UNIT- 4

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I. SIGNALS

1. Definition of signal:

A signal is defined as a physical quantity that varies with time, space or any other independent variable or variables.

* If a signal depends on only one variable then it is known as one dimensional signal.

Ex: power signal, speech signal, ECG. Variation of room temperature.

* If a signal depends on two independent variables, then it is known as two-dimensional signal.

Ex: pictures, x-ray and sonogram.

* If a signal depends on many independent variables then it is known as Multi dimensional signal.

2. Classification of signals:

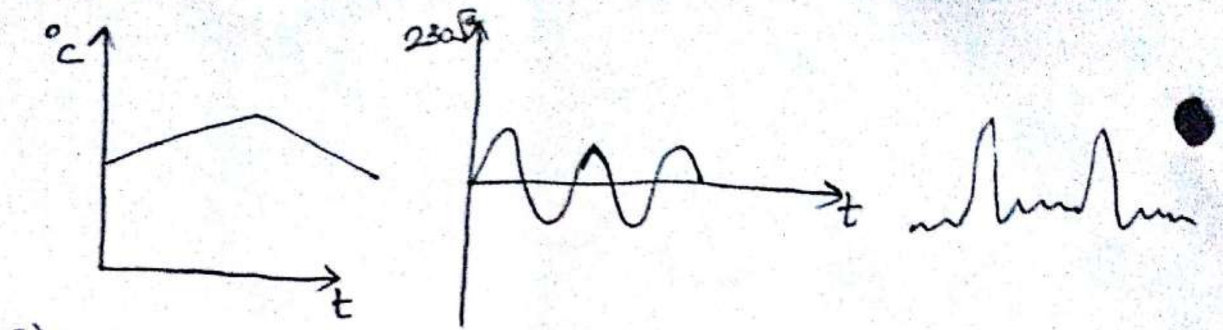
There are 3 types : 1. Continuous time signals
2. Discrete time signals
3. Digital signals

a. Continuous time signals:

Continuous time signals are defined for every value of time t and is represented by $x(t)$.

It is also called as analog signal.

Ex: AC power supply, ECG waveform, room temperature variation

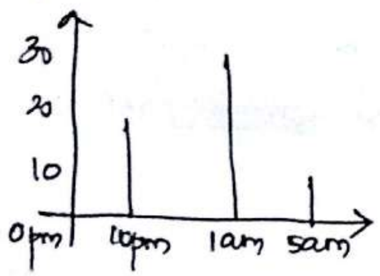


a) room temp. Variation   b) AC power supply   c) ECG

B. Discrete time signals:

* The signals that are defined at discrete instant of time are known as discrete time signals.

* The discrete time signals are continuous in amplitude and discrete in time and denoted by $x(n)$.



pressure signal

Ex: These signals occur in business, economics, science & engineering.

In business (or) economics, the discrete time variable may be the day, month or year.

* sampling of continuous time signal will give you discrete time signal, denoted by

$x(nT) = x(t)|_{t=nT}$

where T = sampling period

n = integer ranging from $-\infty$ to ∞ is called time index.

The instants at which the signal appears are called sampling instants. $n = 0, \pm 1, \pm 2, \ldots$

C. Digital signals:

Digital signal $x(n)$ is a discrete time signal that can have only a finite number of different values.

A binary signal is a digital signal, whose values are equal to 1 (or) 0 that is $x(n) = 0$ or 1 for $n = -\infty$ to ∞

A signal that is discretized in time and quantized in amplitude is known as digital signal.

Problems:

1. Sketch the continuous time signal $x(t) = 2e^{-2t}$ for an interval $0 \le t \le 2$ sample the continuous time signal with a sampling period of $T = 0.2s$ and sketch the discrete time signal.

Solution:

Given: $x(t) = 2e^{-2t}$

$x(0) = 2$

$x(0.2) = 2e^{-2(0.2)} = 1.3406$

$x(0.4) = 0.8987$

$x(0.6) = 0.6024$

$x(0.8) = 0.4038$

$x(1) = 0.2707$

$x(1.2) = 0.1814$

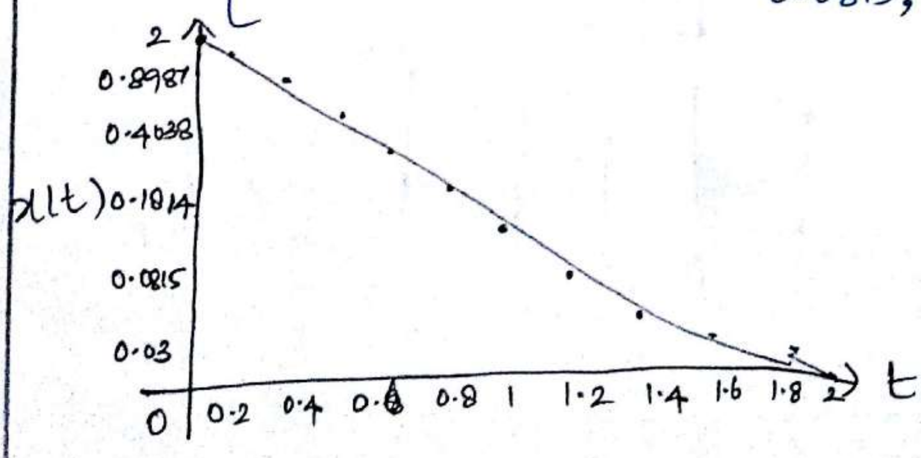
$x(1.4) = 0.1216$

$x(1.6) = 0.0815$

$x(1.8) = 0.0546$

$x(2.0) = 0.0366$

$x(t) = \{2, 1.3406, 0.8987, 0.6024, 0.4038, 0.2707, 0.1814, 0.1216, 0.0815, 0.0546, 0.0366\}$

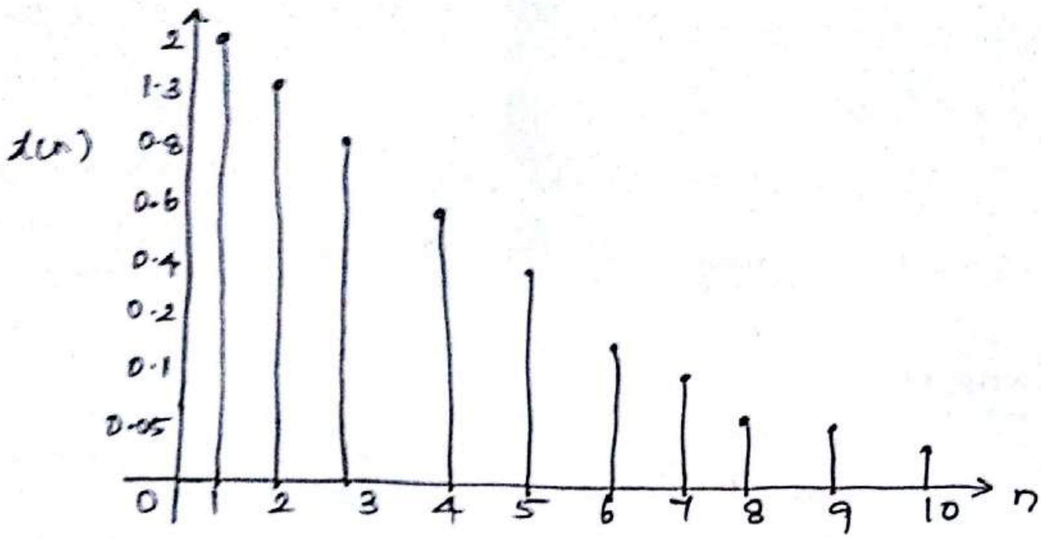


Discrete time Signal:

$T = 0.2s,\ t = nT$

$\therefore x(n) = 2e^{-2(0.2n)}$

$x(n) = \{2, 1.34, 0.89, 0.6, 0.4, 0.27, 0.18, 0.12, 0.08, 0.05, 0.036\}$



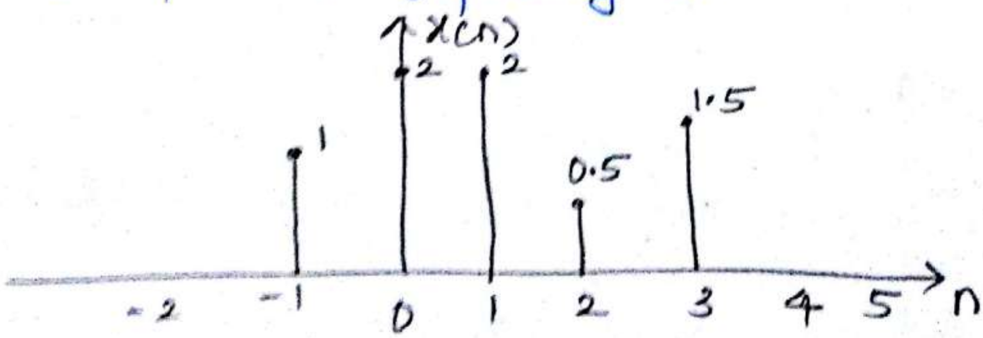
3. Representation of discrete time signals:

There are four different types

1. Graphical Representation
2. Functional Representation
3. Tabular Representation
4. Sequence Representation

a. Graphical Representation:

Let us consider a signal $x(n)$ with values $x(-1) = 1, x(0) = 2, x(1) = 2, x(2) = 0.5$ & $x(3) = 1.5$. This can be represented graphically as.



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b) Functional Representation:

$$x(n) = \begin{cases} 1 & \text{for } n=-1 \\ 2 & \text{for } n=0,1 \\ 0.5 & \text{for } n=2 \\ 1.5 & \text{for } n=3 \\ 0 & \text{otherwise} \end{cases}$$

(for same values in graphical Representation)

c) Tabular Representation:

n	-1	0	1	2	3
$x(n)$	1	2	2	0.5	1.5

d) sequence Representation:

A finite duration sequence with time origin ($n=0$) indicated by the symbol ↑ is represented as

$$x(n) = \{1, \underset{\uparrow}{2}, 2, 0.5, 1.5\}$$

A finite duration sequence that satisfies the condition $x(n)=0$ for $n<0$ can be represented as

$$x(n) = \{2, 4, 6, 8, -3\}$$

4. Elementary Signals:

The different elementary signals are

1. unit step
2. unit Ramp
3. unit Impulse
4. Exponential
5. Sinusoidal signal
6. complex exponential signal.

Continuous time Signals

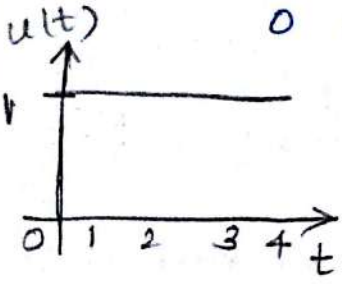
a) unit Step function

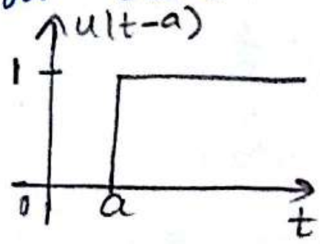
It is defined as

$$u(t) = 1 \quad \text{for } t \geq 0$$
$$\quad\;\; 0 \quad \text{for } t < 0$$

Delayed unit step is defined as

$$u(t-a) = 1 \quad \text{for } t \geq a$$
$$\qquad\quad 0 \quad \text{for } t < a$$





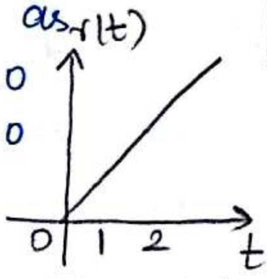
b) unit ramp function

It is defined as

$$r(t) = t \quad \text{for } t \geq 0$$
$$\qquad 0 \quad \text{for } t < 0$$

(or)

$$r(t) = t\, u(t)$$



* The ramp can be obtained by integrating unit step,

$$r(t) = \int u(t)\,dt = \int dt = t$$

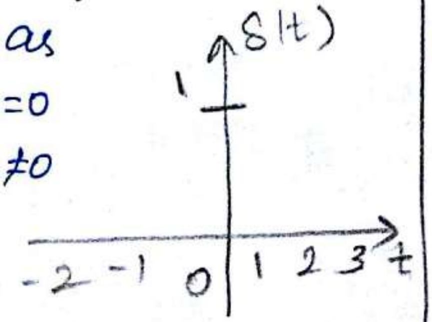
* The unit step can be obtained by differentiating ramp,

$$u(t) = \frac{d}{dt} r(t) = \frac{d}{dt}(t) = 1$$

c) unit impulse function

It is defined as

$$\delta(t) = [illegible] \quad \text{for } t = 0$$
$$\qquad 0 \quad \text{for } t \neq 0$$



Discrete time Signals

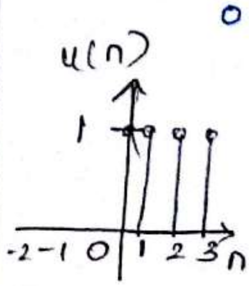
a) unit step sequence

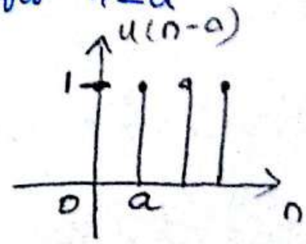
It is defined as

$$u(n) = 1 \quad \text{for } n \geq 0$$
$$\qquad 0 \quad \text{for } n < 0$$

Delayed unit step is

$$u(n-a) = 1 \quad \text{for } n \geq a$$
$$\qquad\quad 0 \quad \text{for } n < a$$

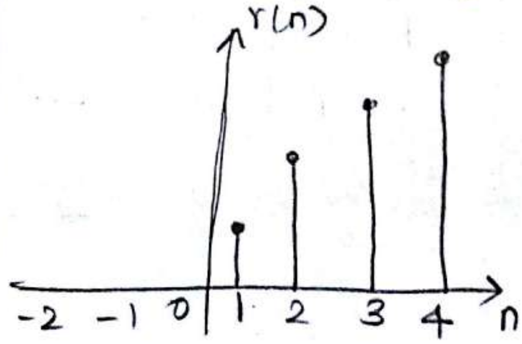




b) unit ramp sequence:

It can be defined as

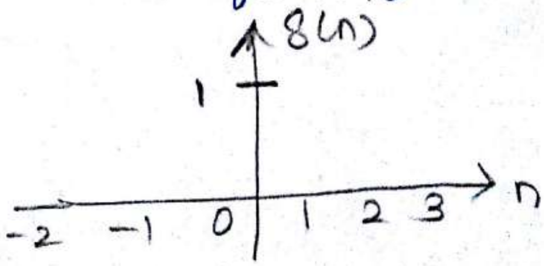
$$r(n) = n \quad \text{for } n \geq 0$$
$$\qquad 0 \quad \text{for } n < 0$$



c) unit impulse sequence

It is defined as

$$\delta(n) = [illegible] \quad \text{for } n = 0$$
$$\qquad 0 \quad \text{for } n \neq 0$$



Continuous time signals	Discrete time signals

Continuous time signals

d) Exponential sequence

It is defined as

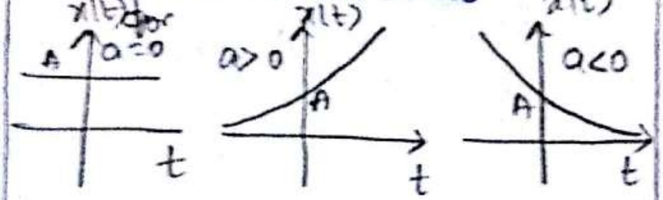
$$x(t) = Ae^{at}$$

A & a ⇒ real.

If a = +ve, x(t) is growing exponential

a = −ve, x(t) is decaying exponential

a = 0, x(t) is constant



Discrete time signals

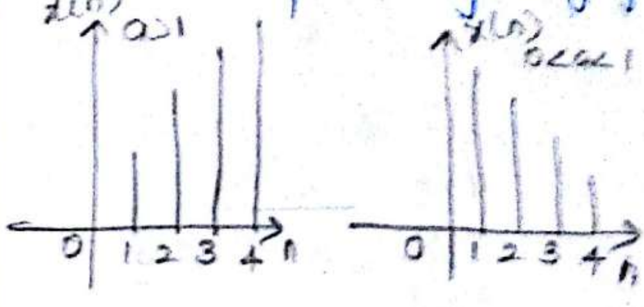
d) Exponential sequence

It is defined as

$x(n) = a^n$ for n

when $a>1$ ⇒ exponentially growing

$0<a<1$ ⇒ exponentially decaying



Continuous time signals

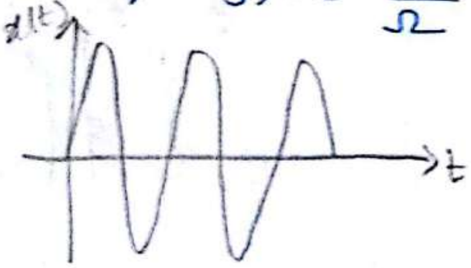
c) Sinusoidal signal

It is given by

$$x(t) = A\sin(\Omega t + \phi)$$

A ⇒ amplitude, ϕ = phase angle

Ω ⇒ frequency, $T = \frac{2\pi}{\Omega}$



Discrete time signals

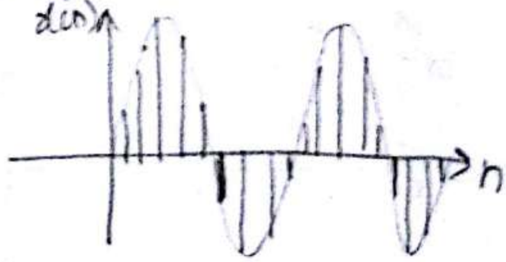
e) Sinusoidal signal

It is given by

$$x(n) = A\cos(\omega_0 n + \phi)$$

ω_0 = frequency

ϕ = phase



Continuous time signals

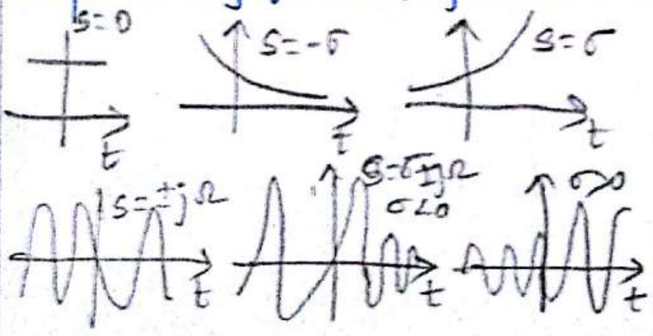
f) complex exponential signal

Given by, $x(t) = e^{(\sigma + j\Omega)t}$

$x(t) = e^{\sigma t} \cdot e^{j\Omega t}$

$x(t) = e^{\sigma t}[\cos\Omega t + j\sin\Omega t]$

1. $\sigma = 0$ & $\Omega = 0$, $x(t) = 1$, x(t) ⇒ DC
2. $\Omega = 0$, $s = \sigma$, $x(t) = e^{\sigma t}$, which decays exponentially for $\sigma < 0$ & grows for $\sigma > 0$



Discrete time signals

f) Complex Exponential sequence

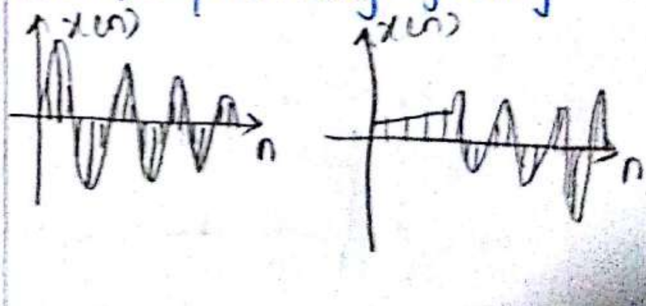
$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

$x(n) = a^n\cos(\omega_0 n + \phi) + j a^n \sin(\omega_0 n + \phi)$

$|a| = 1$, real & imaginary parts of complex exponential sequence are sinusoidal

$|a| < 1$ ⇒ exponentially decays

$|a| > 1$ ⇒ exponentially growing

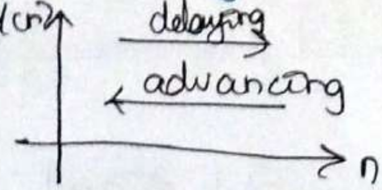


5. Basic set of operation on signals:

a) shifting:

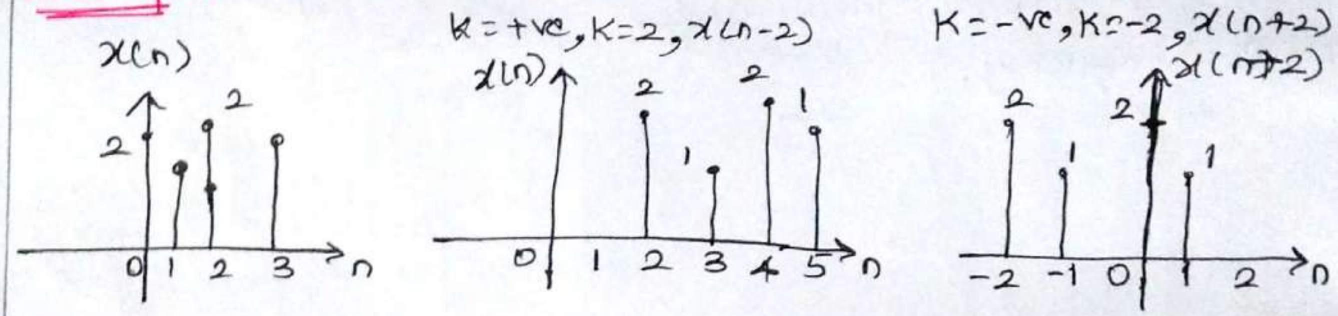
Shifting operation takes the input sequence and shift the values by an integer increment of the independent variable the shifting may delay (or) advance the sequence in time.

Consider $y(n) = x(n-k)$ where $x(n)$ = input signal
$y(n)$ = output signal



If k = +ve, shifting delays the sequence (ie) shift right
k = -ve, shifting advances the sequence (ie) shift left

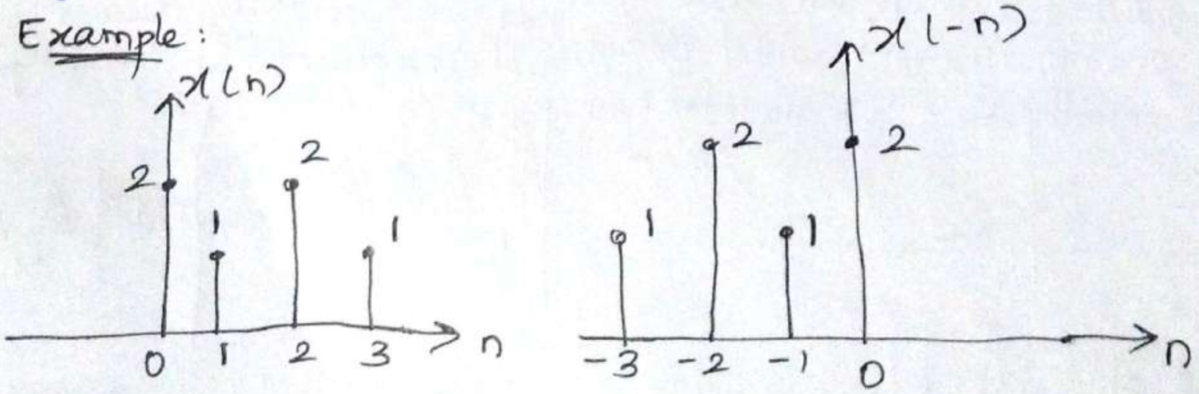
Example:



b) Time Reversal (folding (or) Transpose):

The time reversal of sequence can be obtained by folding the sequence about $n=0$. It is denoted by $x(-n)$

Example:



c) Time scaling:

Time scaling is obtained by replacing 'n' by 'λn' in the sequence $x(n)$.

Example:

$x(n) = \{1, 2, 3, 4, \underset{\uparrow}{5}, 4, 3, 2, 1\}$

$y(n) = x(2n)$

$\therefore y(0) = x(0) = 5$ $\qquad y(-1) = x(-2) = 3$

$y(1) = x(2) = 3$ $\qquad y(-2) = x(-4) = 1$

$y(2) = x(4) = 1$

$\therefore y(n) = \{1, 3, \underset{\uparrow}{5}, 3, 1\}$

d) Scalar Multiplication:

Signal $x(n)$ is multiplied by a scale factor 'a'.

$y(n) = a\,x(n)$

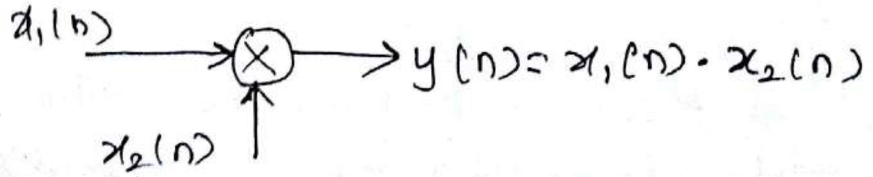
Example: $x(n) = \{1, 2, 1, -1\}$ & $a = 2$

then $y(n) = 2\{1, 2, 1, -1\}$

$y(n) = \{2, 4, 2, -2\}$

e) Signal Multiplier:

It multiplies two different signals.



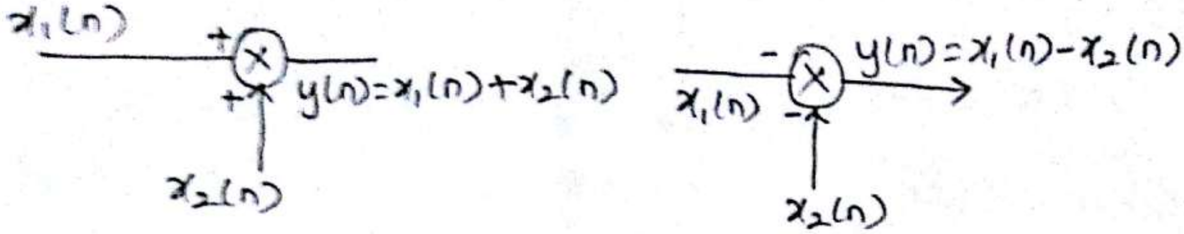
Example: $x_1(n) = \{-1, 1, 2, 3\}$ $\quad x_2(n) = \{1, 0, 2, 1\}$

then $y(n) = \{-1, 0, 4, 3\}$

7) Addition / Subtraction:

It adds their values at every instant.

It subtracts their values at every instant.



Example: $x_1(n) = \{1, 3, 2, 1\}$ & $x_2(n) = \{1, -2, 3, 2\}$

$y(n) = x_1(n) + x_2(n) = \{2, 1, 5, 3\}$

$y(n) = x_1(n) - x_2(n) = \{0, 5, -1, -1\}$

6. Classification of Signals:

The signals can be classified into

1. Deterministic and Random signals
2. Causal and Non-causal signals
3. Even and odd signals
4. periodic and aperiodic signals
5. Energy and power signals.

1. Deterministic and Random signals:

A deterministic signal is a signal exhibiting no uncertainity of its Magnitude and phase at any given instant of time. Its instantaneous value can be accurately determined by a mathematical equation.

Ex: $A \sin \Omega t$

A random signal is a signal characterized by uncertainity about its actual occurence.

Ex: noise

2. Causal and Non-causal Signals:

A signal $x(n)$ is said to be causal if $x(n)=0$ for $n<0$, otherwise the signal is non-causal.

for an anti-causal signal $x(n)=0$ for $n>0$.

3. Even and odd signals:

Symmetric or Even signal: A signal is said to be symmetric (even) if it satisfies the condition

$$x(-n) = x(n) \text{ for all } n$$

Antisymmetric or odd signal: A signal is said to be anti-symmetric (odd) if it satisfies the condition

$$x(-n) = -x(n) \text{ for all } n$$

* Any signal can be expressed as sum of even and odd components

$$x(n) = x_o(n) + x_e(n)$$

Where $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

Problem:

1. $x(n) = \{2, 1, \underset{\uparrow}{2}, -1, 3\}$. find the odd & even component for the signal $x(n)$.

Solution :

Given: $x(0) = 2, x(1) = -1, x(2) = 3, x(-1) = 1, x(-2) = -2$

$x(n) = x_o(n) + x_e(n)$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$, $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

Odd sequence will be,

$x_o(0) = \frac{1}{2}[x(0) - x(-0)] = \frac{1}{2}[2-2] = 0$

$x_o(1) = \frac{1}{2}[x(1) - x(-1)] = \frac{1}{2}[-1-1] = -1$

$x_o(2) = \frac{1}{2}[x(2) - x(-2)] = \frac{1}{2}[3+2] = 2.5$

$\cancel{x_o(3) = \frac{1}{2}[x(3)}$

$x_o(-1) = \frac{1}{2}[x(-1) - x(1)] = \frac{1}{2}[1+1] = 1$

$x_o(-2) = \frac{1}{2}[x(-2) - x(2)] = \frac{1}{2}[-2-3] = -2.5$

$\therefore$ odd sequence, $x_o(n) = \{-2.5, 1, \underset{\uparrow}{0}, -1, 2.5\}$

Even sequence will be,

$x_e(0) = \frac{1}{2}[x(0) + x(-0)] = \frac{1}{2}[2+2] = 2$

$x_e(1) = \frac{1}{2}[x(1) + x(-1)] = \frac{1}{2}[-1+1] = 0$

$x_e(2) = \frac{1}{2}[x(2) + x(-2)] = \frac{1}{2}[3-2] = 0.5$

$x_e(-1) = \frac{1}{2}[x(-1) + x(1)] = \frac{1}{2}[1-1] = 0$

$x_e(-2) = \frac{1}{2}[x(-2) + x(2)] = \frac{1}{2}[-2+3] = 0.5$

Even Sequence, $x_e(n) = \{0.5, 0, \underset{\uparrow}{2}, 0, 0.5\}$

4. periodic and aperiodic signals:

A signal $x(n)$ is said to be periodic with period N if and only if

$x(N+n) = n$ for all n

The smallest value of N for which the above equation hold is known as fundamental period. otherwise the signal is aperiodic signal.

$$\omega_0 N = 2\pi m$$

$$\omega_0 = \frac{2\pi m}{N}$$

$$= 2\pi \left[\frac{m}{N}\right]$$

$$N = 2\pi \left[\frac{m}{\omega_0}\right]$$

* To find N, smallest value of m is chosen that will make $N = 2\pi\left(\frac{m}{\omega_0}\right)$ as integer.

* The sum of two periodic signals $x_1(n), x_2(n)$ with period N_1 & N_2 may (or) not be periodic depending on relationship between N_1 & N_2.

* If the sum of two signals is to be periodic, then the ratio of time periods $\frac{N_1}{N_2}$ must be rational number (or) integer. otherwise it is not periodic.

Problems: check the signals are periodic or not

1. $x(n) = \cos(2\pi n)$

$$\omega_0 = 2\pi$$

$$N = \frac{2\pi m}{\omega_0} = \frac{2\pi}{2\pi} m = m$$

When $m=1$, $N=1$ & it is periodic

2. $x(n) = 12\cos(20n)$

$$\omega_0 = 20$$

$$N = \frac{2\pi m}{\omega_0} = \frac{2\pi m}{20} = \frac{\pi m}{10}$$

for all values of m, N is not an integer.
So aperiodic.

3. $x(n) = e^{j\frac{3\pi}{5}(n+\frac{1}{2})}$

$$\omega_0 = \frac{3\pi}{5}$$

$$N = \frac{2\pi m}{\omega_0} = \frac{2\pi m}{(3\pi/5)} = \frac{10}{3} m$$

The smallest value of m for which N is an integer is 3

$$N = \left(\frac{10}{3}\right)3 = 10$$

$$N = 10$$

$\therefore$ the fundamental period $N=10$ & periodic

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4) $x(n) = e^{j\left(\frac{2\pi}{3}\right)n} + e^{j\left(\frac{3\pi}{4}\right)n}$

$$\omega_1 = \frac{2\pi}{3} \Rightarrow N_1 = \frac{2\pi m}{\omega_1} = \frac{2\pi m}{(2\pi/3)} = 3m$$

for $m=1 \Rightarrow N_1 = 3$

$$\omega_2 = \frac{3\pi}{4} \Rightarrow N_1 = \frac{2\pi m}{(3\pi/4)} = \frac{8}{3}m$$

for $m=3 \Rightarrow N_2 = 8$

$$\therefore \frac{N_1}{N_2} = \frac{3}{8} \Rightarrow 8N_1 = 3N_2$$

$N = 24$

The given signal is periodic with time period 24.

5. Energy and power signals:

Consider a voltage $V(t)$ across a resistance R producing a current $i(t)$. The instantaneous power dissipated in the resistance R is defined by

$$p(t) = V(t)\, i(t)$$
$$= V(t)\, \frac{V(t)}{R}$$
$$p(t) = \frac{V^2(t)}{R}$$
$$p(t) = i^2(t) R \quad — \quad (1)$$

Integrate eq (1) over the interval $|t| \le T$, we can express the total energy and average power of a signal as

$$E = \lim_{T \to \infty} \int_{-T}^{T} i^2(t) R \, dt \quad (R = 1\Omega)$$

$$p = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} i^2(t)\, dt \text{ watts}$$

The square root of p is known as r.m.s value of the signal.

for a discrete time signal $x(n)$, the energy E is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

The average power of a discrete time signal $x(n)$ is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

* E = finite and p = 0 ⇒ Energy signal
* p = finite and E = ∞ ⇒ power signal
* signal which does not satisfy the above are neither energy nor power signal.

List of formula:

a) Infinite summation formulas

a) $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a|<1$

d) $\sum_{n=-\infty}^{\infty} 1 = \infty$

b) $\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2} \quad |a|<1$

c) $\sum_{n=0}^{\infty} n^2 a^n = \frac{a^2+a}{(1-a)^3} \quad |a|<1$

b) Finite summation formulas:

a) $\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{(1-a)}, \; a \neq 1$

b) $\sum_{n=0}^{N} 1 = N+1$

c) $\sum_{n=N_1}^{N_2} 1 = N_2 - N_1 + 1$

d) $\sum_{n=0}^{N} n = N\frac{(N+1)}{2}$

e) $\sum_{n=0}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$

f) $\sum_{n=0}^{N} n^3 = \frac{N^2(N+1)^2}{4}$

g) $e^{-jx} = \cos x - j\sin x$

h) $e^{jx} = \cos x + j\sin x$

i) $\left| e^{j(\omega+\theta)} \right| = 1$

j) $\sum_{n=-N}^{N} 1 = 2N+1$

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Problems:

check the Signals are Energy (or) power signal

1. $x(n) = \left(\frac{1}{3}\right)^n u(n)$

Solution:

$$E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2 = \sum_{n=-\alpha}^{\alpha} \left[\left(\frac{1}{3}\right)^n u(n)\right]^2$$

$u(n) = 1$ for $n>0$
$0 \quad n<0$

$$\therefore E = \sum_{n=0}^{\alpha} \left[\left(\frac{1}{3}\right)^n (1)\right]^2$$

By

$$\sum_{n=0}^{\alpha} a^n = \frac{1}{1-a} \quad a<1$$

$$= \sum_{n=0}^{\alpha} \left(\frac{1}{3}\right)^{2n}$$

$$= \sum_{n=0}^{\alpha} \left(\frac{1}{9}\right)^n$$

$$= \frac{1}{1-\frac{1}{9}}$$

$$E = \frac{9}{8}$$

$$P = \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[\left(\frac{1}{3}\right)^n u(n)\right]^2$$

$$= \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n$$

$$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a} \quad a \neq 1$$

$$= \lim_{N\to\alpha} \frac{1}{2N+1} \frac{1-\left(\frac{1}{9}\right)^{N+1}}{1-\frac{1}{9}}$$

$$P = 0$$

Energy is finite & power is 0, Energy signal

2) $x(n) = \sin\left(\frac{\pi}{4}n\right)$

<u>Solution:</u>

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} \left[\sin\left(\frac{\pi}{4}n\right)\right]^2$$

$$= \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{\pi}{4}n\right) = \sum_{n=-\infty}^{\infty} \frac{1-\cos\frac{2\pi}{4}n}{2}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} 1 - \cos\frac{n\pi}{2} \qquad \left[\cos\frac{n\pi}{2} = 0\right]$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} 1 - 0 \qquad \left[\sum_{n=-\infty}^{\infty} 1 = \infty\right]$$

$$\therefore E = \frac{1}{2}[\infty] = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \sin^2\left(\frac{\pi}{4}n\right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos\frac{n\pi}{2}}{2}$$

$$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1 - \cos\frac{n\pi}{2} \qquad \left[\cos\frac{n\pi}{2} = 0\right]$$

$$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1 \qquad \left[\sum_{n=-N}^{N} 1 = 2N+1\right]$$

$$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \times 2N+1$$

$P = \frac{1}{2}$ $\quad\therefore$ power signal because $E = \infty, P = \frac{1}{2}$

3) $x(n) = e^{2n} u(n)$

$$E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2 = \sum_{n=-\alpha}^{\alpha} \left[e^{2n} u(n)\right]^2$$

$u(n) = 1$ for $n \geq 0$
$\phantom{u(n) = {}} 0 \quad n < 0$

$$= \sum_{n=0}^{\alpha} \left(e^{2n}\right)^2 = \sum_{n=0}^{\alpha} e^{4n}$$

$$= 1 + e^4 + e^8 + \cdots + \alpha = \alpha$$

$E = \alpha$

$$P = \lim_{N \to \alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} |e^{2n} u(n)|^2$$

$$= \lim_{N \to \alpha} \frac{1}{2N+1} \sum_{n=0}^{N} e^{4n}$$

$$= \lim_{N \to \alpha} \frac{1}{2N+1} \left[\frac{e^{4(N+1)} - 1}{e^{4-1} - 1}\right]$$

$P = \alpha$

$E = P = \alpha$, $x(n)$ is neither Energy nor power signal.

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II. SYSTEMS :

Definition:

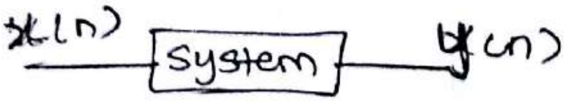
A system is defined as a physical device that performs an operation for an input signal and produces another signal as output.

Ex: Motor, amplifier, filter, boiler and turbine

The relation between the input and corresponding output y(t) of a system has the form

$y(n)$ = operation on $x(n)$

$y(n) = T[x(n)]$



1. classification of Systems:

The systems are classified as follows

1. continuous time and discrete time systems
2. Static and dynamic systems
3. Linear and Non linear systems
4. Time variant and Time invariant systems
5. Stable and unstable systems.
6. causal and non causal systems
7. Recursive and non recursive systems

1. Continuous time and discrete time Systems:

A continuous time system is one which operates on a continuous time signal and produces a continuous time output signal.

Ex: amplifier, Motor etc.

A discrete time system is one which operates on a discrete time signal and produces a discrete time output signal.

2. Static and Dynamic Systems:

A system is called static (or) memoryless if its output at any instant depends on the input at that instant but not on the past or future values of input.

otherwise the system is said to be dynamic or with Memory.

Ex: $y(n) = n x(n) \Rightarrow$ Static

$y(n) = x(n-1) \Rightarrow$ Dynamic

Problems:

find whether the system is static or dynamic

1) $y(n) = n x(n) + b x^3(n)$

The o/p depends on present input, the system is static

2) $y(n) = x(n) + 3x(n-1)$

The o/p depends on both present and past input, the system is dynamic

3. Causal and Non-Causal systems:

A system is said to be causal if the output depends on the present and past inputs to output depends on the present and past output but does not depends on future input to output.

A non causal system is one whose output depends on future input (or) output.

Ex: $y(n) = nx(n) + x(n-3) \Rightarrow$ causal

$y(n) = x(2n) \Rightarrow$ non causal

Problems:

1) $y(n) = x(n) + \frac{1}{x(n-1)}$

Solution:

$n=0 \Rightarrow y(0) = x(0) + \frac{1}{x(-1)}$

$n=1 \Rightarrow y(1) = x(1) + \frac{1}{x(0)}$

$n=-1 \Rightarrow y(-1) = x(-1) + \frac{1}{x(-2)}$

for all values of n, the output depends on present and past values of input.

$\therefore$ The system is causal.

2) $y(n) = x(n^2)$

Solution: $n=0,\ y(0) = x(0)$

$n=1,\ y(1) = x(1)$

$n=2,\ y(2) = x(4)$

The output depends on future values.

The system is non causal

4. Linear and Non-linear Systems:

A system that obeys the superposition principle is said to be a linear system.

Superposition principle states that the response to a weighted sum of input signals be equal to the corresponding weighted sum of the outputs of the system to each of the individual input signals.

$$T[a x_1(n) + b x_2(n)] = aT[x_1(n)] + bT[x_2(n)]$$

Procedure:

1. Compute $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$ & $a_1 y_1(n) + a_2 y_2(n)$
2. Let $y_3(n)$ be the response for $x_3(n)$
3. Check whether $y_3(n) = a_1 y_1(n) + a_2 y_2(n)$.

   If they are equal then the system is linear. otherwise it is Non-linear.

Problems:

1. Check the system is linear or not

$y(n) = n x(n)$

Solution:

$$a_1 y_1(n) + a_2 y_2(n) = a_1 n x_1(n) + a_2 n x_2(n) \quad -①$$

$$y_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$y_3(n) = n[a_1 x_1(n) + a_2 x_2(n)] \quad -②$$

① = ②, system is linear.

2) $y(n) = x^2(n)$

Solution:

$$a_1 y_1(n) + a_2 y_2(n) = a_1 x_1^2(n) + a_2 x_2^2(n) \quad -①$$

$$y_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$= [a_1 x_1(n) + a_2 x_2(n)]^2$$

$$y_3(n) = a_1^2 x_1^2(n) + a_2^2 x_2^2(n) + 2a_1 a_2 x_1(n) x_2(n) \quad -②$$

① ≠ ②, System is non-linear.

5) Time Variant and Time Invariant System:

A system is said to be time invariant (or) shift invariant, if the characteristics of the system do not change with time.

$y(n,k) = y(n-k) \Rightarrow$ Time invariant

otherwise Time Variant

Problems:

1) $y(n) = x(2n)$

Solution:

If the input is delayed by K units

$$y(n) = x(2n-k) \quad — ①$$

If the output is delayed by K units

$$y(n-k) = x(2(n-k))$$

$$y(n-k) = x(2n-2k) \quad -②$$

① ≠ ② $\Rightarrow$ Time Variant system

2) $y(n) = x^2(n-1)$

Solution:

If I/p is delayed by K units,

$y(n) = x^2(n-1-K)$ — ①

If o/p is delayed by K units,

$y(n-K) = x^2(n-1-K)$ — ②

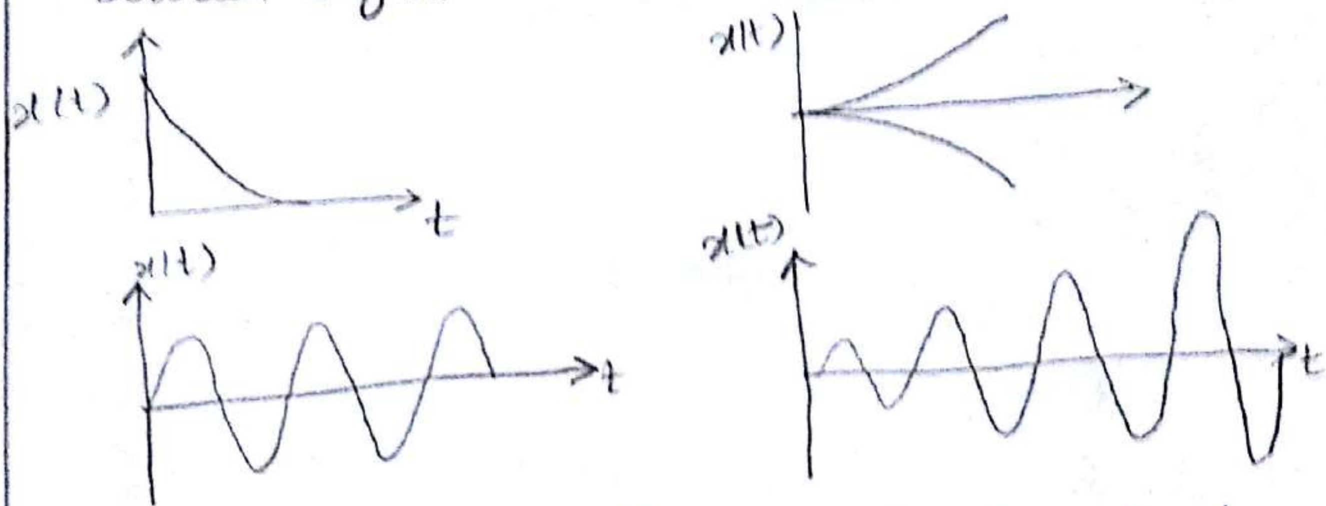
① = ②, system is time invariant.

6) stable and unstable systems:

A system is said to be bounded input and bounded output stable if and only if every bounded input produces a bounded output.

Examples of bounded & unbounded signals

Bounded signal unbounded signal



The output is bounded if the impulse response of the system satisfies the condition

$$\sum_{K=-\infty}^{\infty} |h(K)| < \infty$$

u

Problems:

1) Test whether the systems are ~~bounded~~ stable or not.

$$h(n) = 2^n u(n-3)$$

Solution:

For a system to be stable, $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

$$\therefore \sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |2^n u(n-3)|$$

$u(n-3) = 1$

$$= \sum_{n=3}^{\infty} |2^n|$$

$$= 2^3 + 2^4 + \cdots + 2^{\infty}$$

$$= \infty$$

$\therefore$ system is unstable

2) $h(n) = \left(\frac{1}{2}\right)^n u(n)$

Solution:

$$\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(n)$$

$u(n) = 1 \quad n \geq 0$
$\ 0 \quad n < 0$

$$\therefore \ = \sum_{n=0}^{\infty} \left(1/2\right)^n$$

By formula, $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a| < 1$

$$= \frac{1}{1-(1/2)} = \frac{1}{0.5} = 2 < \infty$$

$\therefore$ The given system is stable

7. Recursive & non-Recursive Systems:

* A system whose output $y(n)$ at time n depends on any number of past output values as well as present & past input is called as Recursive System.

Ex: $y(n) = F\{y(n-1), y(n-2), \ldots y(n-N), x(n), x(n-1), x(n-m)\}$

* A system whose output does not depend on past output but depends only on the present and past input is called as non-Recursive System.

Ex: $y(n) = F\{x(n), x(n-1), \ldots, x(n-m)\}$

CONVOLUTION

1. Impulse Response and convolution sum

A discrete time system performs an operation on an input signal based on a predefined criteria to produce a modified output signal.

Let the input signal to the system be $x(n)$ and the output of the system be $y(n)$.

$$x(n) \longrightarrow \boxed{T} \longrightarrow y(n) = T[x(n)]$$

If the input to the system is a unit impulse then the output of the system is known as impulse response of the system denoted by $h(n)$.

$$h(n) = T[\delta(n)]$$

Any arbitary sequence $x(n)$ can be represented as a weighted sum of discrete impulses given by

$$x(n) = \sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)$$

Then the output,

$$y(n) = T\left[\sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)\right]$$

By using the linear property of the system, interchange the system operator T with the summation $x(K)$ yields

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\,T[\delta(n-K)]$$

The response to a shifted impulse response can be denoted by $h(n,k)$

$$h(n,k) = T\left[\delta(n-k)\right]$$

$$\therefore y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n,k)$$

for a time invariant systems, $h(n,k) = h(n-k)$

$$\therefore y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \quad \text{--- (1)}$$

The eq (1) is called the convolution sum & given by

$$y(n) = x(n) * h(n)$$

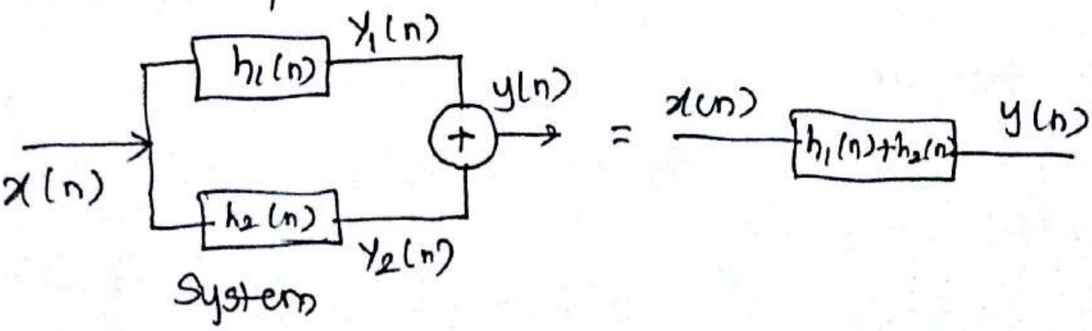
Where $*$ => Convolution Sum.

2. Properties of Convolution Sum:

1. The distributive property:

$$x(n) * \left[h_1(n) + h_2(n)\right] = x(n) * h_1(n) + x(n) * h_2(n)$$

Consider two LTI systems with $h_1(n)$ & $h_2(n)$ connected in parallel as shown in fig.



$$y_1(n) = x(n) * h_1(n)$$

$$y_2(n) = x(n) * h_2(n)$$

$$y(n) = y_1(n) + y_2(n)$$

$$= x(n) * h_1(n) + x(n) * h_2(n)$$

$$= \sum_{K=-\infty}^{\infty} x(K) h_1(n-K) + \sum_{K=\infty}^{\infty} x(K) h_2(n-K)$$

$$= \sum_{K=-\infty}^{\infty} x(K) \left[h_1(n-K) + h_2(n-K) \right]$$

$$= \sum_{K=-\infty}^{\infty} x(K) h(n-K)$$

$$= x(n) * h(n)$$

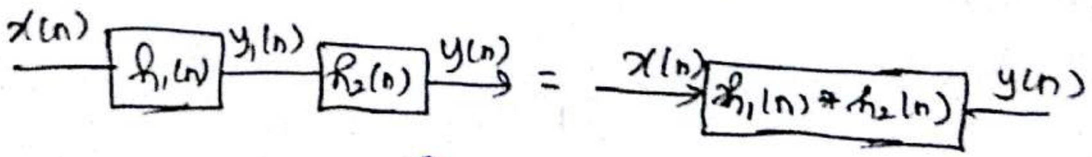
where $h(n) = h_1(n) + h_2(n)$

* If two systems are connected in parallel the overall impulse response is equal to the sum of two impulse responses.

2. The Associative property:

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

consider two LTI systems with impulse responses $h_1(n)$ & $h_2(n)$ connected in parallel.



$$y_1(n) = x(n) * h_1(n)$$

$$= \sum_{K=-\infty}^{\infty} x(K) h_1(n-K)$$

$$y(n) = y_1(n) * h_2(n)$$

$$= \left[\sum_{k=-\infty}^{\infty} x(k)\, h_1(n-k) \right] * h_2(n)$$

$$= \sum_{p=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(p)\, h_1(k-p)\, h_2(n-k)$$

Let $l = k-p$ then

$$y(n) = \sum_{p=-\infty}^{\infty} x_1(p) \sum_{l=-\infty}^{\infty} h_1(l)\, h_2(n-p-l)$$

$$= \sum_{p=-\infty}^{\infty} x_1(p)\, h(n-p)$$

$$y(n) = x(n) * h(n)$$

where $h(n) = \sum_{l=-\infty}^{\infty} h_1(l)\, h_2(n-l)$

$$h(n) = h_1(n) * h_2(n)$$

* The Impulse response of two LTI systems connected in cascade is the convolution of individual impulse responses.

3. The commutative property:

$$h_1(n) * h_2(n) = h_2(n) * h_1(n)$$

$$h_1(n) * h_2(n) = \sum_{k=-\infty}^{\infty} h_1(k)\, h_2(n-k)$$

Let $n-k = p$

$$= \sum_{p=-\infty}^{\infty} h_1(n-p)\, h_2(p)$$

$= h_2(n) * h_1(n)$

$$\therefore \boxed{h_1(n) * h_2(n) = h_2(n) * h_1(n)}$$

4. The shifting property:

If $x(n) * h(n) = y(n)$

then $x(n-k) * h(n-m) = y(n-k-m)$

5. The convolution with an impulse:

$x(n) * \delta(n) = x(n)$

3. Types of convolution:

Two types of convolution

1- circular convolution

2. Linear convolution

1. Circular Convolution:

In circular convolution if $x(n)$ contains 'L' number of samples, $h(n)$ has 'M' number of samples and $L > M$, than we perform circular convolution between the two using $N = \max(L, M)$ by adding $L-M$ number of zero samples to the sequence $h(n)$, so that both the sequences are periodic with period N.

It can not be used to find the response of a linear filter without zero padding.

Zero padding and its uses:

Let 'L' be the length of the sequence $x(n)$. If we want to find 'N' point DFT of the sequence $x(n)$ $(N > L)$, then $(N-L)$ number of zeros have to be added to the sequence.

* Get Better display of the frequency spectrum
* with zero padding the DFT can be used in linear filtering.

Methods to perform circular convolution:

There are 3 methods.

1. concentric circle method
2. Matrix Method
3. Tabular Method

1. concentric circle method:

Consider $x_1(n)$ & $x_2(n)$, circular convolution of these two sequences

$x_3(n) = x_1(n) \, (N) \, x_2(n)$ can be found by following steps.

step 1: Graph N samples of $x_1(n)$ as equally spaced points around an outer circle in counter clockwise direction.

Step 2 : Start at the same point as $x_1(n)$ graph N samples of $x_2(n)$ as equally spaced points around on inner circle in clockwise direction.

Step 3 : Multiply corresponding samples on the two circles and sum the products to produce output.

Step 4 : Rotate the inner circle one sample at a time counter clockwise direction and go to step 3 to obtain the next values of output.

Step 5 : Repeat step 4 until the inner circle first sample lines up with the first sample of the extension circle once again.

1. find the circular convolution of two finite duration sequences.

$x_1(n) = \{1, -1, -2, 3, -1\}$ $x_2(n) = \{1, 2, 3\}$

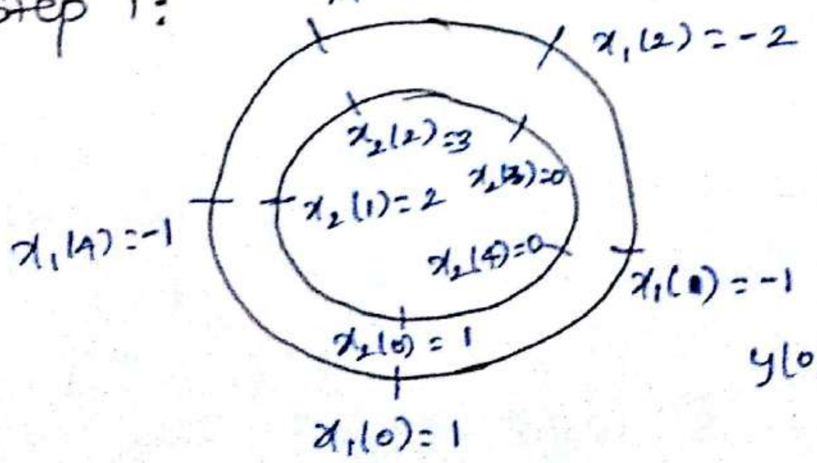
Solution :

Both sequences must be of same length.

$\therefore x_1(n) = \{1, -1, -2, 3, -1\}$

$x_2(n) = \{1, 2, 3, 0, 0\}$

Step 1 :



$\therefore y(0) = 1(1) + 0(-1) + 0(-2) + 3(3) + 2(-1)$

$y(0) = 8$

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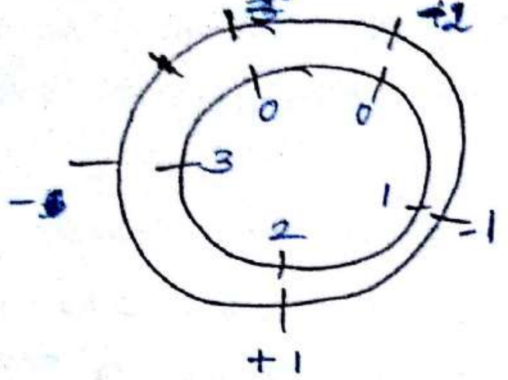
Step 2:

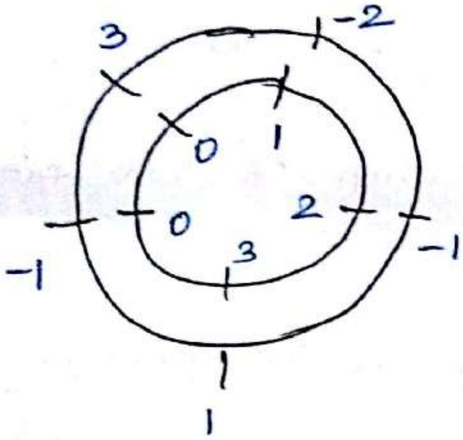
Rotate the three circle in Counter clock wise direction by one sample



$$y(1) = 2(1) + 1(-1) + 0(-2) + 0(3) + (-1)(3)$$
$$= -2$$

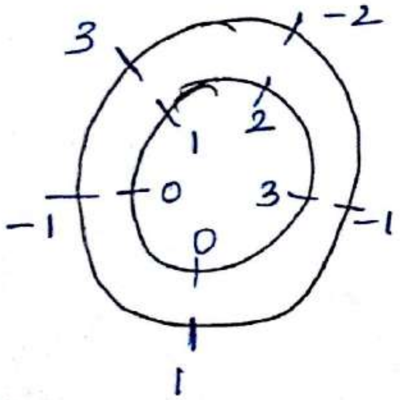
Step 3:

Rotate one sample



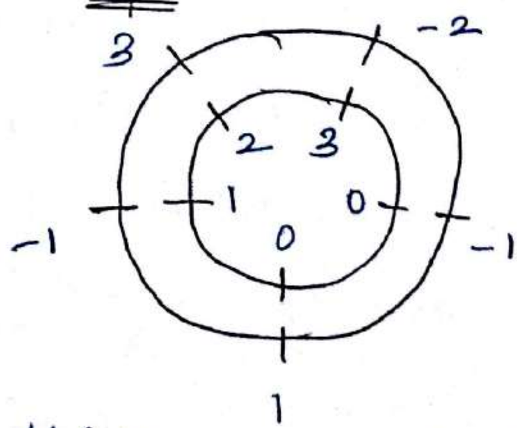
$$y(2) = 3(1) + 2(-1) + 1(-2) + 0(3) + 0(-1)$$
$$y(2) = -1$$

Step 4:



$$y(3) = 0(1) + 3(-1) + 2(-2) + 1(3) + (-1)0$$
$$y(3) = -4$$

Step 5:



$$y(4) = 0(1) + 0(-1) + 3(-2) + 2(3) + 1(-1)$$
$$y(4) = -1$$
$$\boxed{y(5) = y(0)}$$

$$\therefore y(n) = \{8, -2, -1, -4, -1\}$$

2. Matrix Method:

Concentric circle method is simple but time consuming. Matrix method avoids this complication.

In matrix method, the data sequences $x_1(n)$ & $x_2(n)$ are in the form of matrices.

1. Find the circular convolution by Matrix Method

$$x_1(n) = \{\underset{\uparrow}{1},3,5,7\} \quad x_2(n) = \{\underset{\uparrow}{2},4,6,8\}$$

Solution:

$$\begin{bmatrix} 2 & 8 & 6 & 4 \\ 4 & 2 & 8 & 6 \\ 6 & 4 & 2 & 8 \\ 8 & 6 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 2+24+30+28 \\ 4+6+40+42 \\ 6+12+10+56 \\ 8+18+20+14 \end{bmatrix}$$

$$\therefore y(n) = \begin{bmatrix} 84 \\ 92 \\ 84 \\ 60 \end{bmatrix}$$

2. $x_1(n) = \{1,2,-3,4,-5\} \quad x_2(n) = \{-2,4,6\}$

Solution:

$$y(n) = \begin{bmatrix} -2 & 0 & 0 & 6 & 4 \\ 4 & -2 & 0 & 0 & 6 \\ 6 & 4 & -2 & 0 & 0 \\ 0 & 6 & 4 & -2 & 0 \\ 0 & 0 & 6 & 4 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \\ 20 \\ -8 \\ 8 \end{bmatrix}$$

3. Tabular Method:

The steps are,

1. change the index n in the sequences $x_1(n)$ & $x_2(n)$ to get $x_1(m)$ & $x_2(m)$ and then represent the sequences as two rows of tabular array.

2. fold one of the sequence.

Let us fold $x_2(m)$ to get $x_2(-m)$

3. periodically extend $x_2(-m)$. Here the periodicity is N, where N = length of the given sequence

4. shift the sequence $x_2(-m)$ q times to get sequence $x_2(q-m)_N$,

if q is +ve then right shift

if q is -ve then left shift

5. $$x_3(q) = \sum_{m=0}^{N-1} x_1(m)\, x_2(q-m)_N$$

$$x_3(q) = \sum_{m=0}^{N-1} x_1(m)\, x_2(q(m))$$

6. Sum of the sample of the product sequence gives the sample $x_3(q)$.

The above procedure is repeated for all possible values of n to get the sequence $x_3(n)$.

1\. $x_1(n) = \{2, 1, 2, -1\}$ & $x_2(n) = \{1, 2, 3, 4\}$. find circular convolution of two sequence using tabulation method.

Solution:

using tabulation method, Here $N = 4$

m	-3	-2	-1	0	1	2	3
$x_1(m)$				2	1	2	-1
$x_2(m)$				1	2	3	4
$x_2(-m)$	4	3	2	1	4	3	2
$x_2(-m+1)$		4	3	2	1	4	3
$x_2(-m+2)$			4	3	2	1	4
$x_2(-m+3)$				4	3	2	1

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2(n-m)_N$$

At $n=0$, $x_3(0) = 1(4) + 2(3) + 2(1) + -1(2)$

$= 10$

$n=1$, $x_3(1) = 2(2) + 1(1) + 2(4) + (-1)(3)$

$x_3(1) = 10$

$n=2$, $x_3(2) = 2(3) + 1(2) + 2(1) + (-1)(4)$

$= 6$

$n=3$, $x_3(3) = 2(4) + 1(3) + 2(2) + (-1)(1)$

$= 14$

$\therefore x_3(n) = \{10, 10, 6, 14\}$

2. $x(n) = \{1, 0.5\}$ $h(n) = \{0.5, 1\}$

find circular convolution.

Solution:

By tabulation Method, $N=2$

m	-1	0	1
$x_1(m)$	0	1	0.5
$h(m)$	0	0.5	1
$h(-m)$	1	0.5	1
$h(-m+1)$	0	1	0.5

$n=0 \Rightarrow x_3(0) = 1(0.5) + (0.5)(1) + 0(1)$

$x_3(0) = 1$

$n=1 \Rightarrow x_3(1) = 1(1) + 0.5(0.5) + 0 \times 0$

$= 1.25.$

$\therefore x_3(n) = \{1, 1.25\}$

2. Linear Convolution:

The response or output $y(n)$ of a LTI system for any arbitrary input is given by convolution of input $x(n)$ & the impulse response $h(n)$ of the system.

If the input $x(n)$ has N_1 samples and impulse response $h(n)$ has N_2 samples then output sequence $y(n)$ will be a finite duration sequence of (N_1+N_2-1) samples.

The convolution results in a non periodic sequence. Hence this is also called as periodic convolution.

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\alpha}^{\alpha} x(k)\, h(n-k)$$

(or)

$$y(n) = x(n) * h(n) = \sum_{k=-\alpha}^{\alpha} h(k)\, x(n-k)$$

Properties of Linear convolution:

1. commutative property: $y(n) = x(n) * h(n) = h(n) * x(n)$
2. Associative property:
$$[x(n) * h_1(n)] * h_2(n) = x(n) [h_1(n) * h_2(n)]$$
3. Distributive property:
$$x(n) * [h_1(n) + h_2(n)] = [x(n) * h_1(n)] + [x(n) * h_2(n)]$$

Methods to perform linear convolution:

3 Methods : 1. Graphical Method
2. Matrix Method
3. Tabular Method

1. Graphical Method:

Let $x_1(n)$ & $x_2(n)$ be the input sequence & $x_3(n)$ be the output sequence

a) change the index 'n' of i/p seq to 'm' to get $x_1(m)$ & $x_2(m)$

b) Sketch the graphical representation of the input sequences $x_1(m)$ & $x_2(m)$

c) Let us fold $x_2(m)$ to get $x_2(-m)$ and sketch $x_2(-m)$

d) shift $x_2(-m)$ to the left graphically so that the product of $x_1(m)$ & shifted $x_2(-m)$ gives only one zero sample. Now Multiply $x_1(m)$ & $x_2(-m)$ to get a product of sequence & sum up the samples of product sequence, which is the first sample of output sequence.

e) To get the next sample of output sequence, shift $x_2(-m)$ of previous step to one position right & multiply the shifted sequence with $x_1(m)$ to get a product sequence. Now the sum of the samples of product sequence gives the second sample of output sequence.

f) To get the subsequent samples of output sequence, the step (e) is repeated until we get a non-zero product sequence.

1. Determine linear convolution whose Input x(n) & impulse response h(n) are given by

$$x(n)=\{1,2,3,1\} \quad h(n)=\{1,\underset{\uparrow}{2},1,-1\}$$

Solution:

x(n) starts at $n=0$

h(n) starts at $n=-1$

∴ output sequence starts at $n=0-1=-1$

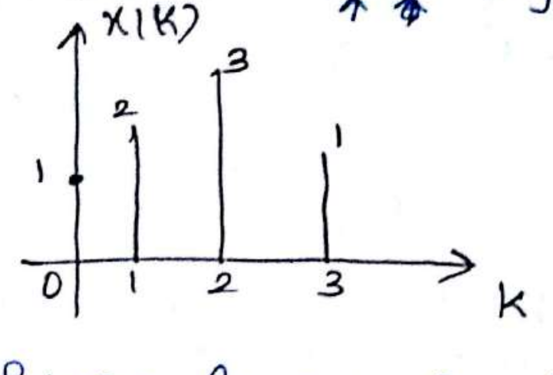
No. of samples in $x(n)=N_1=4$

" " $h(n)=N_2=4$

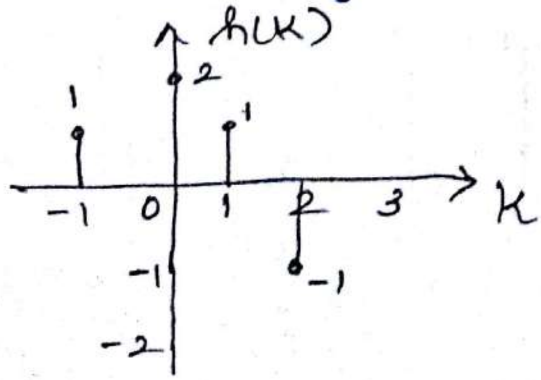
∴ no. of samples in $y(n)=N_1+N_2-1$

$y(n) = 4+4-1 = 7$ samples

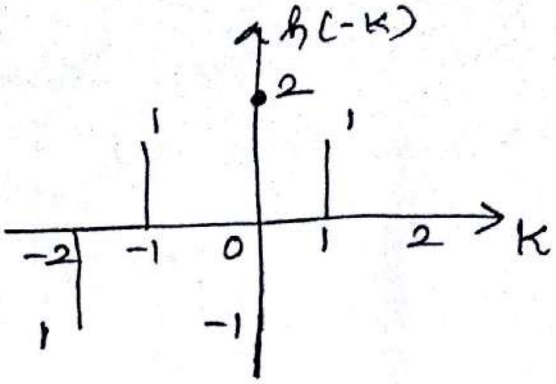
Now, $x(n)=\{\underset{\uparrow}{1},2,3,1\}$ $x(k)=\{\underset{\uparrow}{1},2,3,1\}$



$h(n)=\{1,2,1,-1\}$ $h(k)=\{1,2,1,-1\}$



Respone $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$



By convolution formula,

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{k=-\infty}^{\infty} x(k)\, h_n(k)$$

where $n=0$, $y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(0-k)$

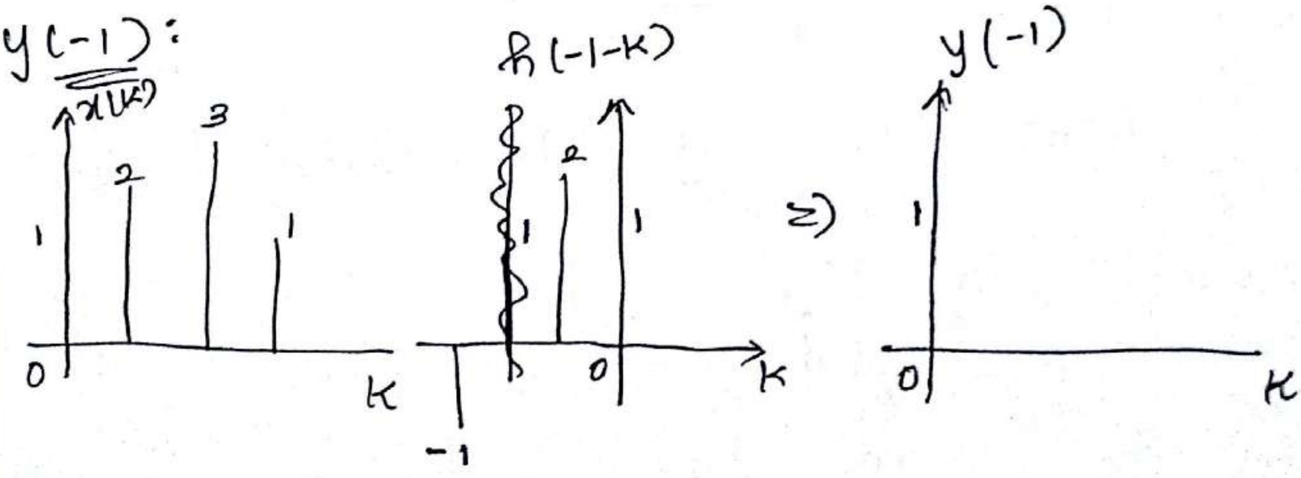
$n=-1$, $y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$

$n=1$, $y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$

$n=2$, $y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$

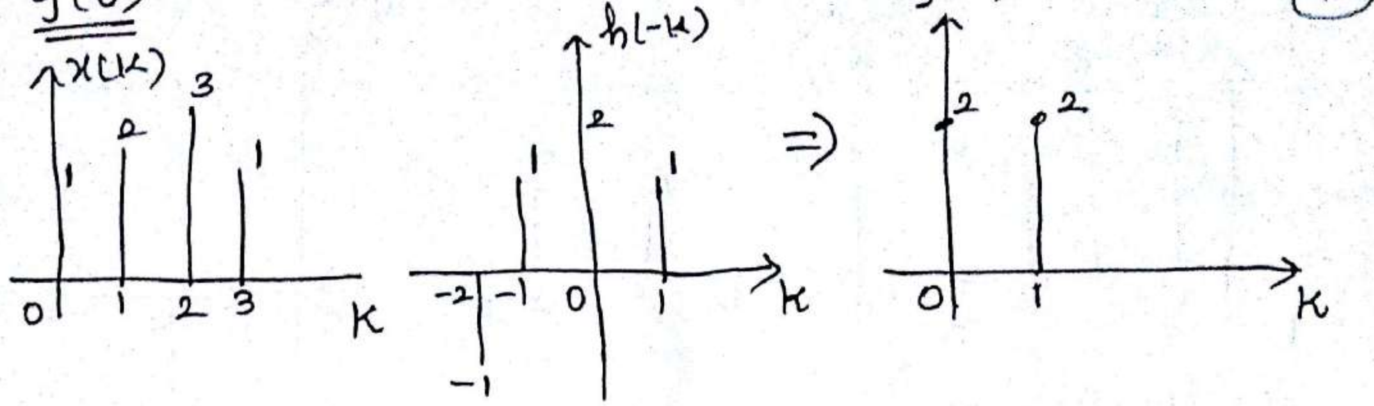
upto $n=7$.

y(-1):



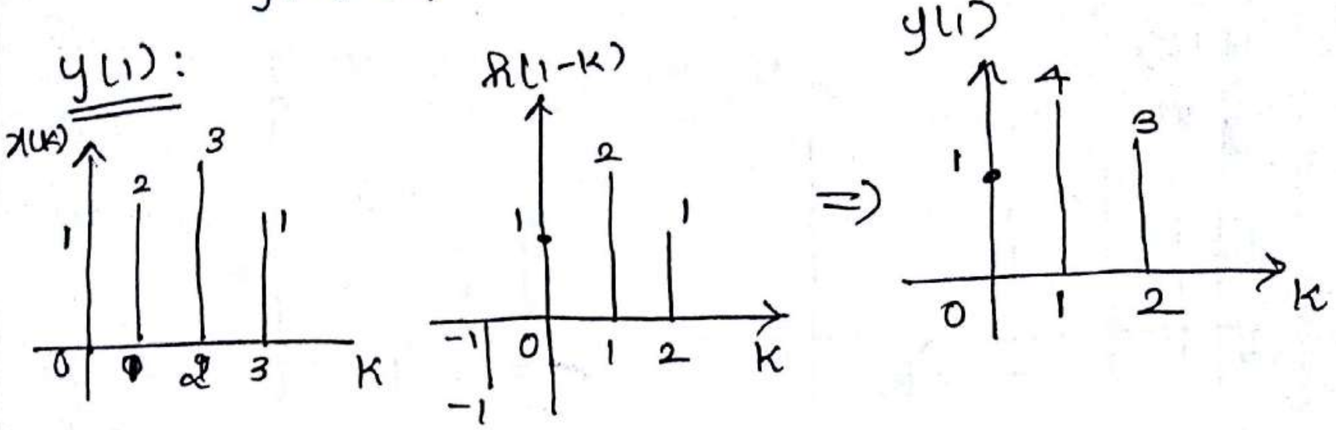
$y(-1) = 1$

$y(0)$



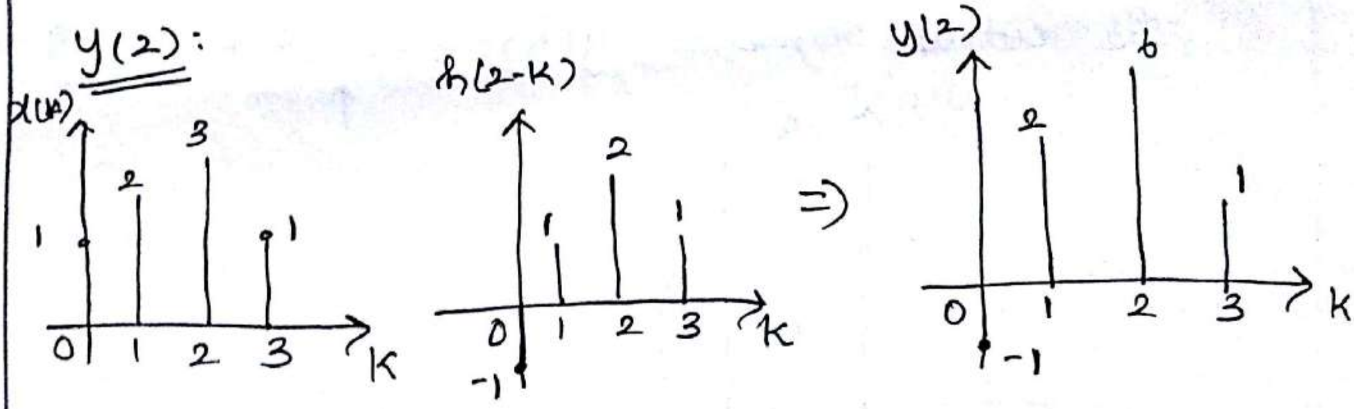
$y(0) = 4$

$y(1)$:



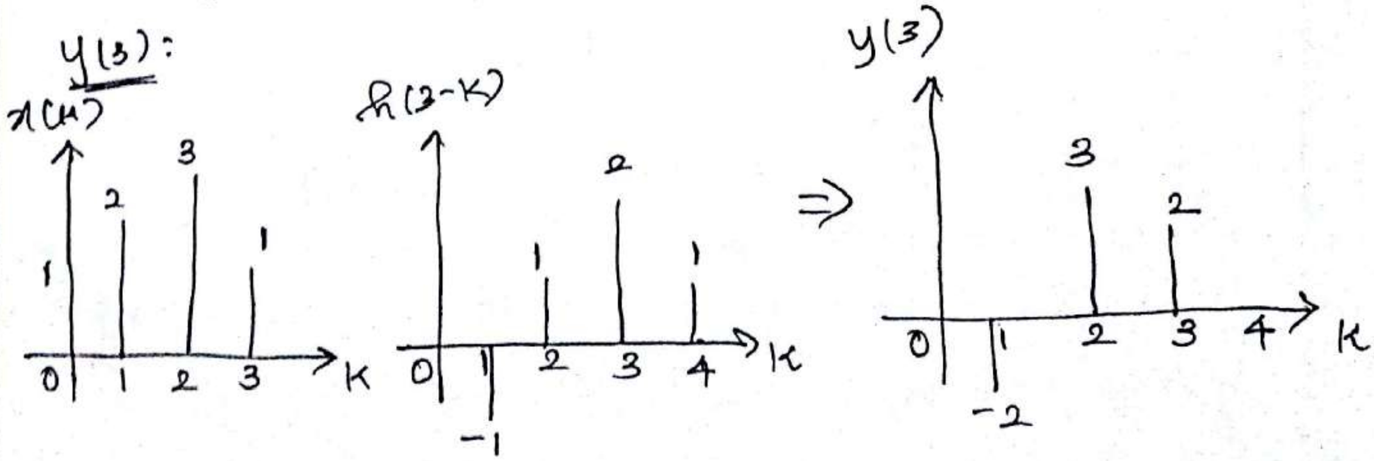
$y(1) = 1 + 4 + 3 = 8$

$y(2)$:



$y(2) = 2 + 6 + 1 - 1 = 8$

$y(3)$:



$y(3) = -2 + 3 + 2 = 3$

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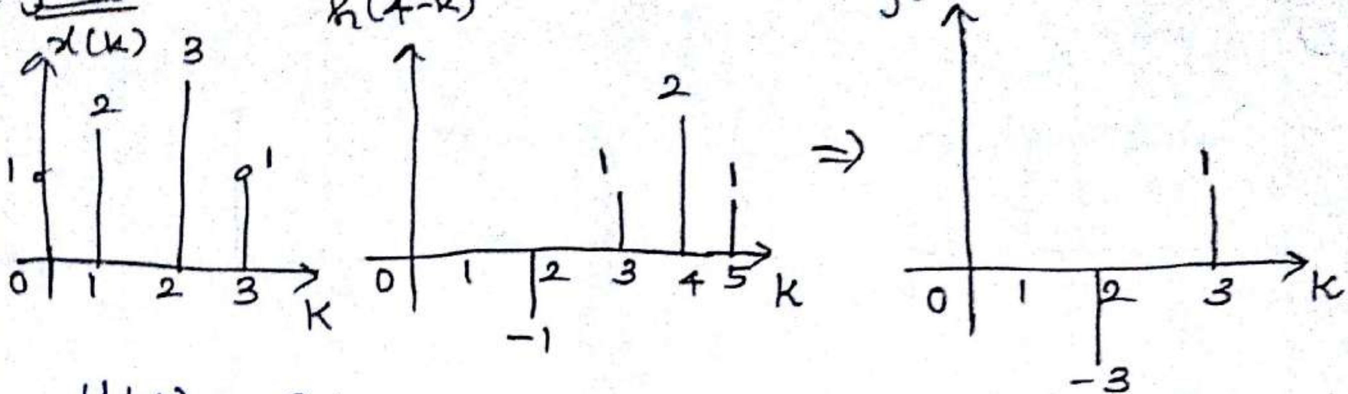
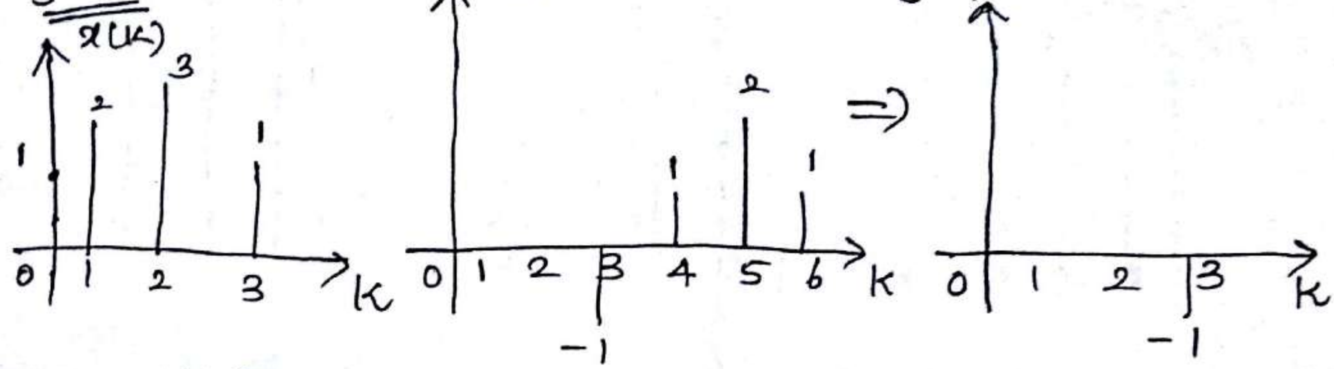
y(4):



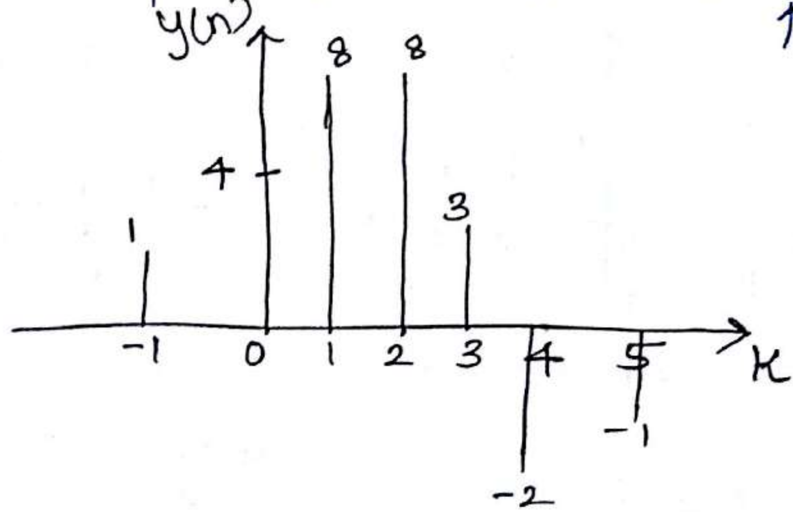
$y(4) = -3 + 1 = -2$

y(5):



$y(5) = -1$

$\therefore$ The output sequence $y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$



2. Determine the response of the LTI system whose input $x(n)$ & impulse response $h(n)$ are given by $x(n) = \{1, 2, 0.5, 1\}$ & $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$ (use tabular & Matrix Method)

Solution:

Tabular Method (No periodicity)

$N = N_1 + N_2 - 1 = 8 - 1 = 7$ samples (so $m = 0$ to 7 in +ve axis)

m	-3	-2	-1	0	1	2	3	4	5	6
$x(m)$				1	2	0.5	1			
$h(m)$			1	2	1	-1				
$h(-m)$		-1	1	2	1					
$h(-m-1)$	-1	1	2	1						
$h(-m)$		-1	1	2	1					
$h(-m+1)$			-1	1	2	1				
$h(-m+2)$				-1	1	2	1			
$h(-m+3)$					-1	1	2	1		
$h(-m+4)$						-1	1	2	1	
$h(-m+5)$							-1	1	2	1

$$y(n) = \sum_{k=-\infty}^{\infty} x(m)\, h(n-m)$$

when $n = -1$ (origin of $h(n)$),

$$y(-1) = \sum_k x(m)\, h(-m) = (-1) \times 0 + (1) \times 0 + 0\,(2) + 1\,(1) + 0 \times 0.5 + 1 \times 0$$

$$y(-1) = 1$$

$n = 0$, $y(0) = 4$

$n = 1$, $y(1) = 5.5$

$n = 2$, $y(2) = 3$

$n=3, y(3)=0.5$

$n=4, y(4)=0.5$

$n=5, y(5)=-1$

$\therefore y(n)=\{1, \underset{\uparrow}{4}, 5.5, 3, 0.5, 0.5, -1\}$

Matrix Method:

$x(n)$	$h(n)$: 1	2	1	-1
1	1	2	1	-1
2	2	4	2	-2
0.5	0.5	1	0.5	-0.5
1	1	2	1	-1

$N = N_1 + N_2 - 1 = 4+4-1$

$N = 7$ samples

$y(-1)=1$, $y(0)=2+2=4$, $y(1)=0.5+4+1=5.5$

$y(2)=1+1+2-1=3$, $y(3)=2+0.5-2=0.5$

$y(4)=1-0.5=0.5$, $y(5)=-1$

$\therefore y(n)=\{1, \underset{\uparrow}{4}, 5.5, 3, 0.5, 0.5, -1\}$

Linear convolution using circular convolution:

The linear convolution output can also be obtained by circular convolution.

This can be done by $N = N_1 + N_2 - 1$, no. of samples will be found in $x(n)$ & $h(n)$, if not add appropriate number of zeros to make it equal to length of $x(n)$ & $h(n)$ as N.

Example:

1. obtain linear convolution by circular convolution.
$x(n) = \{1, 2, 3, 1\}$ $h(n) = \{1, 1, 1\}$

Solution:

$N = N_1 + N_2 - 1 = 4 + 3 - 1 = 6$ samples

Here $x(n)$ has 4 samples, so add 2 zeros
$h(n)$ has 3 samples, so add 3 zeros.

$\therefore x(n) = \{1, 2, 3, 1, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0\}$

Circular convolution:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 6 \\ 4 \\ 1 \end{bmatrix}$$

$\therefore$ output of Linear Convolution using circular convolution is $y(n) = \{1, 3, 6, 6, 4, 1\}$

Circular Convolution using Linear Convolution

1. Determine the circular convolution by using linear convolution for $h(n) = \{1, 1, 1\}$ & $x(n) = \{1, 2, 3, 1\}$

Solution:

Linear Convolution:

$N = N_1 + N_2 - 1 = 4 + 3 - 1 = 6$ samples

Matrix Method:

$x(n)$ \ $h(n)$	1	1	1
1	1	1	1
2	2	2	2
3	3	3	3
1	1	1	1

$\therefore y(n) = \{1, 3, 6, 6, 4, 1\}$

To obtain the circular convolution output add the last two element with the first two element.

$y(n) = \{1, 3, 6, 6, 4, 1\}$

+

+

$\therefore y(n) = \{5, 4, 6, 6\}$

Difference between Linear and circular Convolution:

S.No	Linear Convolution	Circular convolution
1.	Length of input sequences can be different	Length of input sequences should be same
2.	Zero padding is not required	If the length of input sequences are different then zero padding is required
3.	Length of y(n) is (output) $N = N_1 + N_2 - 1$	Length of y(n) is (output) $N = \max(L, M)$
4.	Input sequences need not be periodic	Atleast one of the input sequence should be periodic
5.	output sequence is non-periodic	output sequence is periodic.

CORRELATION:

correlation is used to find out the similarities between two signals.

Ex: communication circuit

* It is of two types

1. Auto Correlation
2. cross correlation

1. <u>Auto</u> - corr<u>elation</u> :

It is used to find out the similarity between the signals from the same input.

$$\gamma_{xx}(n) = \sum_{n=-\alpha}^{\alpha} x(n)\, x(n-1)$$

(or)

$$\gamma_{xx}(n) = x(n) * x(-n)$$

2. <u>Cross</u> - corr<u>elation</u> :

It is used to find out the similarity between the signals from different input.

1. $$\gamma_{xy}(n) = \sum_{n=-\alpha}^{\alpha} x(n)\, y(n-1)$$

$$\gamma_{xy}(n) = x(n) * y(-n)$$

2. $$\gamma_{yx}(n) = \sum_{n=-\alpha}^{\alpha} x(n-1)\, y(n)$$

$$\gamma_{yx}(n) = y(n) * x(-n)$$

$$\therefore \boxed{\gamma_{xy}(n) = \gamma_{yx}(-n)}$$

Problems:

1. find the correlation of the following sequences:

$x(n) = \{-2, 3, -2, \underset{\uparrow}{4}, 2, 3\}$ $y(n) = \{-2, 3, \underset{\uparrow}{1}, 2, 3, 4\}$

Solution:

1. Auto correlation:

$\gamma_{xx}(n) = x(n) * x(-n)$

$\gamma_{yy}(n) = y(n) * y(-n)$

Matrix method: $x(-n) = \{3, 2, \underset{\uparrow}{4}, -2, 3, -2\}$

$x(n)$ \ $x(-n)$	3	2	4	-2	3	-2
-2	-6	-4	-8	4	-6	4
3	9	6	12	-6	9	-6
-2	-6	-4	-8	4	-6	4
4	12	8	16	-8	12	-8
2	6	4	8	-4	6	-4
3	9	6	12	-6	9	-6

$\gamma_{xx}(n) = \{-6, 5, -8, 24, -6, \underset{\uparrow}{46}, -6, 24, -8, 5, -6\}$

$r_{yy}(n)$:

$y(n) = \{-2, 3, \underset{\uparrow}{1}, 2, 3, 4\}$

$y(-n) = \{4, 3, 2, \underset{\uparrow}{1}, 3, -2\}$

$y(n)$ \ $y(-n)$	4	3	2	1	3	-2
-2	-8	-6	-4	-2	-6	4
3	12	9	6	3	9	-6
1	4	3	2	1	3	-2
2	8	6	4	2	6	-4
3	12	9	6	3	9	-6
4	16	12	8	4	12	-8

$r_{yy}(n) = \{-8, 6, 9, 15, 17, \underset{\uparrow}{43}, 17, 15, 9, 6, -8\}$

2. **Cross correlation:**

$r_{xy}(n) = x(n) * y(-n)$

$x(n) = \{-2, 3, -2, \underset{\uparrow}{4}, 2, 3\}$

$y(-n) = \{4, 3, 2, \underset{\uparrow}{1}, 3, -2\}$

$n_1 = -3, \; n_2 = -3$ (starting point)

$\therefore n_1 + n_2 = -6$

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$x(n)$ \ $y(-n)$	4	3	2	1	3	-2
-2	-8	-6	-4	-2	-6	4
3	12	9	6	3	9	-6
-2	-8	-6	-4	-2	-6	4
4	16	12	8	4	12	-8
2	8	6	4	2	6	-4
3	12	9	6	3	9	-6

$$r_{xy}(n) = \{-8, 6, -3, 14, 13, 37, \underset{\uparrow}{5}, 24, 1, 5, -6\} \quad -①$$

$r_{yx}(n) = y(n) * x(-n)$

$y(n) = \{-2, 3, \underset{\uparrow}{1}, 2, 3, 4\}$ $\qquad n_1 = -2, n_2 = -2$

$x(-n) = \{3, 2, \underset{\uparrow}{4}, -2, 3, -2\}$ $\qquad \therefore n_1 + n_2 = -4$

$y(n)$ \ $x(-n)$	3	2	4	-2	3	-2
-2	-6	-4	-8	4	-6	4
3	9	6	12	-6	9	-6
1	3	2	4	-2	3	-2
2	6	4	8	-4	6	-4
3	9	6	12	-6	9	-6
4	12	8	16	-8	12	-8

from ① + ②
$r_{xy}(n) = r_{yx}(-n)$

$$r_{yx}(n) = \{-6, 5, 1, 24, \underset{\uparrow}{5}, 37, 13, 14, -3, 6, -8\} \quad -②$$

Circular Correlation:

Circular correlation is given by

$$\gamma_{xy}(K) = \frac{1}{N}\sum_{K=0}^{N-1} x(K)\, y(n-K) \quad K=0,1,\ldots N-1$$

N = length of sequence

$$\gamma_{yx}(K) = \frac{1}{N}\sum_{K=0}^{N-1} y(K)\, x(n-K)$$

1. find circular correlation of the sequence $x(n) = \{1,2,3,4\}$ $h(n) = \{1,2,2,1\}$

Solution: $N = N_1 + N_2 - 1 = 8 - 1 = 7$ samples

n	0	1	2	3	4	5	6	7
$x(n)$	1	2	3	4	1	2	3	4
$h(n)$	1	2	2	1				
$h(n-1)$		1	2	2	1			
$h(n-2)$			1	2	2	1		
$h(n-3)$				1	2	2	1	

$$\gamma_{xy}(0) = \frac{1}{4}\{1+4+6+4\} = \frac{15}{4}$$

$$\gamma_{xy}(1) = \frac{1}{4}\{2+6+8+1\} = \frac{17}{4}$$

$$\gamma_{xy}(2) = \frac{1}{4}\{3+8+2+2\} = \frac{15}{4}$$

$$\gamma_{xy}(3) = \frac{1}{4}\{4+2+4+3\} = \frac{13}{4}$$

$$\therefore \gamma_{xy}(n) = \left\{\frac{15}{4}, \frac{17}{4}, \frac{15}{4}, \frac{13}{4}\right\}$$

Deconvolution:

* Deconvolution is used in all measurements.
* Deconvolution is applicable only for finite one-sided signal

To find input sequence $x(n)$,

$$x(n) = \frac{y(n) - \sum_{K=0}^{(n-1)} x(K)\, h(n-K)}{h(0)}$$

1. The output of the sequence is

$y(n) = \{\underset{\uparrow}{1}, 5, 10, 11, 8, 4, 1\}$ $h(n) = \{\underset{\uparrow}{1}, 2, 1\}$

find the input value of sequence.

Solution:

$$y(n) = x(n) + h(n) - 1$$

$$N = N_1 + N_2 - 1$$

$$7 = N_1 + 3 - 1$$

$$\boxed{N_1 = 5}$$

$$x(n) = \frac{y(n) - \sum_{K=0}^{n-1} x(K)\, h(n-K)}{h(0)}$$

$$n=0 \Rightarrow x(0) = \frac{y(0)}{h(0)} = \frac{1}{1} = 1$$

$$x(1) = \frac{y(1) - \sum_{K=0} x(0)\, h(1-0)}{h(0)} = \frac{5 - (1)(2)}{1}$$

$$x(1) = 3$$

$$x(2) = \frac{y(2) - \sum_{k=0}^{1} x(k)\, h(2-k)}{h(0)}$$

$$= \frac{y(2) - [x(0)\,h(2) + x(1)\,h(1)]}{h(0)}$$

$$x(2) = \frac{10 - [1\times1 + 3\times2]}{1} = \frac{10-7}{1}$$

$$x(2) = 3$$

$$x(3) = \frac{y(3) - [x(0)\,h(3) + x(1)\,h(2) + x(2)\,h(1)]}{h(0)}$$

$$= \frac{11 - [1\times0 + 3\times1 + 3\times2]}{1} = \frac{11-9}{1}$$

$$x(3) = 2$$

$$x(4) = \frac{y(4) - [x(0)\,h(4) + x(1)\,h(3) + x(2)\,h(2) + x(3)\,h(1)]}{h(0)}$$

$$= \frac{8 - [1\times0 + 3\times0 + 3\times1 + 2\times2]}{1} = 8 - (3+4)$$

$$x(4) = 1$$

$$\therefore x(n) = \{\underset{\uparrow}{1}, 3, 3, 2, 1\}$$