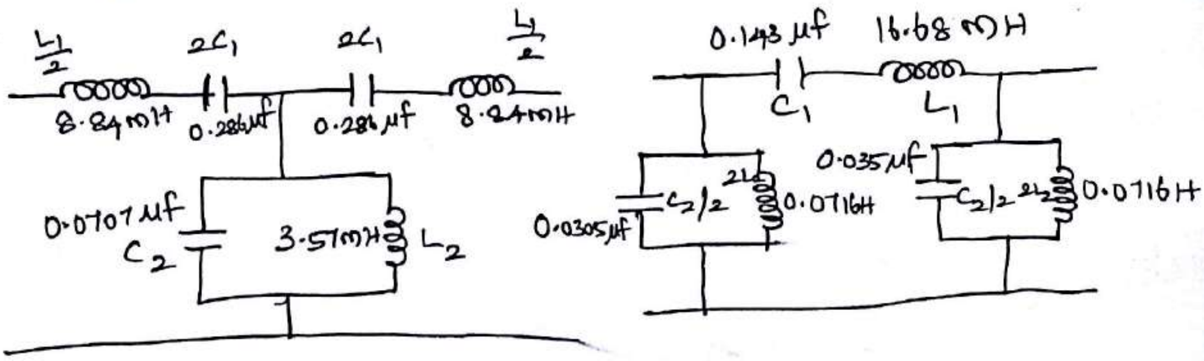
$$2C_1 = 2 \times 0.143\,\mu F = 0.286\,\mu F$$

* Shunt → $C_2 = 0.0707\,\mu f$ & $L_2 = 3.57\,mH$
* for the constant K, π section filter elements are

$$C_1 = 0.143\,\mu F, \; L_1 = 16.68\,mH$$

$$\frac{C_2}{2} = \frac{0.0707\,\mu f}{2} = 0.035\,\mu f$$

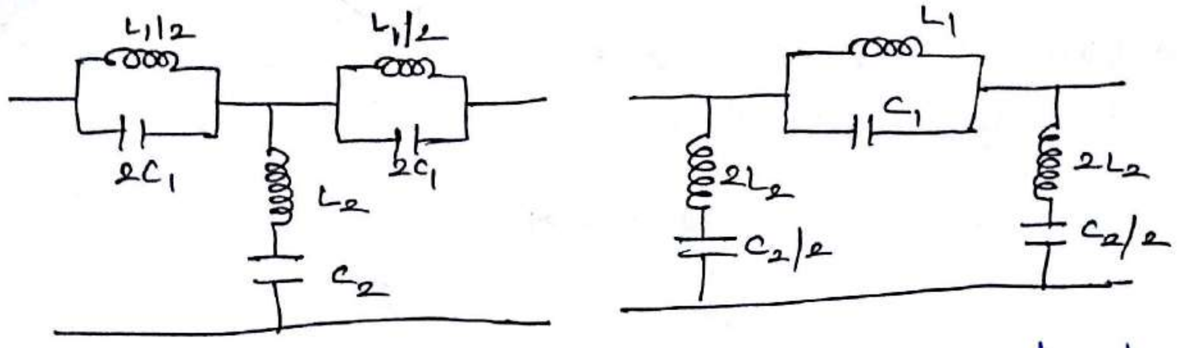
$$2L_2 = 2 \times 0.0358 = 0.0716\,H$$



4) BAND ELIMINATION FILTER:

* A band stop filter is one which passes without attenuation all frequencies less than the lower cut-off frequency f_1 and greater than the upper cut-off frequency f_2. frequencies lying between f_1 & f_2 are attenuated.

* It can be realized by connecting a low pass filter in parallel with a high pass section, in which the cut-off frequency of low pass filter is below that of a high pass filter.



* The design procedure is same as band pass filter.

$$Z_1 . Z_2 = K^2 \quad -(1)$$

The impedance ~~of~~ Z_1 is given by

$$Z_1 = j\omega L_1 || \frac{1}{j\omega C_1}$$

$$Z_1 = \frac{j\omega L_1}{1-\omega^2 L_1 C_1}$$ (same as Band pass filter)

$$Z_2 = j\omega L_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)$$

$$Z_1 Z_2 = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)\left(\frac{j\omega L_1}{1-\omega^2 L_1 C_1}\right)$$

$$= \frac{-L_2}{C_1}\left(\frac{\omega^2 L_1 C_1 - 1}{1-\omega^2 L_2 C_2}\right) \quad -(2)$$

* As for the band pass filter, the series and shunt arms are chosen to resonate at the same frequency ω_0.

* The condition of equal resonant frequencies

$$\frac{\omega_0 L_1}{2} = \frac{1}{2\omega_0 C_1} \Rightarrow \text{series arm}$$

$$\therefore \omega_0^2 = \frac{1}{L_1 C_1} \quad - (3)$$

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2} \Rightarrow \text{shunt arm}$$

$$\omega_0^2 = \frac{1}{L_2 C_2} \quad - (4)$$

$$(3) = (4) \Rightarrow \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

$$\Rightarrow \boxed{L_1 C_1 = L_2 C_2} \quad - (5)$$

sub (5) in (2)

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2 \quad - (5a)$$

* In pass band, $-1 < \frac{Z_1}{4Z_2} < 0$

* At cut off frequency, $\frac{Z_1}{4Z_2} = -1$

$$Z_1 = -4Z_2$$

$$\times Z_2 \Rightarrow Z_1 Z_2 = -4Z_2^2$$

$$-4Z_2^2 = k^2 \Rightarrow \boxed{Z_2 = \pm j\frac{k}{2}} \quad - (6)$$

* By resonant frequency condition,

$$\left(\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right) = -\left(\frac{1}{j\omega_2 C_2} + j\omega_2 L_2\right)$$

$$j\left(\omega_1 L_1 - \frac{1}{\omega_1 C_1}\right) = -j\left(\omega_2 L_1 - \frac{1}{\omega_2 C_1}\right)$$

$$\omega_1 L_1 - \frac{1}{\omega_1 C_1} = \frac{1}{\omega_2 C_1} - \omega_2 L_1$$

$$\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 C_1} = \frac{1 - \omega_2^2 L_1 C_1}{\omega_2 C_1}$$

$$(\omega_1^2 L_1 C_1 - 1)\,\omega_2 \varphi_1 = (1 - \omega_2^2 L_1 C_1)\,\omega_1 \varphi_1$$

$$(\omega_1^2 L_1 C_1 - 1) = \frac{\omega_1}{\omega_2}(1 - \omega_2^2 L_1 C_1) \quad -(7)$$

from (3), $L_1 C_1 = \frac{1}{\omega_0^2}$ $-(8)$

Sub (8) in (7)

$$\left(\frac{\omega_1^2}{\omega_0^2} - 1\right) = \frac{\omega_1}{\omega_2}\left(1 - \frac{\omega_2^2}{\omega_0^2}\right)$$

$$(\omega_1^2 - \omega_0^2)\,\omega_2 = \omega_1(\omega_0^2 - \omega_2^2)$$

$$\omega_1^2\omega_2 - \omega_0^2\omega_2 = \omega_1\omega_0^2 - \omega_2^2\omega_1$$

$$-\omega_0^2\omega_2 - \omega_1\omega_0^2 = -\omega_2^2\omega_1 - \omega_1^2\omega_2$$

$$\omega_0^2(\omega_1 + \omega_2) = \omega_2\omega_1(\omega_2 + \omega_1)$$

$$\omega_0^2 = \frac{\omega_1\omega_2\,\cancel{(\omega_2 + \omega_1)}}{\cancel{(\omega_2 + \omega_1)}}$$

$$\omega_0^2 = \omega_1\omega_2$$

Sub $\omega = 2\pi f$,

$$(2\pi f_0)^2 = (2\pi f_1)(2\pi f_2)$$

$$f_0^2 = \frac{\cancel{4\pi^2} f_1 f_2}{\cancel{4\pi^2}}$$

$$f_0^2 = f_1 f_2$$

$$\boxed{f_0 = \sqrt{f_1 f_2}} \quad -(9)$$

Design:

$$Z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = j\frac{K}{2}$$

$$\frac{1 - \omega_1^2 L_2 C_2}{\omega_1 C_2} = \frac{K}{2}$$

$$1 - \omega_1^2 C_2 L_2 = \omega_1 C_2 \frac{K}{2}$$

from ④, $L_2 C_2 = \frac{1}{\omega_0^2}$

Sub this in above eqn,

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{K}{2} \omega_1 C_2$$

Sub $\omega = 2\pi f$,

$$1 - \frac{f_1^2}{f_0^2} = \frac{K}{2} \times 2\pi f_1 \times C_2$$

$$1 - \left(\frac{f_1}{f_0}\right)^2 = K\pi f_1 C_2$$

$$C_2 = \frac{1}{K\pi f_1}\left[1 - \left(\frac{f_1}{f_0}\right)^2\right]$$

from ⑤, $f_0 = \sqrt{f_1 f_2}$

$$\therefore C_2 = \frac{1}{K\pi f_1}\left[1 - \frac{f_1^2}{f_1 f_2}\right] = \frac{1}{K\pi f_1}\left[1 - \frac{f_1}{f_2}\right]$$

$$= \frac{1}{K\pi f_1}\left[\frac{f_2 - f_1}{f_2}\right]$$

$$\therefore \boxed{C_2 = \frac{1}{K\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right]} \quad -⑩$$

from ④, $\omega_0^2 = \frac{1}{L_2 C_2}$

$$L_2 = \frac{1}{\omega_0^2 C_2} = \frac{1}{\omega_0^2} \times \frac{\pi K f_1 f_2}{(f_2 - f_1)} \quad (f_0^2 = f_1 f_2)$$

$$\therefore = \frac{1}{4\pi^2 f_0^2} \times \frac{\pi K f_0^2}{(f_2 - f_1)}$$

$$\boxed{L_2 = \frac{K}{4\pi(f_2 - f_1)}} \quad -⑪$$

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from (5a), $K^2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$

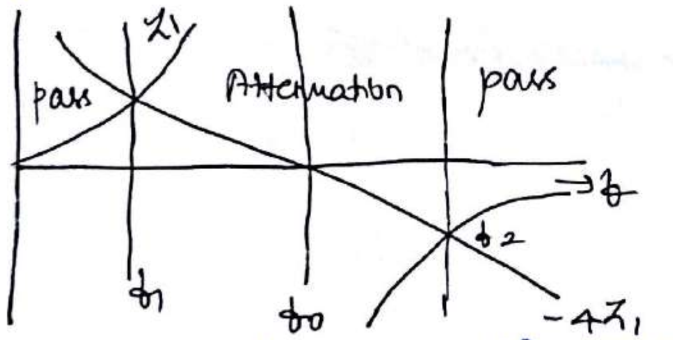
$$L_1 = C_2 K^2 = K^2 \times \frac{1}{K\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right]$$

$$\boxed{L_1 = \frac{K}{\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right]} \quad \text{— (12)}$$

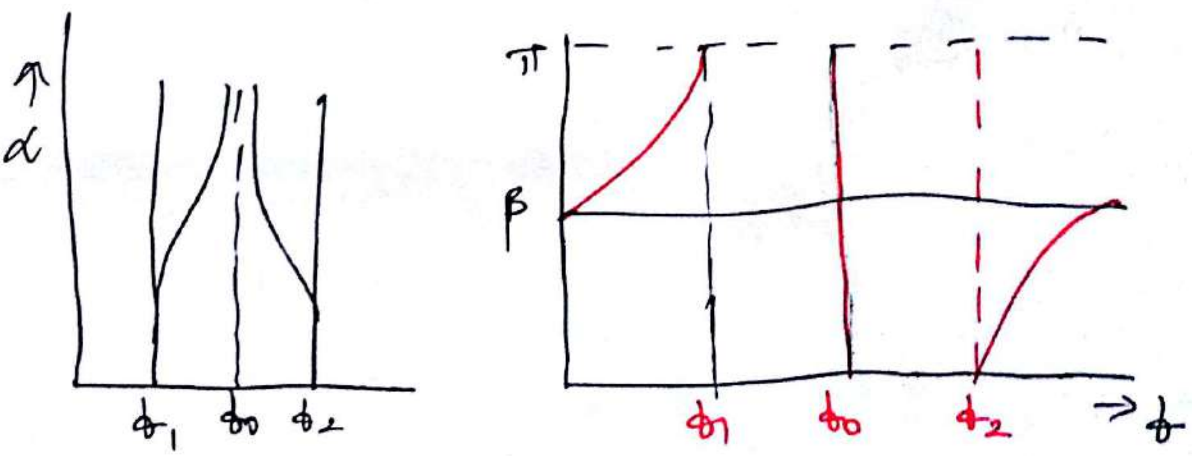
$$C_1 = \frac{L_2}{K^2} = \frac{K}{K^2 4\pi (f_2 - f_1)}$$

$$\boxed{C_1 = \frac{1}{4\pi K (f_2 - f_1)}} \quad \text{— (13)}$$

* the variation of the reactances with respect to frequency is shown on figure.



* The Variation of α, β with respect to frequency is shown on figure.



Problem:

1. Design a band elimination filter having a design impedance of 600 Ω and cut-off frequencies $f_1 = 2\,KHz$ & $f_2 = 6\,KHz$.

Solution:

Given: $K = 600\,\Omega$, $f_1 = 2\,KHz$, $f_2 = 6\,KHz$

$$L_1 = \frac{K}{\pi}\left[\frac{f_2 - f_1}{f_2 f_1}\right] = \frac{600}{\pi}\left[\frac{4\,KHz}{2\times10^3 \times 6\times10^3}\right]$$

$L_1 = 63\,mH$

$$C_1 = \frac{1}{4\pi K (f_2 - f_1)} = \frac{1}{4\times\pi\times600\times4\times10^3}$$

$C_1 = 0.033\,\mu f$

$$L_2 = \frac{K}{4\pi (f_2 - f_1)} = \frac{600}{4\,\pi\times4\times10^3}$$

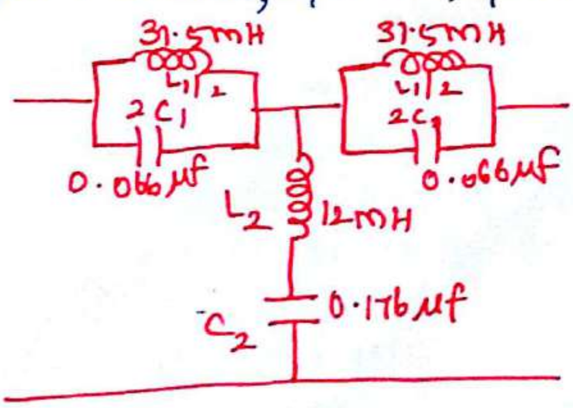
$L_2 = 12\,mH$

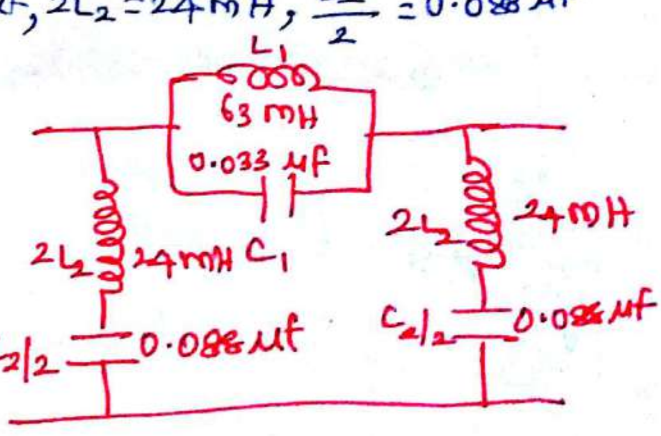
$$C_2 = \frac{1}{K\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] = \frac{1}{600\times\pi}\left[\frac{4\times10^3}{2\times10^3\times6\times10^3}\right]$$

$C_2 = 0.176\,\mu f$.

for T section, $\frac{L_1}{2} = 31.5\,mH$, $2C_1 = 0.066\,\mu f$, $L_2 = 12\,mH$, $C_2 = 0.176\,\mu f$

for π section, $L_1 = 63\,mH$, $C_1 = 0.033\,\mu f$, $2L_2 = 24\,mH$, $\frac{C_2}{2} = 0.088\,\mu f$





Limitations of Constant K filters:

There are 2 disadvantages

1. Ideally the attenuation should change sharply in the attenuation band. But in all K filter, the attenuation changes gradually in the stop band. Hence frequencies near cut-off frequency are passed through the filter.

2. In pass band, the output of filter should remain constant. This indicates that the characteristic impedance should remain constant. It is observed that characteristic impedance varies with frequency from value R_0 (ie) design impedance value, throughout the pass band. Hence, filter can not be terminated properly.

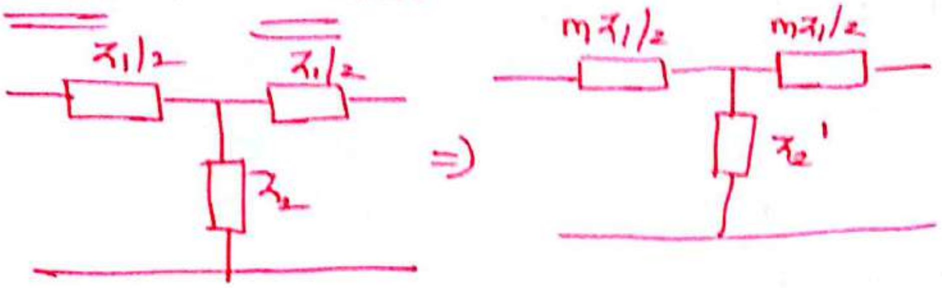
M) Derived Filters:

* The first disadvantage of K filter can be overcome by connecting two or more prototype sections (K) of same type (either all T or all π) in cascade. In such a cascade connection, attenuation to the frequencies in pass band remains zero ideally, but attenuation to the frequencies in entire attenuation band considerably increases.

* But due to the resistance in components used in cascade connection, the attenuation in pass band slightly increases instead of being zero.

* To fulfill this, it is necessary to design a new section having same f_c as of K type but different attenuation characteristics in attenuation band. Also same f_c is obtained must have same Z_0.

M- derived T - section:



$$Z_{0T} = Z_{0T}' \quad \text{—} \; (1)$$

where Z_{0T}' is the characteristic impedance of modified T network.

$$Z_{0T} \text{ of T network} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$\therefore \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{m^2 Z_1^2}{4} + m Z_1 Z_2'}$$

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + m Z_1 Z_2'$$

$$m Z_1 Z_2' = \frac{Z_1^2}{4} + Z_1 Z_2 - \frac{m^2 Z_1^2}{4}$$

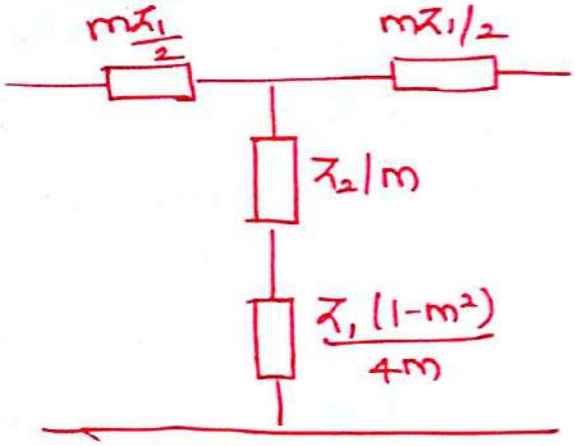
$$m Z_1 Z_2' = \frac{Z_1^2}{4}(1-m^2) + Z_1 Z_2$$

$$Z_2' = \frac{Z_1^2}{Z_1 m}(1-m^2) + \frac{Z_1 Z_2}{m Z_1}$$

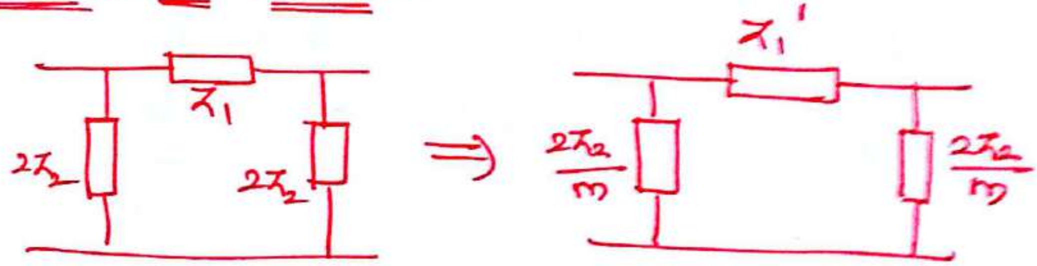
$$Z_2' = \frac{Z_1}{m}(1-m^2) + \frac{Z_2}{m} \quad \text{—} \; (2)$$

from eq (2), Z_2' consists of two impedances in series as shown in the figure.

$\frac{1-m^2}{4m}$ should be positive to realize the impedance Z_2' physically (ie) $0 < m < 1$. Thus m derived section can be obtained from the prototype by modifying the series and shunt arms.



Modified π section:



* The characteristic impedance is given by

$$Z_{0\pi} = Z'_{0\pi}$$

$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

$$\therefore \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1' \frac{Z_2}{m}}{1 + \frac{Z_1'}{\frac{4Z_2}{m}}}}$$

$$\frac{Z_1 Z_2}{\frac{4Z_2 + Z_1}{4Z_2}} = \frac{Z_1' Z_2/m}{\frac{4Z_2 + Z_1' m}{4Z_2}}$$

$$\frac{4Z_1 Z_2^2}{4Z_2 + Z_1} = \frac{4Z_1' Z_2^2}{4Z_2 m + Z_1' m^2}$$

$$4Z_1' Z_2^2 = \frac{(4Z_1 Z_2^2)(4m Z_2 + Z_1' m^2)}{4Z_2 + Z_1}$$

$$\left(\frac{Z_1 Z_2^2}{4}\right)(4Z_2 m + Z_1' m^2) = \left(\frac{Z_1' Z_2^2}{4}\right)(4Z_2 + Z_1)$$

$$4Z_1 Z_2 m + Z_1 Z_1' m^2 - 4Z_2 Z_1' - Z_1' Z_1 = 0$$

$$Z_1'(m^2 Z_1 - 4Z_2 - Z_1) = -4Z_1 Z_2 m$$

$$Z_1' = \frac{-4Z_1 Z_2 m}{m^2 Z_1 - 4Z_2 - Z_1}$$

$\div$ by $4m$, $$Z_1' = \frac{Z_1 Z_2}{\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{mZ_1}{4}}$$

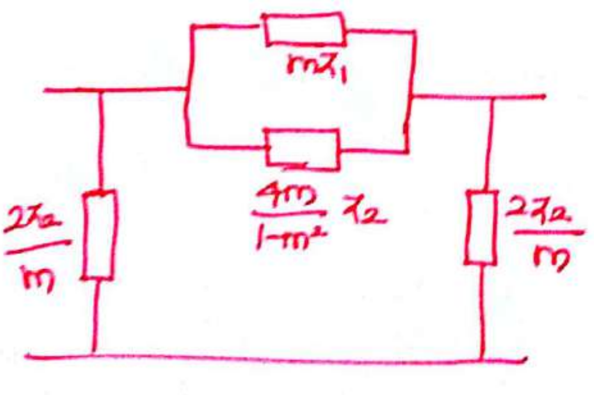
$$\therefore Z_1' = \frac{Z_1 Z_2}{\frac{Z_2}{m} + \frac{Z_1}{4m}(1-m^2)}$$

$\times$ by $\frac{4m^2}{(1-m^2)}$ $\Rightarrow$ $$Z_1' = \frac{Z_1 Z_2 \frac{4m^2}{1-m^2}}{\frac{Z_2 4m^2}{m(1-m^2)} + Z_1 m}$$

$$Z_1' = \frac{Z_1 Z_2 \frac{4m^2}{(1-m^2)}}{\frac{Z_2 4m^2}{m(1-m^2)} + Z_1 m}$$

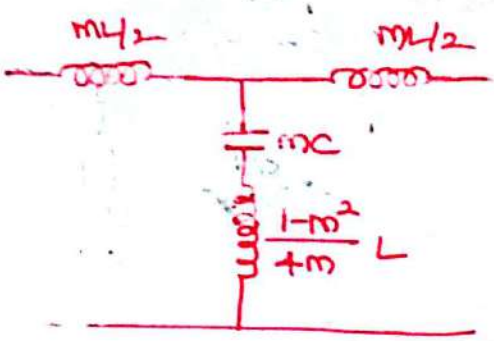
$$Z_1' = \frac{mZ_1 \frac{Z_2 4m}{(1-m^2)}}{mZ_1 + \frac{Z_2 4m}{(1-m^2)}}$$

* It appears that the series arm of the m derived π section is a parallel combination of mZ_1 & $\frac{Z_2 4m}{(1-m^2)}$



M- Derived Low pass filter:

Using T section:



* The shunt arm is to be chosen so that it is resonant at some frequency f_∞ above cut-off frequency f_c.

* If the shunt arm is series resonant, its impedance will be minimum or zero

* Therefore the output is zero and will correspond to infinite attenuation at this particular frequency. Thus at f_∞.

$$\therefore \frac{1}{m\omega_r C} = \frac{1-m^2}{4m}\omega_r L,$$

where $\omega_r \Rightarrow$ resonant frequency

$$\omega_r^2 = \frac{4}{(1-m^2)LC}$$

$$\omega = 2\pi f \Rightarrow 4\pi^2 f_r^2 = \frac{4}{(1-m^2)LC}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} = f_\infty \quad -①$$

Since the cut off frequency for the low pass filter is $f_c = \frac{1}{\pi\sqrt{LC}}$ $\quad -②$

sub ② in ①

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}} \quad -③$$

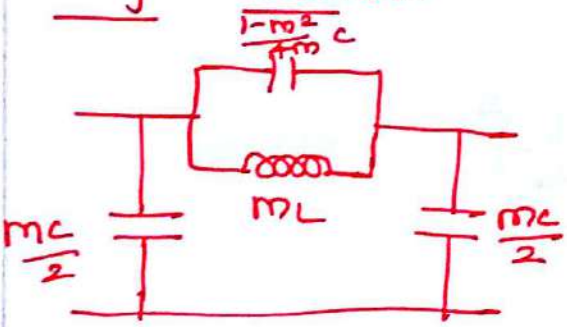
$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} \quad -④$$

* If a sharp cut-off is desired f_∞ should be near to f_c.

* from eq (5), for the smaller the value of m, f_∞ comes close to f_c.

* from eq (4), if f_c & f_∞ are specified, the necessary Value of m may then be calculated.

Using π-section:



* The inductance and capacitance in the series arm constitute a resonant circuit.

* Thus at f_∞ a frequency corresponds to infinite attenuation at f_∞

* $$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4m}\right)\omega_r C}$$

$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

Sub $\omega_r = 2\pi f_r$

$$\therefore f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} \quad — (5)$$

Since, $$f_c = \frac{1}{\pi\sqrt{LC}} \quad — (6)$$

Sub (2) in (1)

$$f_r = \frac{f_c}{\sqrt{1-m^2}} = f_\infty \quad — (7)$$

* for both m derived low pass networks for a positive value of m $(0 < m < 1)$, $f_\infty > f_c$.

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* Eq (4) & (7) can be used to choose the value of m, knowing f_c & f_r.

* The variation of attenuation for a low pass m-derived section can be verified from

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \quad \text{for } f_c < f < f_\infty.$$

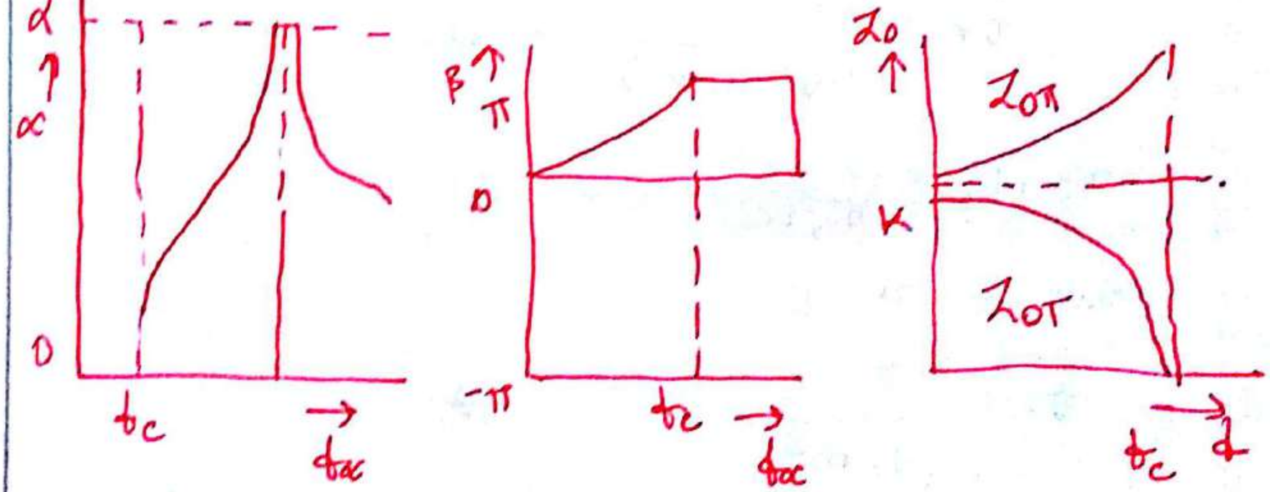
for $Z_1 = j\omega L$ & $Z_2 = -j/\omega C$

$$\therefore \alpha = 2\cosh^{-1} \frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_\infty}\right)^2}}$$

$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

$$\beta = 2\sin^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_c}\right)^2(1-m^2)}}$$

* Figure shows the variation of α, β and Z_0 with respect to frequency.



Design Equations:

1. $m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$

2. $L = \frac{K}{\pi f_c}$ (Same as LPF)

3. $C = \frac{1}{\pi K f_c}$

4. $\frac{mL}{2}$ 5. mC 6. $\frac{1-m^2}{4m}L$ 7. $\frac{mC}{2}$

8. $\frac{1-m^2}{4m} \times C$ 9. mL 10. $f_c = \frac{1}{\pi\sqrt{LC}}$

Problems:

1. Design a m-derived low pass filter having cut-off frequency of 1KHz, design impedance of 400Ω and the resonant frequency 1100 Hz.

Solution:

Given: $K = 400\Omega$, $f_c = 1000\ Hz$; $f_\infty = 1100\ Hz$

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1-\left(\frac{1000}{1100}\right)^2}$$

$$m = 0.416$$

$$L = \frac{K}{\pi f_c} = \frac{400}{\pi \times 1000} = 127.32\ mH$$

$$C = \frac{1}{\pi f_c K} = \frac{1}{\pi \times 400 \times 1000} = 0.795\ \mu f$$

T section elements are

$$\frac{mL}{2} = \frac{0.416 \times 127.32 \times 10^{-3}}{2} = 26.48\ mH$$

$mC = 0.416 \times 0.795 \times 10^{-6}$

$mC = 0.33\mu f$

$$\frac{1-m^2}{4m} L = \frac{1-(0.416)^2}{4 \times 0.416} \times 127.32 \times 10^{-3}$$

$= 63.27\ mH$

π section elements are:

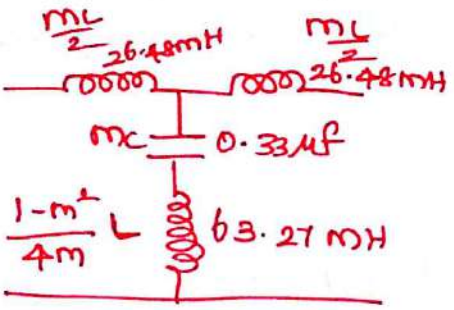
$$\frac{mC}{2} = \frac{0.416 \times 0.795 \times 10^{-6}}{2} = 0.165\ \mu f$$

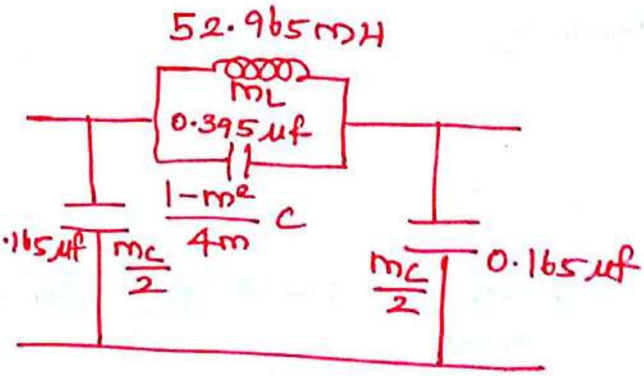
$$\frac{1-m^2}{4m} \times C = \frac{1-(0.416)^2}{4 \times 0.416} \times 0.795 \times 10^{-6}$$

$= 0.395\ \mu f$

$mL = 0.416 \times 127.32 \times 10^{-3}$

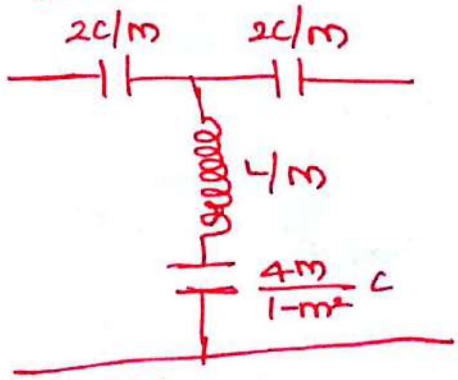
$mL = 52.965\ mH$





M derived High pass filter:

using T Section:



* If the shunt arm in T section is series resonant, it offers minimum or zero impedance.
* Therefore the output is zero and thus at resonance frequency or the frequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

$$\omega_r^2 = \frac{1}{\frac{L}{m} \frac{4m}{1-m^2} C} = \frac{1-m^2}{4LC}$$

$$f_r^2 = \frac{1-m^2}{4LC}$$

$$f_r = f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}} \quad — (1)$$

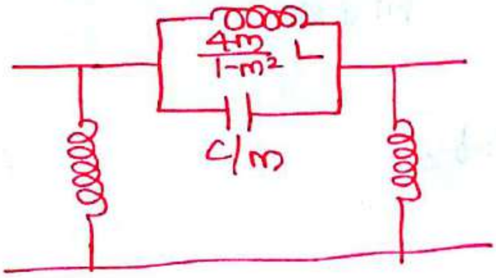
The cutt off frequency of a Hpf is $f_c = \frac{1}{4\pi\sqrt{LC}}$ — (2)

Sub (2) in (1)

$$f_\alpha = f_r = f_c\sqrt{1-m^2} \quad — (3)$$

$$m = \sqrt{1-\left(\frac{f_\alpha}{f_c}\right)^2} \quad — (4)$$

Using π section:



* The resonant circuit is constituted by the series arm inductance and capacitance. Thus at f_α.

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m} C}$$

$$\omega_r^2 = \frac{1-m^2}{4LC}$$

$$f_r = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}} = f_\alpha \quad — (5)$$

$$f_r = f_\alpha = f_c\sqrt{1-m^2} \quad — (6)$$

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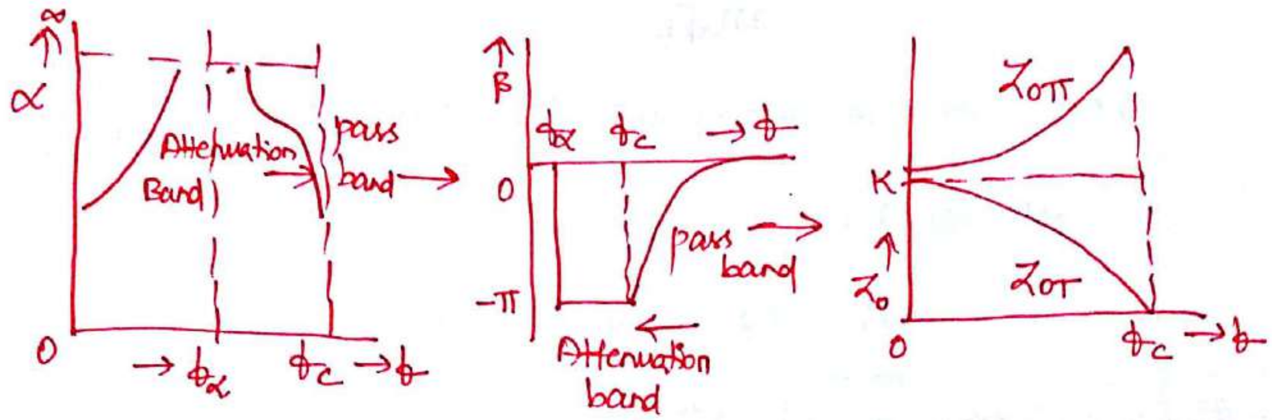
$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} \quad — \ (7)$$

* Thus the frequency corresponding to infinite attenuation is the same for both sections.

* The variation of α, β and Z_0 with frequency is shown in the figure.



Design Equations:

1. $m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$ 2. $f_c = \frac{1}{4\pi\sqrt{LC}}$

3. $L = \frac{K}{4\pi f_c}$ 4. $C = \frac{1}{4K\pi f_c}$

5. $\frac{2C}{m}$ 6. $\frac{L}{m}$ 7. $\frac{4m}{1-m^2}C$

8. $\frac{2L}{m}$ 9. $\frac{4m}{1-m^2}L$ 10. $\frac{C}{m}$

Problem:

1. Design a m-derived high pass filter with a cut-off frequency of 10KHz, design impedance of 500Ω and $m = 0.4$

Solution:

for high pass filter,

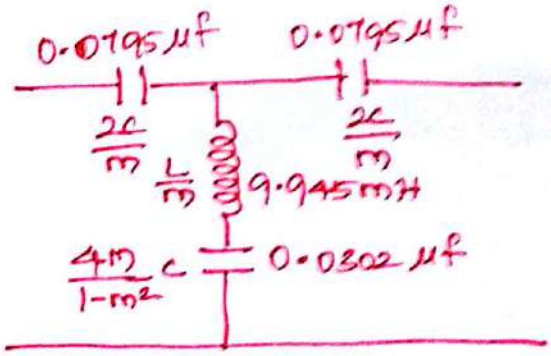
$$L = \frac{K}{4\pi f_c} = \frac{500}{4 \times \pi \times 10 \times 10^3}$$

$$L = 3.978\,mH$$

$$C = \frac{1}{4\pi K f_c} = \frac{1}{4 \times \pi \times 500 \times 10 \times 10^3}$$

$$C = 0.0159\,\mu f$$

T section elements,



$$\frac{2C}{m} = \frac{2 \times 0.0159 \times 10^{-6}}{0.4}$$

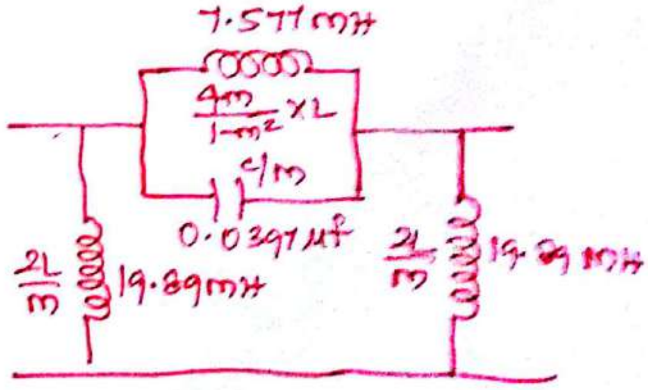
$$\frac{2C}{m} = 0.0795\,\mu f$$

$$\frac{L}{m} = \frac{3.978 \times 10^{-3}}{0.4}$$

$$\frac{L}{m} = 9.945\,mH$$

$$\frac{4m}{1-m^2} C = 0.0302\,\mu f$$

π section elements,



$$\frac{2L}{m} = \frac{2 \times 0.0159 \times 10^{-3}}{0.4}$$

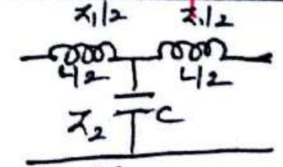
$$\frac{2L}{m} = 19.89\,mH$$

$$\frac{4m}{1-m^2} \times L = 7.577\,mH$$

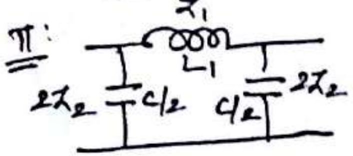
$$\frac{C}{m} = 0.0397\,\mu f$$

Constant K LPF

T:



π:



a) $Z_1 Z_2 = K^2 = \frac{L}{C}$

$K = \sqrt{\frac{L}{C}}$

b) $\frac{Z_1}{4Z_2} = -1$

$f_c = \frac{1}{\pi\sqrt{LC}}$ (cut off freq)

c) Characteristic Impedance

$Z_{0T} = K\sqrt{1-\left(f/f_c\right)^2}$

$Z_{0\pi} = \frac{K}{\sqrt{1-(f/f_c)^2}}$

d) $C = \frac{1}{\pi f_c K}$

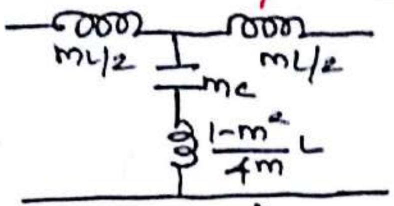
$L = \frac{K}{\pi f_c}$

c) $\alpha = 2\cosh^{-1}(f/f_c)$

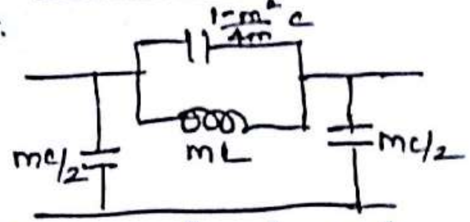
$\beta = 2\sin^{-1}(f/f_c)$

M derived LPF

T:



π:



a) Resonant freq f_r (or) f_α:

$f_r = f_\alpha = \frac{1}{\pi\sqrt{LC(1-m^2)}}$

b) cut off freq f_c : $f_c = \frac{1}{\pi\sqrt{LC}}$

c) $m = \sqrt{1-\left(\frac{f_c}{f_\alpha}\right)^2}$

d) $C = \frac{1}{\pi f_c K}$

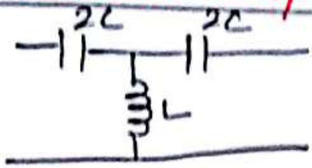
e) $L = \frac{K}{\pi f_c}$

f) $\alpha = 2\cosh^{-1}\frac{m(f/f_c)}{\sqrt{1-(f/f_\alpha)^2}}$

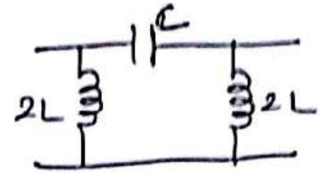
g) $\beta = 2\sin^{-1}\frac{m(f/f_c)}{\sqrt{1-(f/f_c)^2(1-m^2)}}$

Constant K HPF

T:



π:



a) $Z_1 Z_2 = K^2 = \frac{L}{C}$

$K = \sqrt{L/C}$

b) $\frac{Z_1}{4Z_2} = -1$

cut off freq, $f_c = \frac{1}{4\pi\sqrt{LC}}$

c) Characteristic Impedance

$Z_{0T} = K\sqrt{1-(f_c/f)^2}$

$Z_{0\pi} = \frac{K}{\sqrt{1-(f_c/f)^2}}$

d) $C = \frac{1}{4\pi f_c K}$

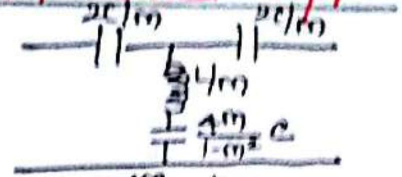
$L = \frac{K}{4\pi f_c}$

e) $\alpha = 2\cosh^{-1}(f_c/f)$

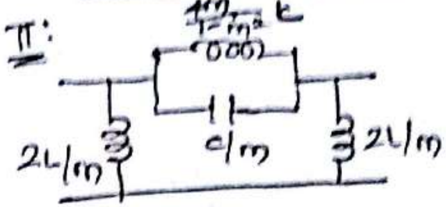
$\beta = 2\sin^{-1}(f_c/f)$

M derived HPF

T:



π:



a) Resonant freq f_r or f_α

$f_r = f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$

b) cut off freq f_c:

$f_c = \frac{1}{4\pi\sqrt{LC}}$

c) $m = \sqrt{1-\left(\frac{f_\alpha}{f_c}\right)^2}$

d) $C = \frac{1}{4\pi f_c K}$

$L = \frac{K}{4\pi f_c}$

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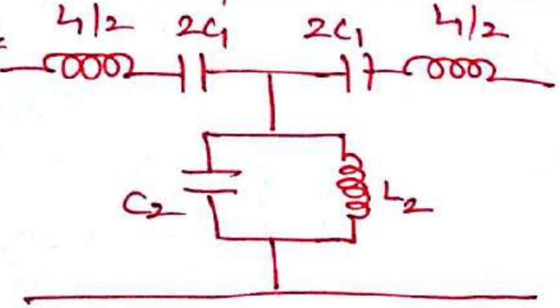
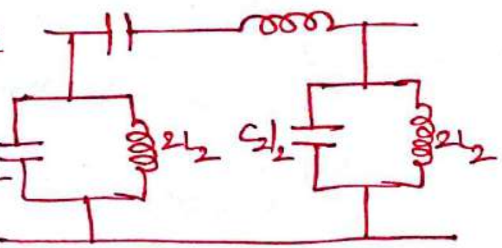
Band pass filter

T:



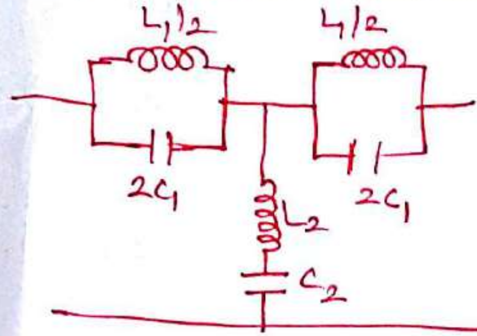
Π:



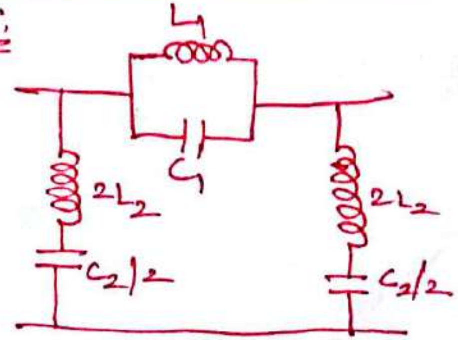
1. $Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = K^2$
2. $Z_1 = -j2K$
3. $f_0 = \sqrt{f_1 f_2}$
4. $C_1 = \frac{f_2 - f_1}{4\pi K f_1 f_2}$ 5. $L_1 = \frac{K}{\pi (f_2 - f_1)}$
6. $C_2 = \frac{1}{\pi K (f_2 - f_1)}$ 7. $L_2 = \frac{(f_2 - f_1) K}{4\pi f_1 f_2}$

Band stop filter

T:



Π:



1. $Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = K^2$
2. $Z_1 = K \frac{j}{2}$
3. $f_0 = \sqrt{f_1 f_2}$
4. $C_2 = \frac{1}{K\pi} \left[\frac{f_2 - f_1}{f_1 f_2} \right]$ 5. $L_2 = \frac{K}{4\pi (f_2 - f_1)}$
6. $C_1 = \frac{1}{4\pi K (f_2 - f_1)}$ 7. $L_1 = \frac{K}{\pi} \left(\frac{f_2 - f_1}{f_1 f_2} \right)$