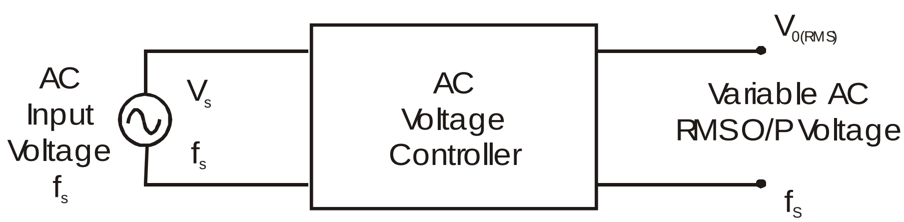
AC VOLTAGE CONTROLLER CIRCUITS
(RMS VOLTAGE CONTROLLERS)

AC voltage controllers (ac line voltage controllers) are employed to vary the RMS value of the alternating voltage applied to a load circuit by introducing Thyristors between the load and a constant voltage ac source. The RMS value of alternating voltage applied to a load circuit is controlled by controlling the triggering angle of the Thyristors in the ac voltage controller circuits.

In brief, an ac voltage controller is a type of thyristor power converter which is used to convert a fixed voltage, fixed frequency ac input supply to obtain a variable voltage ac output. The RMS value of the ac output voltage and the ac power flow to the load is controlled by varying (adjusting) the trigger angle ‘α’



There are two different types of thyristor control used in practice to control the ac power flow

- On-Off control
- Phase control

These are the two ac output voltage control techniques.

In On-Off control technique Thyristors are used as switches to connect the load circuit to the ac supply (source) for a few cycles of the input ac supply and then to disconnect it for few input cycles. The Thyristors thus act as a high speed contactor (or high speed ac switch).

PHASE CONTROL

In phase control the Thyristors are used as switches to connect the load circuit to the input ac supply, for a part of every input cycle. That is the ac supply voltage is chopped using Thyristors during a part of each input cycle.

The thyristor switch is turned on for a part of every half cycle, so that input supply voltage appears across the load and then turned off during the remaining part of input half cycle to disconnect the ac supply from the load.

By controlling the phase angle or the trigger angle 'α' (delay angle), the output RMS voltage across the load can be controlled.

The trigger delay angle 'α' is defined as the phase angle (the value of ωt) at which the thyristor turns on and the load current begins to flow.

Thyristor ac voltage controllers use ac line commutation or ac phase commutation. Thyristors in ac voltage controllers are line commutated (phase commutated) since the input supply is ac. When the input ac voltage reverses and becomes negative during the negative half cycle the current flowing through the conducting thyristor decreases and falls to zero. Thus the ON thyristor naturally turns off, when the device current falls to zero.

Phase control Thyristors which are relatively inexpensive, converter grade Thyristors which are slower than fast switching inverter grade Thyristors are normally used.

For applications upto 400Hz, if Triacs are available to meet the voltage and current ratings of a particular application, Triacs are more commonly used.

Due to ac line commutation or natural commutation, there is no need of extra commutation circuitry or components and the circuits for ac voltage controllers are very simple.

Due to the nature of the output waveforms, the analysis, derivations of expressions for performance parameters are not simple, especially for the phase controlled ac voltage controllers with RL load. But however most of the practical loads are of the RL type and hence RL load should be considered in the analysis and design of ac voltage controller circuits.

TYPE OF AC VOLTAGE CONTROLLERS

The ac voltage controllers are classified into two types based on the type of input ac supply applied to the circuit.

- Single Phase AC Controllers.
- Three Phase AC Controllers.

Single phase ac controllers operate with single phase ac supply voltage of 230V RMS at 50Hz in our country. Three phase ac controllers operate with 3 phase ac supply of 400V RMS at 50Hz supply frequency.

Each type of controller may be sub divided into

- Uni-directional or half wave ac controller.
- Bi-directional or full wave ac controller.

In brief different types of ac voltage controllers are

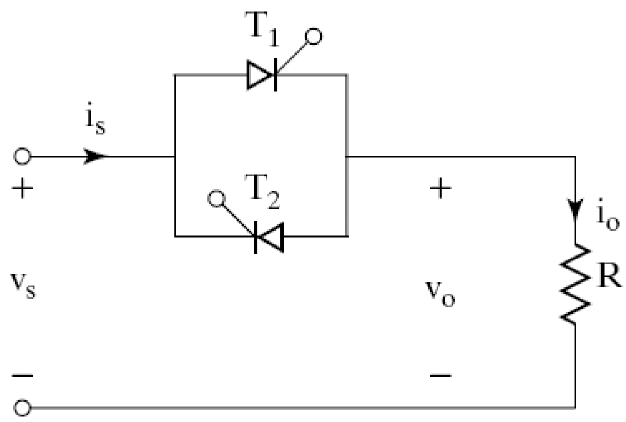
- Single phase half wave ac voltage controller (uni-directional controller).
- Single phase full wave ac voltage controller (bi-directional controller).
- Three phase half wave ac voltage controller (uni-directional controller).
- Three phase full wave ac voltage controller (bi-directional controller).

APPLICATIONS OF AC VOLTAGE CONTROLLERS

- Lighting / Illumination control in ac power circuits.
- Induction heating.
- Industrial heating & Domestic heating.
- Transformer tap changing (on load transformer tap changing).
- Speed control of induction motors (single phase and poly phase ac induction motor control).
- AC magnet controls.

PRINCIPLE OF ON-OFF CONTROL TECHNIQUE (INTEGRAL CYCLE CONTROL)

The basic principle of on-off control technique is explained with reference to a single phase full wave ac voltage controller circuit shown below. The thyristor switches T_1 and T_2 are turned on by applying appropriate gate trigger pulses to connect the input ac supply to the load for 'n' number of input cycles during the time interval t_{ON}. The thyristor switches T_1 and T_2 are turned off by blocking the gate trigger pulses for 'm' number of input cycles during the time interval t_{OFF}. The ac controller ON time t_{ON} usually consists of an integral number of input cycles.



$R = R_L$ **= Load Resistance**

Fig.: Single phase full wave AC voltage controller circuit

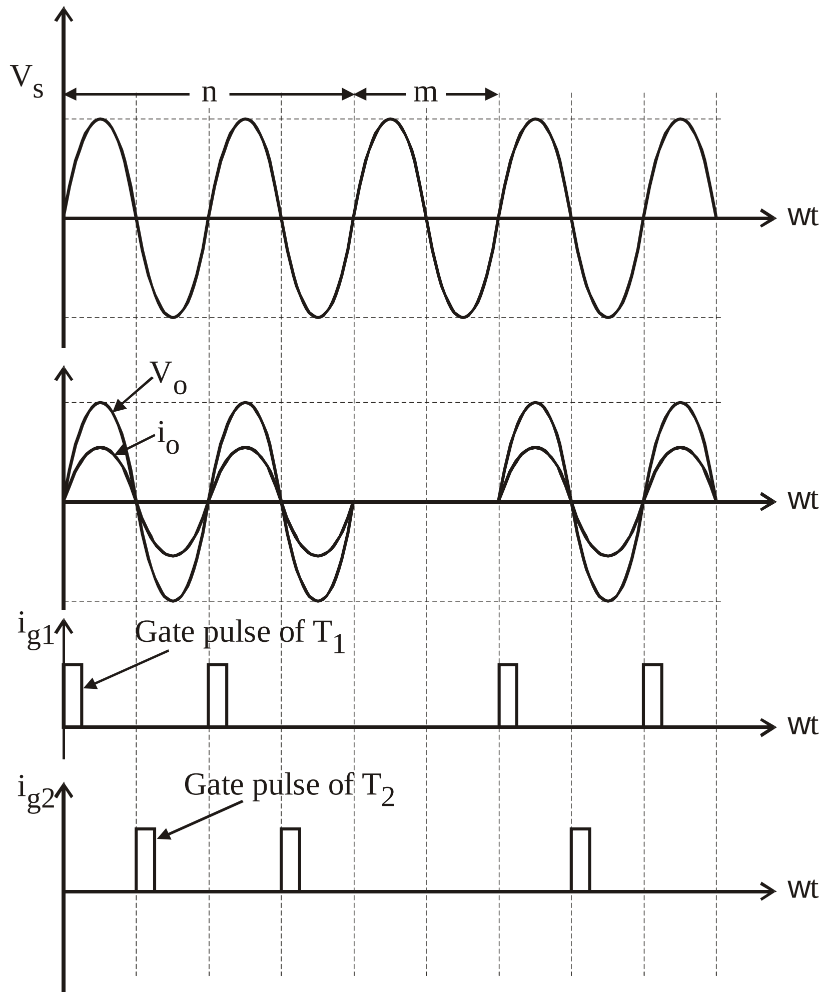


Fig.: Waveforms

Example

Referring to the waveforms of ON-OFF control technique in the above diagram,

$n =$ Two input cycles. Thyristors are turned ON during t_{ON} for two input cycles.

$m =$ One input cycle. Thyristors are turned OFF during t_{OFF} for one input cycle

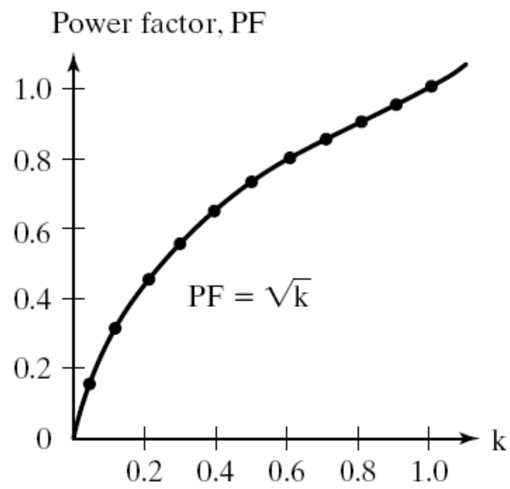


Fig.: Power Factor

Thyristors are turned ON precisely at the zero voltage crossings of the input supply. The thyristor T_1 is turned on at the beginning of each positive half cycle by applying the gate trigger pulses to T_1 as shown, during the ON time t_{ON} . The load current flows in the positive direction, which is the downward direction as shown in the circuit diagram when T_1 conducts. The thyristor T_2 is turned on at the beginning of each negative half cycle, by applying gating signal to the gate of T_2, during t_{ON}. The load current flows in the reverse direction, which is the upward direction when T_2 conducts. Thus we obtain a bi-directional load current flow (alternating load current flow) in a ac voltage controller circuit, by triggering the thyristors alternately.

This type of control is used in applications which have high mechanical inertia and high thermal time constant (Industrial heating and speed control of ac motors). Due to zero voltage and zero current switching of Thyristors, the harmonics generated by switching actions are reduced.

For a sine wave input supply voltage,

$$v_s = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t$$

$V_S =$ RMS value of input ac supply = $\dfrac{V_m}{\sqrt{2}}$ = RMS phase supply voltage.

If the input ac supply is connected to load for ‘n’ number of input cycles and disconnected for ‘m’ number of input cycles, then

$$t_{ON} = n \times T, \qquad t_{OFF} = m \times T$$

Where $T = \frac{1}{f}$ = input cycle time (time period) and

f = input supply frequency.

t_{ON} = controller on time = $n \times T$.

t_{OFF} = controller off time = $m \times T$.

T_O = Output time period = $\left(t_{ON} + t_{OFF}\right) = \left(nT + mT\right)$.

We can show that,

Output RMS voltage $V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$

Where $V_{i(RMS)}$ is the RMS input supply voltage = V_S .

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT VOLTAGE, FOR ON-OFF CONTROL METHOD.

Output RMS voltage $V_{O(RMS)} = \sqrt{\frac{1}{\omega T_O}\int\limits_{\omega t=0}^{\omega t_{ON}} V_m^{\ 2} Sin^2\omega t.d\left(\omega t\right)}$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{\omega T_O}\int\limits_{0}^{\omega t_{ON}} Sin^2\omega t.d\left(\omega t\right)}$$

Substituting for $Sin^2\theta = \frac{1-Cos2\theta}{2}$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{\omega T_O}\int\limits_{0}^{\omega t_{ON}} \left[\frac{1-Cos2\omega t}{2}\right] d\left(\omega t\right)}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{2\omega T_O}\left[\int\limits_{0}^{\omega t_{ON}} d\left(\omega t\right) - \int\limits_{0}^{\omega t_{ON}} Cos2\omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{2\omega T_O}\left[\left(\omega t\right)\Big/_{0}^{\omega t_{ON}} - \frac{Sin2\omega t}{2}\Big/_{0}^{\omega t_{ON}}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{2\omega T_O}\left[\left(\omega t_{ON} - 0\right) - \frac{\sin 2\omega t_{ON} - \sin 0}{2}\right]}$$

Now t_{ON} = An integral number of input cycles; Hence

$t_{ON} = T, 2T, 3T, 4T, 5T, ...$ & $\omega t_{ON} = 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, ...$

Where T is the input supply time period (T = input cycle time period). Thus we note that $\sin 2\omega t_{ON} = 0$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}\cancel{\omega} t_{ON}}{2\cancel{\omega} T_O}} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{t_{ON}}{T_O}}$$

$$V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$$

Where $V_{i(RMS)} = \frac{V_m}{\sqrt{2}} = V_S =$ RMS value of input supply voltage;

$\frac{t_{ON}}{T_O} = \frac{t_{ON}}{t_{ON} + t_{OFF}} = \frac{nT}{nT + mT} = \frac{n}{\left(n+m\right)} = k$ = duty cycle (d).

$V_{O(RMS)} = V_S\sqrt{\frac{n}{\left(m+n\right)}} = V_S\sqrt{k}$

PERFORMANCE PARAMETERS OF AC VOLTAGE CONTROLLERS

- **RMS Output (Load) Voltage**

$$V_{O(RMS)} = \left[\frac{n}{2\pi\left(n+m\right)}\int_0^{2\pi} V_m^{\ 2}\sin^2\omega t.d\left(\omega t\right)\right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{n}{\left(m+n\right)}} = V_{i(RMS)}\sqrt{k} = V_S\sqrt{k}$$

$$V_{O(RMS)} = V_{i(RMS)}\sqrt{k} = V_S\sqrt{k}$$

Where $V_S = V_{i(RMS)}$ = RMS value of input supply voltage.

- **Duty Cycle**

$$k = \frac{t_{ON}}{T_O} = \frac{t_{ON}}{\left(t_{ON} + t_{OFF}\right)} = \frac{nT}{\left(m+n\right)T}$$

Where, $k = \dfrac{n}{(m+n)}$ = duty cycle (d).

- **RMS Load Current**

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{V_{O(RMS)}}{R_L}; \quad \text{for a resistive load } Z = R_L.$$

- **Output AC (Load) Power**

$$P_O = I^2_{O(RMS)} \times R_L$$

- **Input Power Factor**

$$PF = \frac{P_O}{VA} = \frac{\text{output load power}}{\text{input supply volt amperes}} = \frac{P_O}{V_S I_S}$$

$$PF = \frac{I^2_{O(RMS)} \times R_L}{V_{i(RMS)} \times I_{in(RMS)}}; \qquad I_S = I_{in(RMS)} = \text{RMS input supply current.}$$

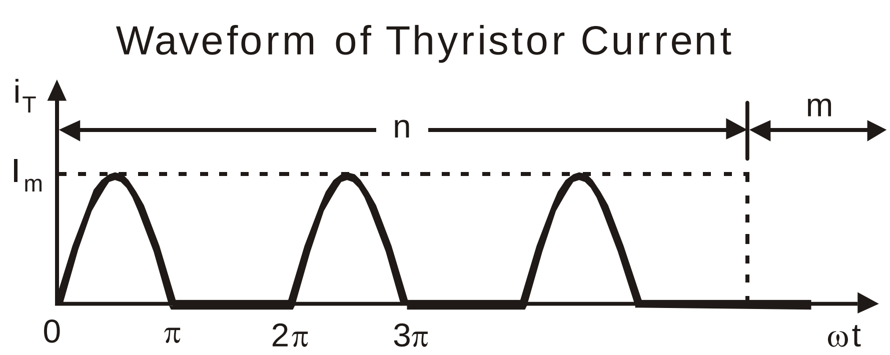
The input supply current is same as the load current $I_{in} = I_O = I_L$

Hence, RMS supply current = RMS load current; $I_{in(RMS)} = I_{O(RMS)}$.

$$PF = \frac{I^2_{O(RMS)} \times R_L}{V_{i(RMS)} \times I_{in(RMS)}} = \frac{V_{O(RMS)}}{V_{i(RMS)}} = \frac{V_{i(RMS)}\sqrt{k}}{V_{i(RMS)}} = \sqrt{k}$$

$$PF = \sqrt{k} = \sqrt{\frac{n}{m+n}}$$

- **The Average Current of Thyristor** $I_{T(Avg)}$



$$I_{T(Avg)} = \frac{n}{2\pi(m+n)} \int_0^{\pi} I_m \sin\omega t.d(\omega t)$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \int_0^{\pi} \sin\omega t.d(\omega t)$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \left[-\cos\omega t \Big/_0^{\pi} \right]$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \left[-\cos\pi + \cos 0 \right]$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \left[-(-1) + 1 \right]$$

$$I_{T(Avg)} = \frac{n}{2\pi(m+n)} \left[2I_m \right]$$

$$I_{T(Avg)} = \frac{I_m n}{\pi(m+n)} = \frac{k.I_m}{\pi}$$

$$k = \text{duty cycle} = \frac{t_{ON}}{(t_{ON} + t_{OFF})} = \frac{n}{(n+m)}$$

$$I_{T(Avg)} = \frac{I_m n}{\pi(m+n)} = \frac{k.I_m}{\pi},$$

Where $I_m = \frac{V_m}{R_L}$ = maximum or peak thyristor current.

- **RMS Current of Thyristor** $I_{T(RMS)}$

$$I_{T(RMS)} = \left[\frac{n}{2\pi(n+m)} \int_0^{\pi} I_m^2 \sin^2\omega t.d(\omega t) \right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{2\pi(n+m)} \int_0^{\pi} \sin^2\omega t.d(\omega t) \right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{2\pi(n+m)} \int_0^{\pi} \frac{(1-\cos 2\omega t)}{2} d(\omega t) \right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)} \left\{ \int_0^\pi d(\omega t) - \int_0^\pi \cos 2\omega t . d(\omega t) \right\} \right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)} \left\{ (\omega t) \Big/_0^\pi - \left(\frac{\sin 2\omega t}{2} \right) \Big/_0^\pi \right\} \right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)} \left\{ (\pi - 0) - \left(\frac{\sin 2\pi - \sin 0}{2} \right) \right\} \right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)} \{ \pi - 0 - 0 \} \right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2 \pi}{4\pi(n+m)} \right]^{1/2} = \left[\frac{nI_m^2}{4(n+m)} \right]^{1/2}$$

$$I_{T(RMS)} = \frac{I_m}{2} \sqrt{\frac{n}{(m+n)}} = \frac{I_m}{2} \sqrt{k}$$

$$I_{T(RMS)} = \frac{I_m}{2} \sqrt{k}$$

PRINCIPLE OF AC PHASE CONTROL

The basic principle of ac phase control technique is explained with reference to a single phase half wave ac voltage controller (unidirectional controller) circuit shown in the below figure. The half wave ac controller uses one thyristor and one diode connected in parallel across each other in opposite direction that is anode of thyristor T_1 is connected to the cathode of diode D_1 and the cathode of T_1 is connected to the anode of D_1. The output voltage across the load resistor 'R' and hence the ac power flow to the load is controlled by varying the trigger angle 'α'. The trigger angle or the delay angle 'α' refers to the value of ωt or the instant at which the thyristor T_1 is triggered to turn it ON, by applying a suitable gate trigger pulse between the gate and cathode lead.The thyristor T_1 is forward biased during the positive half cycle of input ac supply. It can be triggered and made to conduct by applying a suitable gate trigger pulse only during the positive half cycle of input supply. When T_1 is triggered it conducts and the load current flows through the thyristor T_1, the load and through the transformer secondary winding.

By assuming T_1 as an ideal thyristor switch it can be considered as a closed switch when it is ON during the period $\omega t = \alpha$ to π radians. The output voltage across the load follows the input supply voltage when the thyristor T_1 is turned-on and when it conducts from $\omega t = \alpha$ to π radians. When the input supply voltage decreases to zero at $\omega t = \pi$, for a resistive load the load current also falls to zero at $\omega t = \pi$ and hence the thyristor T_1 turns off at $\omega t = \pi$. Between the time period $\omega t = \pi$ to 2π, when the supply voltage reverses and becomes negative the diode D_1 becomes forward biased and hence turns ON and conducts. The load current flows in the opposite direction during $\omega t = \pi$ to 2π radians when D_1 is ON and the output voltage follows the negative half cycle of input supply.

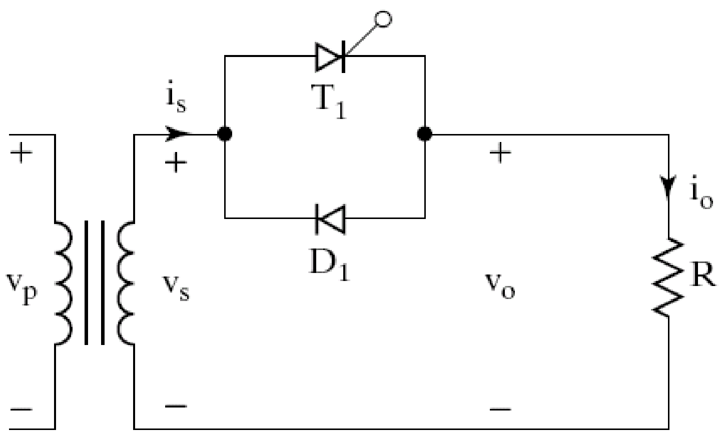
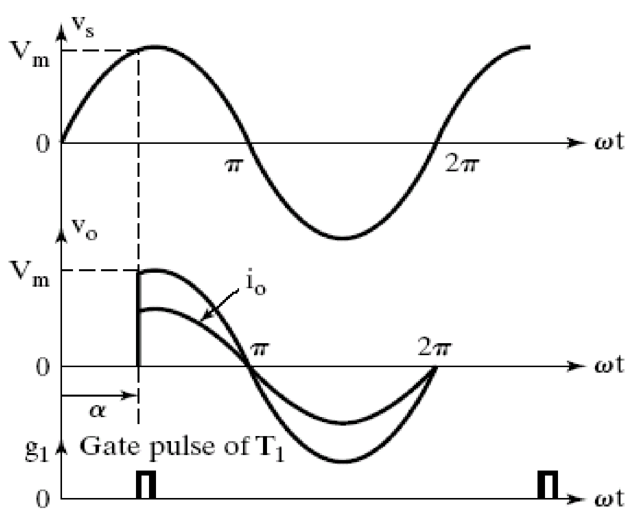


Fig.: Halfwave AC phase controller (Unidirectional Controller)



Equations

Input AC Supply Voltage across the Transformer Secondary Winding.

$$v_s = V_m \sin \omega t$$

$V_S = V_{in(RMS)} = \dfrac{V_m}{\sqrt{2}}$ = RMS value of secondary supply voltage.

Output Load Voltage

$v_o = v_L = 0$; for $\omega t = 0$ to α

$v_o = v_L = V_m \sin \omega t$; for $\omega t = \alpha$ to 2π.

Output Load Current

$i_o = i_L = \dfrac{v_o}{R_L} = \dfrac{V_m \sin \omega t}{R_L}$; for $\omega t = \alpha$ to 2π.

$i_o = i_L = 0$; for $\omega t = 0$ to α.

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE $V_{O(RMS)}$

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{2\pi} V_m{}^2 \sin^2 \omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{2\pi}\left[\int_{\alpha}^{2\pi} \left(\frac{1-\cos 2\omega t}{2}\right).d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{4\pi}\left[\int_{\alpha}^{2\pi} \left(1-\cos 2\omega t\right).d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\int_{\alpha}^{2\pi} d\left(\omega t\right) - \int_{\alpha}^{2\pi} \cos 2\omega t.d\omega t\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\left(\omega t\right)\Big/_{\alpha}^{2\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(2\pi-\alpha\right) - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(2\pi-\alpha\right) - \left\{\frac{\sin 4\pi}{2} - \frac{\sin 2\alpha}{2}\right\}} \qquad ;\sin 4\pi = 0$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{(2\pi - \alpha) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}} \sqrt{(2\pi - \alpha) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_{i(RMS)} \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

Where, $V_{i(RMS)} = V_S = \frac{V_m}{\sqrt{2}}$ = RMS value of input supply voltage (across the transformer secondary winding).

Note: Output RMS voltage across the load is controlled by changing $'\alpha'$ as indicated by the expression for $V_{O(RMS)}$

PLOT OF $V_{O(RMS)}$ VERSUS TRIGGER ANGLE α FOR A SINGLE PHASE HALF-WAVE AC VOLTAGE CONTROLLER (UNIDIRECTIONAL CONTROLLER)

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

By using the expression for $V_{O(RMS)}$ we can obtain the control characteristics, which is the plot of RMS output voltage $V_{O(RMS)}$ versus the trigger angle α . A typical control characteristic of single phase half-wave phase controlled ac voltage controller is as shown below

Trigger angle α in degrees	**Trigger angle α in radians**	$V_{O(RMS)}$
0	0	$V_S = \frac{V_m}{\sqrt{2}}$
30^0	$\pi/6$; $(1\pi/6)$	0.992765 V_S

60^0	$\pi/3 \quad ;\left(2\pi/6\right)$	$0.949868\ V_S$
90^0	$\pi/2 \quad ;\left(3\pi/6\right)$	$0.866025\ V_S$
120^0	$2\pi/3 \quad ;\left(4\pi/6\right)$	$0.77314\ V_S$
150^0	$5\pi/6 \quad ;\left(5\pi/6\right)$	$0.717228\ V_S$
180^0	$\pi \quad ;\left(6\pi/6\right)$	$0.707106\ V_S$

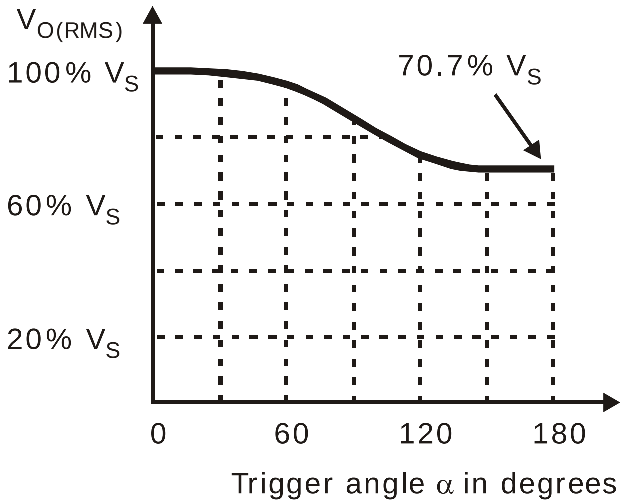


Fig.: Control characteristics of single phase half-wave phase controlled ac voltage controller

Note: We can observe from the control characteristics and the table given above that the range of RMS output voltage control is from 100% of V_S to 70.7% of V_S when we vary the trigger angle α from zero to 180 degrees. Thus the half wave ac controller has the draw back of limited range RMS output voltage control.

TO CALCULATE THE AVERAGE VALUE (DC VALUE) OF OUTPUT VOLTAGE

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{2\pi} V_m \sin\omega t.d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{2\pi} \sin\omega t.d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{2\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos 2\pi + \cos\alpha\right] \qquad ; \quad \cos 2\pi = 1$$

$$V_{dc} = \frac{V_m}{2\pi}\left[\cos\alpha - 1\right] \quad ; \; V_m = \sqrt{2}V_S$$

Hence $V_{dc} = \frac{\sqrt{2}V_S}{2\pi}\left(\cos\alpha - 1\right)$

When $'\alpha'$ is varied from 0 to π. V_{dc} varies from 0 to $\frac{-V_m}{\pi}$

DISADVANTAGES OF SINGLE PHASE HALF WAVE AC VOLTAGE CONTROLLER.

- The output load voltage has a DC component because the two halves of the output voltage waveform are not symmetrical with respect to '0' level. The input supply current waveform also has a DC component (average value) which can result in the problem of core saturation of the input supply transformer.
- The half wave ac voltage controller using a single thyristor and a single diode provides control on the thyristor only in one half cycle of the input supply. Hence ac power flow to the load can be controlled only in one half cycle.
- Half wave ac voltage controller gives limited range of RMS output voltage control. Because the RMS value of ac output voltage can be varied from a maximum of 100% of V_S at a trigger angle $\alpha = 0$ to a low of 70.7% of V_S at $\alpha = \pi$ Radians.

These drawbacks of single phase half wave ac voltage controller can be over come by using a single phase full wave ac voltage controller.

APPLICATIONS OF RMS VOLTAGE CONTROLLER

- Speed control of induction motor (poly phase ac induction motor).
- Heater control circuits (industrial heating).
- Welding power control.
- Induction heating.
- On load transformer tap changing.
- Lighting control in ac circuits.
- Ac magnet controls.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (AC REGULATOR) OR RMS VOLTAGE CONTROLLER WITH RESISTIVE LOAD

Single phase full wave ac voltage controller circuit using two SCRs or a single triac is generally used in most of the ac control applications. The ac power flow to the load can be controlled in both the half cycles by varying the trigger angle $'\alpha'$.

The RMS value of load voltage can be varied by varying the trigger angle $'\alpha'$. The input supply current is alternating in the case of a full wave ac voltage controller and due to the symmetrical nature of the input supply current waveform there is no dc component of input supply current i.e., the average value of the input supply current is zero.

A single phase full wave ac voltage controller with a resistive load is shown in the figure below. It is possible to control the ac power flow to the load in both the half cycles by adjusting the trigger angle $'\alpha'$. Hence the full wave ac voltage controller is also referred to as to a bi-directional controller.

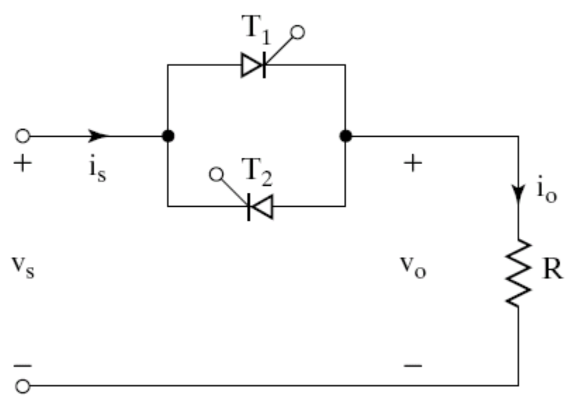


Fig.: Single phase full wave ac voltage controller (Bi-directional Controller) using SCRs

The thyristor T_1 is forward biased during the positive half cycle of the input supply voltage. The thyristor T_1 is triggered at a delay angle of $'\alpha'$ $\left(0 \le \alpha \le \pi \text{ radians}\right)$. Considering the ON thyristor T_1 as an ideal closed switch the input supply voltage appears across the load resistor R_L and the output voltage $v_O = v_S$ during $\omega t = \alpha$ to π radians. The load current flows through the ON thyristor T_1 and through the load resistor R_L in the downward direction during the conduction time of T_1 from $\omega t = \alpha$ to π radians.

At $\omega t = \pi$, when the input voltage falls to zero the thyristor current (which is flowing through the load resistor R_L) falls to zero and hence T_1 naturally turns off . No current flows in the circuit during $\omega t = \pi$ to $(\pi + \alpha)$. The thyristor T_2 is forward biased during the negative cycle of input supply and when thyristor T_2 is triggered at a delay angle $(\pi + \alpha)$, the output voltage follows the negative halfcycle of input from $\omega t = (\pi + \alpha)$ to 2π. When T_2 is ON, the load current flows in the reverse direction (upward direction) through T_2 during $\omega t = (\pi + \alpha)$ to 2π radians. The time interval (spacing) between the gate trigger pulses of T_1 and T_2 is kept at π radians or 180^0. At $\omega t = 2\pi$ the input supply voltage falls to zero and hence the load current also falls to zero and thyristor T_2 turn off naturally.

Instead of using two SCR's in parallel, a Triac can be used for full wave ac voltage control.

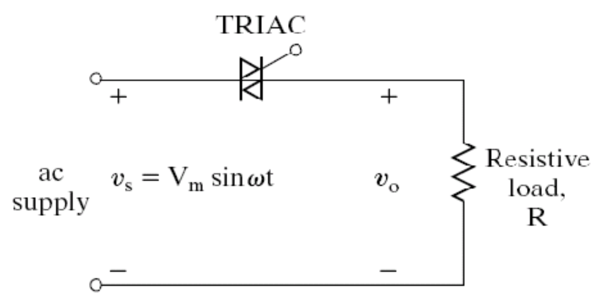


Fig.: Single phase full wave ac voltage controller (Bi-directional Controller) using TRIAC

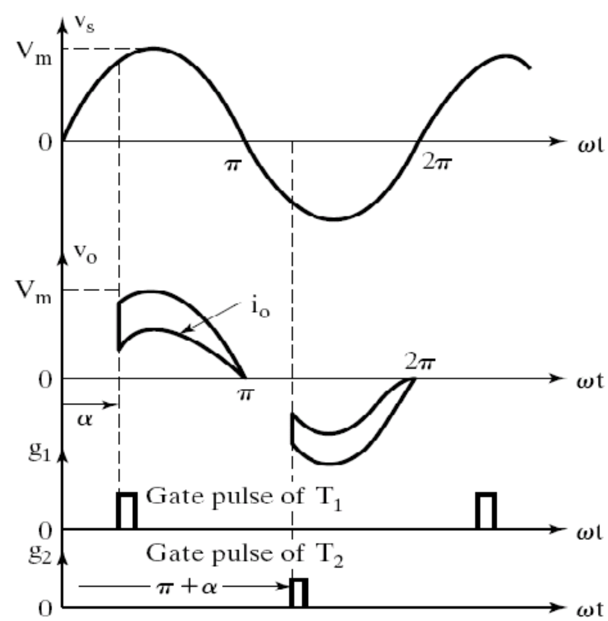


Fig: Waveforms of single phase full wave ac voltage controller

EQUATIONS

Input supply voltage

$$v_S = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t \ ;$$

Output voltage across the load resistor R_L;

$$v_O = v_L = V_m \sin \omega t \ ;$$

$$\text{for } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

Output load current

$$i_O = \frac{v_O}{R_L} = \frac{V_m \sin \omega t}{R_L} = I_m \sin \omega t \ ;$$

$$\text{for } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT (LOAD) VOLTAGE

The RMS value of output voltage (load voltage) can be found using the expression

$$V_{O(RMS)}^2 = V^2_{L(RMS)} = \frac{1}{2\pi} \int_0^{2\pi} v_L^2 d(\omega t);$$

For a full wave ac voltage controller, we can see that the two half cycles of output voltage waveforms are symmetrical and the output pulse time period (or output pulse repetition time) is π radians. Hence we can also calculate the RMS output voltage by using the expression given below.

$$V^2_{L(RMS)} = \frac{1}{\pi} \int_0^{\pi} V_m^2 \sin^2 \omega t . d\omega t$$

$$V^2_{L(RMS)} = \frac{1}{2\pi} \int_0^{2\pi} v_L^2 . d(\omega t) \ ;$$

$v_L = v_O = V_m \sin \omega t$; For $\omega t = \alpha$ to π and $\omega t = (\pi + \alpha)$ to 2π

Hence,

$$V_{L(RMS)}^2 = \frac{1}{2\pi} \left[\int_\alpha^\pi (V_m \sin \omega t)^2 d(\omega t) + \int_{\pi+\alpha}^{2\pi} (V_m \sin \omega t)^2 d(\omega t) \right]$$

$$= \frac{1}{2\pi} \left[V_m^2 \int_\alpha^\pi \sin^2 \omega t . d(\omega t) + V_m^2 \int_{\pi+\alpha}^{2\pi} \sin^2 \omega t . d(\omega t) \right]$$

$$= \frac{V_m^2}{2\pi} \left[\int_\alpha^\pi \frac{1 - \cos 2\omega t}{2} d(\omega t) + \int_{\pi+\alpha}^{2\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t) \right]$$

$$= \frac{V_m^{\;2}}{2\pi \times 2}\left[\int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t.d(\omega t) + \int_{\pi+\alpha}^{2\pi} d(\omega t) - \int_{\pi+\alpha}^{2\pi} \cos 2\omega t.d(\omega t)\right]$$

$$= \frac{V_m^{\;2}}{4\pi}\left[(\omega t)\Big/_{\alpha}^{\pi} + (\omega t)\Big/_{\pi+\alpha}^{2\pi} - \left[\frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi} - \left[\frac{\sin 2\omega t}{2}\right]_{\pi+\alpha}^{2\pi}\right]$$

$$= \frac{V_m^{\;2}}{4\pi}\left[(\pi-\alpha) + (\pi-\alpha) - \frac{1}{2}(\sin 2\pi - \sin 2\alpha) - \frac{1}{2}(\sin 4\pi - \sin 2(\pi+\alpha))\right]$$

$$= \frac{V_m^{\;2}}{4\pi}\left[2(\pi-\alpha) - \frac{1}{2}(0 - \sin 2\alpha) - \frac{1}{2}(0 - \sin 2(\pi+\alpha))\right]$$

$$= \frac{V_m^{\;2}}{4\pi}\left[2(\pi-\alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin 2(\pi+\alpha)}{2}\right]$$

$$= \frac{V_m^{\;2}}{4\pi}\left[2(\pi-\alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin(2\pi+2\alpha)}{2}\right]$$

$$= \frac{V_m^{\;2}}{4\pi}\left[2(\pi-\alpha) + \frac{\sin 2\alpha}{2} + \frac{1}{2}(\sin 2\pi.\cos 2\alpha + \cos 2\pi.\sin 2\alpha)\right]$$

$$\sin 2\pi = 0 \quad \& \quad \cos 2\pi = 1$$

Therefore,

$$V_{L(RMS)}^2 = \frac{V_m^{\;2}}{4\pi}\left[2(\pi-\alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin 2\alpha}{2}\right]$$

$$= \frac{V_m^{\;2}}{4\pi}\left[2(\pi-\alpha) + \sin 2\alpha\right]$$

$$V^2_{\;L(RMS)} = \frac{V_m^{\;2}}{4\pi}\left[(2\pi-2\alpha) + \sin 2\alpha\right]$$

Taking the square root, we get

$$V_{L(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[(2\pi-2\alpha) + \sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{\left[(2\pi-2\alpha) + \sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[(2\pi-2\alpha) + \sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[2\left\{(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right\}\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

$$V_{L(RMS)} = V_{i(RMS)}\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

$$V_{L(RMS)} = V_S\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

Maximum RMS voltage will be applied to the load when $\alpha = 0$, in that case the full sine wave appears across the load. RMS load voltage will be the same as the RMS supply voltage $= \frac{V_m}{\sqrt{2}}$. When α is increased the RMS load voltage decreases.

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi-0)+\frac{\sin 2\times 0}{2}\right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi)+\frac{0}{2}\right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} = V_{i(RMS)} = V_S$$

The output control characteristic for a single phase full wave ac voltage controller with resistive load can be obtained by plotting the equation for $V_{O(RMS)}$

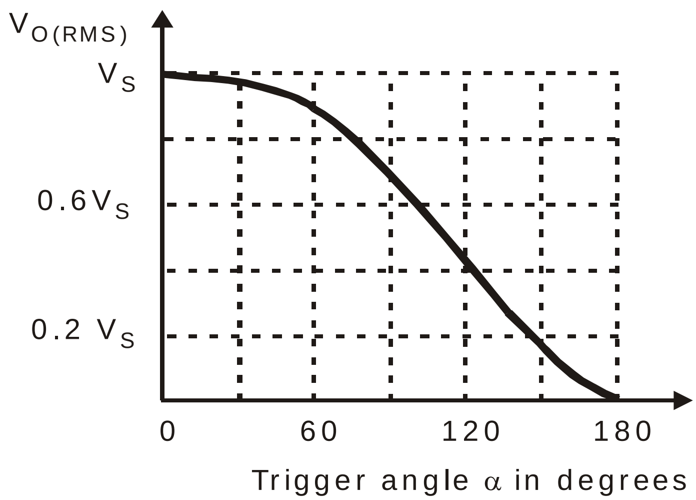
CONTROL CHARACTERISTIC OF SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RESISTIVE LOAD

The control characteristic is the plot of RMS output voltage $V_{O(RMS)}$ versus the trigger angle α; which can be obtained by using the expression for the RMS output voltage of a full-wave ac controller with resistive load.

$$V_{O(RMS)} = V_S\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]} \quad ;$$

Where $V_S = \frac{V_m}{\sqrt{2}} =$ RMS value of input supply voltage

Trigger angle α in degrees	**Trigger angle α in radians**	$V_{O(RMS)}$	**%**
0	0	V_S	100% V_S
30^0	$\pi/6$; $(1\pi/6)$	0.985477 V_S	98.54% V_S
60^0	$\pi/3$; $(2\pi/6)$	0.896938 V_S	89.69% V_S
90^0	$\pi/2$; $(3\pi/6)$	0.7071 V_S	70.7% V_S
120^0	$2\pi/3$; $(4\pi/6)$	0.44215 V_S	44.21% V_S
150^0	$5\pi/6$; $(5\pi/6)$	0.1698 V_S	16.98% V_S
180^0	π ; $(6\pi/6)$	0 V_S	0 V_S



We can notice from the figure, that we obtain a much better output control characteristic by using a single phase full wave ac voltage controller. The RMS output voltage can be varied from a maximum of 100% V_S at $\alpha = 0$ to a minimum of '0' at $\alpha = 180^0$. Thus we get a full range output voltage control by using a single phase full wave ac voltage controller.

Need For Isolation

In the single phase full wave ac voltage controller circuit using two SCRs or Thyristors T_1 and T_2 in parallel, the gating circuits (gate trigger pulse generating circuits) of Thyristors T_1

and T_2 must be isolated. Figure shows a pulse transformer with two separate windings to provide isolation between the gating signals of T_1 and T_2.

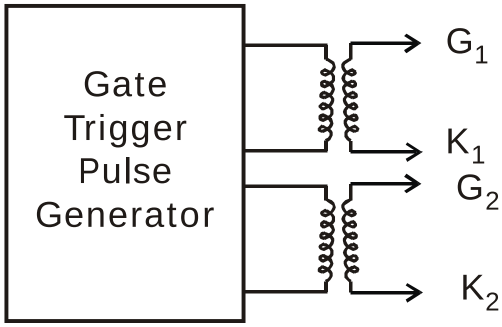


Fig.: Pulse Transformer

SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH COMMON CATHODE

It is possible to design a single phase full wave ac controller with a common cathode configuration by having a common cathode point for T_1 and T_2 & by adding two diodes in a full wave ac controller circuit as shown in the figure below

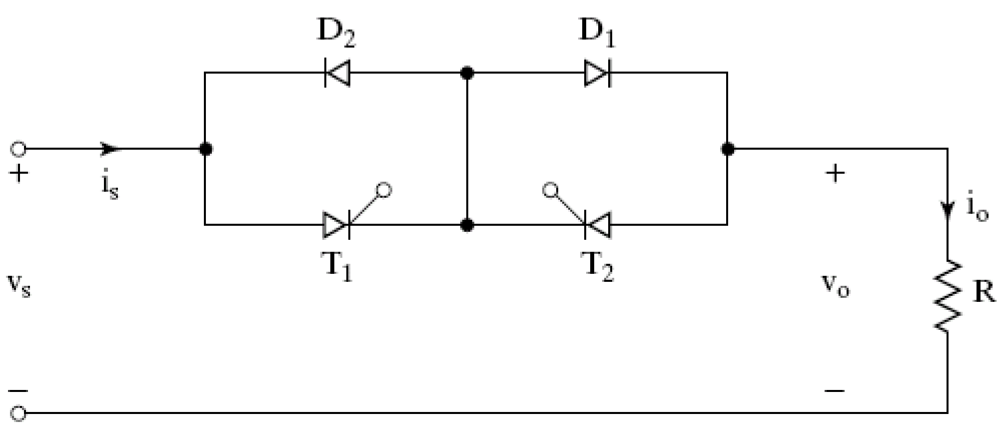


Fig.: Single phase full wave ac controller with common cathode (Bidirectional controller in common cathode configuration)

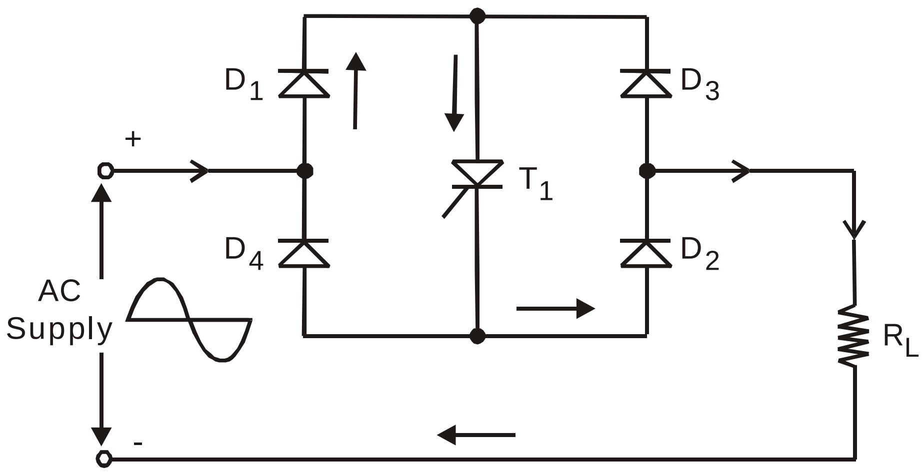
Thyristor T_1 and diode D_1 are forward biased during the positive half cycle of input supply. When thyristor T_1 is triggered at a delay angle α, Thyristor T_1 and diode D_1 conduct together from $\omega t = \alpha$ to π during the positive half cycle.

The thyristor T_2 and diode D_2 are forward biased during the negative half cycle of input supply, when trigged at a delay angle α, thyristor T_2 and diode D_2 conduct together during the negative half cycle from $\omega t = (\pi + \alpha)$ to 2π.

In this circuit as there is one single common cathode point, routing of the gate trigger pulses to the thyristor gates of T_1 and T_2 is simpler and only one isolation circuit is required.

But due to the need of two power diodes the costs of the devices increase. As there are two power devices conducting at the same time the voltage drop across the ON devices increases and the ON state conducting losses of devices increase and hence the efficiency decreases.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER USING A SINGLE THYRISTOR



A single phase full wave ac controller can also be implemented with one thyristor and four diodes connected in a full wave bridge configuration as shown in the above figure. The four diodes act as a bridge full wave rectifier. The voltage across the thyristor T_1 and current through thyristor T_1 are always unidirectional. When T_1 is triggered at $\omega t = \alpha$, during the positive half cycle $(0 \le \alpha \le \pi)$, the load current flows through D_1, T_1, diode D_2 and through the load. With a resistive load, the thyristor current (flowing through the ON thyristor T_1), the load current falls to zero at $\omega t = \pi$, when the input supply voltage decreases to zero at $\omega t = \pi$, the thyristor naturally turns OFF.

In the negative half cycle, diodes $D_3 \& D_4$ are forward biased during $\omega t = \pi$ to 2π radians. When T_1 is triggered at $\omega t = (\pi + \alpha)$, the load current flows in the opposite direction (upward direction) through the load, through D_3, T_1 and D_4. Thus D_3, D_4 and T_1 conduct together during the negative half cycle to supply the load power. When the input supply voltage becomes zero at $\omega t = 2\pi$, the thyristor current (load current) falls to zero at $\omega t = 2\pi$ and the thyristor T_1 naturally turns OFF. The waveforms and the expression for the RMS output voltage are the same as discussed earlier for the single phase full wave ac controller.

But however if there is a large inductance in the load circuit, thyristor T_1 may not be turned OFF at the zero crossing points, in every half cycle of input voltage and this may result in a loss of output control. This would require detection of the zero crossing of the load current waveform in order to ensure guaranteed turn off of the conducting thyristor before triggering the thyristor in the next half cycle, so that we gain control on the output voltage.

In this full wave ac controller circuit using a single thyristor, as there are three power devices conducting together at the same time there is more conduction voltage drop and an increase in the ON state conduction losses and hence efficiency is also reduced.

The diode bridge rectifier and thyristor (or a power transistor) act together as a bidirectional switch which is commercially available as a single device module and it has relatively low ON state conduction loss. It can be used for bidirectional load current control and for controlling the RMS output voltage.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (BIDIRECTIONAL CONTROLLER) WITH RL LOAD

In this section we will discuss the operation and performance of a single phase full wave ac voltage controller with RL load. In practice most of the loads are of RL type. For example if we consider a single phase full wave ac voltage controller controlling the speed of a single phase ac induction motor, the load which is the induction motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

A single phase full wave ac voltage controller circuit (bidirectional controller) with an RL load using two thyristors T_1 and T_2 (T_1 and T_2 are two SCRs) connected in parallel is shown in the figure below. In place of two thyristors a single Triac can be used to implement a full wave ac controller, if a suitable Traic is available for the desired RMS load current and the RMS output voltage ratings.

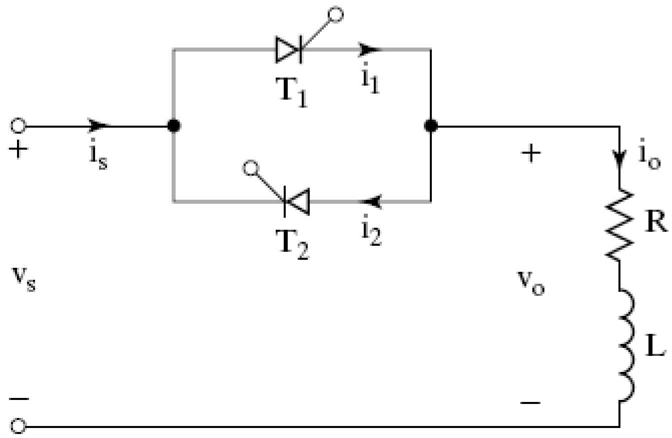


Fig: Single phase full wave ac voltage controller with RL load

The thyristor T_1 is forward biased during the positive half cycle of input supply. Let us assume that T_1 is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to T_1 during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when T_1 is ON. The load current i_O flows through the thyristor T_1 and through the load in the downward direction. This load current pulse flowing through T_1 can be considered as the positive current pulse. Due to the inductance in the load, the load current i_O flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative.

The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β. The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle ϕ. The waveforms of the input supply voltage, the gate trigger pulses of T_1 and T_2, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.

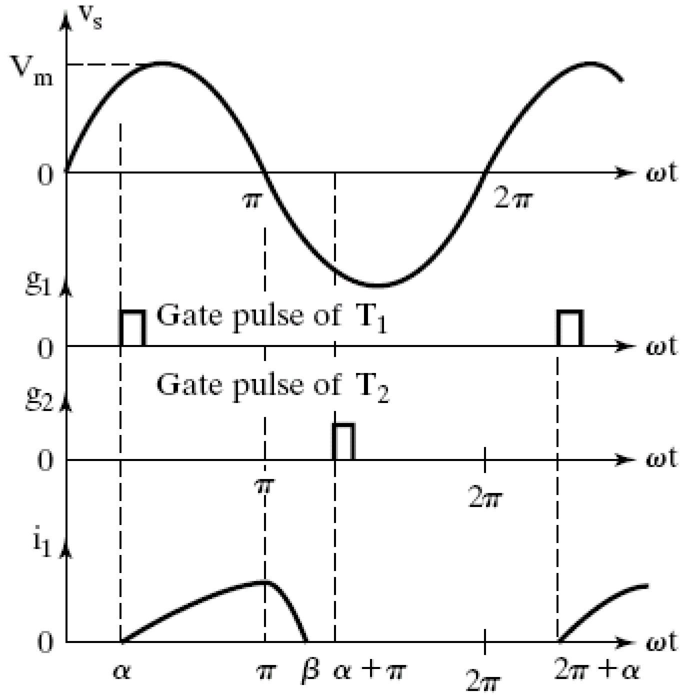


Fig.: Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

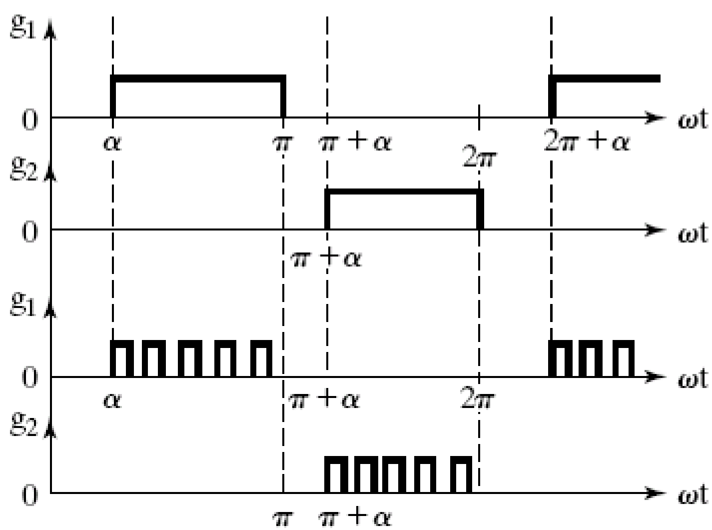


Fig.: Gating Signals

Waveforms of single phase full wave ac voltage controller with RL load for $\alpha > \phi$. Discontinuous load current operation occurs for $\alpha > \phi$ and $\beta < (\pi + \alpha)$;
i.e., $(\beta - \alpha) < \pi$, conduction angle $< \pi$.

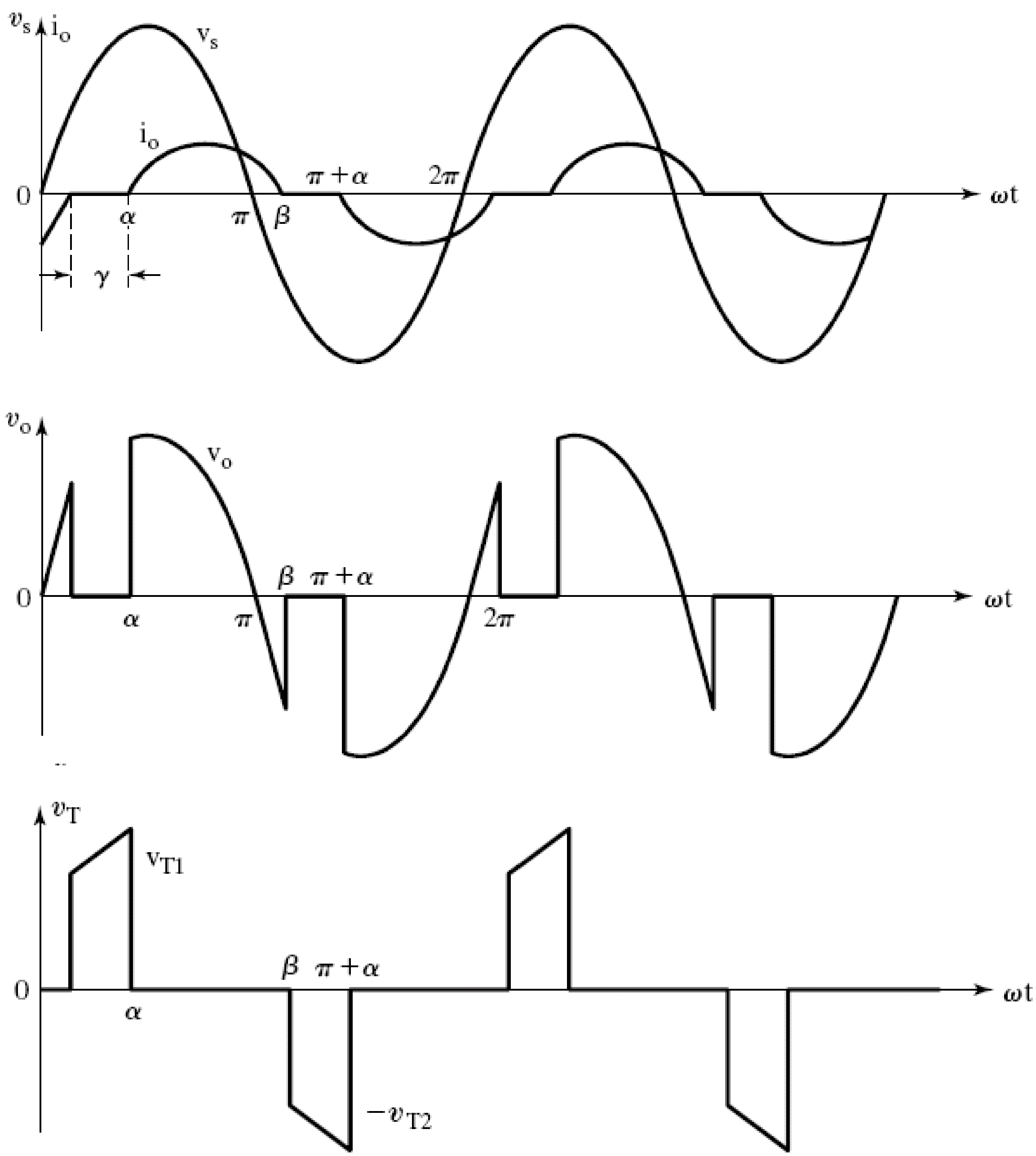


Fig.: Waveforms of Input supply voltage, Load Current, Load Voltage and Thyristor Voltage across T_1

Note

- The RMS value of the output voltage and the load current may be varied by varying the trigger angle α.
- This circuit, AC RMS voltage controller can be used to regulate the RMS voltage across the terminals of an ac motor (induction motor). It can be used to control the temperature of a furnace by varying the RMS output voltage.
- For very large load inductance 'L' the SCR may fail to commutate, after it is triggered and the load voltage will be a full sine wave (similar to the applied input supply voltage and the output control will be lost) as long as the gating signals are applied to the thyristors T_1 and T_2. The load current waveform will appear as a full continuous sine wave and the load current waveform lags behind the output sine wave by the load power factor angle ϕ.

TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING $\omega t = \alpha$ to β WHEN THYRISTOR T_1 CONDUCTS

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

$v_S = V_m \sin \omega t$ = instantaneous value of the input supply voltage.

Let us assume that the thyristor T_1 is triggered by applying the gating signal to T_1 at $\omega t = \alpha$. The load current which flows through the thyristor T_1 during $\omega t = \alpha$ to β can be found from the equation

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \quad ;$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}} \quad ;$$

Where $V_m = \sqrt{2} V_S$ = maximum or peak value of input supply voltage.

$Z = \sqrt{R^2 + \left(\omega L\right)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (power factor angle of load).

$$\tau = \frac{L}{R} = \text{ Load circuit time constant.}$$

Therefore the general expression for the output load current is given by the equation

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-R}{L}t} \quad ;$$

The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_O = 0$, at $\omega t = \alpha$. Hence from the equation for i_O equating i_O to zero and substituting $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

Therefore $\quad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right)$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$A_1 = e^{\frac{+R}{L}t}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$A_1 = e^{\frac{R(\omega t)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

By substituting $\omega t = \alpha$, we get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Substituting the value of constant A_1 from the above equation into the expression for i_O, we obtain

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{L}t} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right] \quad ;$$

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R(\omega t)}{\omega L}} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Therefore we obtain the final expression for the inductive load current of a single phase full wave ac voltage controller with RL load as

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \quad ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

The above expression also represents the thyristor current i_{T1}, during the conduction time interval of thyristor T_1 from $\omega t = \alpha$ to β.

To Calculate Extinction Angle β

The extinction angle β, which is the value of ωt at which the load current i_O falls to zero and T_1 is turned off can be estimated by using the condition that $i_O = 0$, at $\omega t = \beta$

By using the above expression for the output load current, we can write

$$i_O = 0 = \frac{V_m}{Z}\left[\sin\left(\beta - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}(\beta - \alpha)}\right]$$

As $\frac{V_m}{Z} \neq 0$ we can write

$$\left[\sin\left(\beta - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}(\beta - \alpha)}\right] = 0$$

Therefore we obtain the expression

$$\sin\left(\beta - \phi\right) = \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

The extinction angle β can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After β is calculated, we can determine the thyristor conduction angle $\delta = \left(\beta - \alpha\right)$.

β is the extinction angle which depends upon the load inductance value. Conduction angle δ increases as α is decreased for a known value of β.

For $\delta < \pi$ radians, i.e., for $\left(\beta - \alpha\right) < \pi$ radians, for $\left(\pi + \alpha\right) > \beta$ the load current waveform appears as a discontinuous current waveform as shown in the figure. The output load current remains at zero during $\omega t = \beta$ to $(\pi + \alpha)$. This is referred to as discontinuous load current operation which occurs for $\beta < (\pi + \alpha)$.

When the trigger angle α is decreased and made equal to the load impedance angle ϕ i.e., when $\alpha = \phi$ we obtain from the expression for $\sin(\beta - \phi)$,

$$\sin(\beta - \phi) = 0 \;\; ; \;\; \text{Therefore } (\beta - \phi) = \pi \text{ radians.}$$

Extinction angle $\beta = (\pi + \phi) = (\pi + \alpha)$; for the case when $\alpha = \phi$

Conduction angle $\delta = (\beta - \alpha) = \pi \text{ radians} = 180^0$; for the case when $\alpha = \phi$

Each thyristor conducts for 180^0 (π radians) . T_1 conducts from $\omega t = \phi$ to $(\pi + \phi)$ and provides a positive load current. T_2 conducts from $(\pi + \phi)$ to $(2\pi + \phi)$ and provides a negative load current. Hence we obtain a continuous load current and the output voltage waveform appears as a continuous sine wave identical to the input supply voltage waveform for trigger angle $\alpha \le \phi$ and the control on the output is lost.

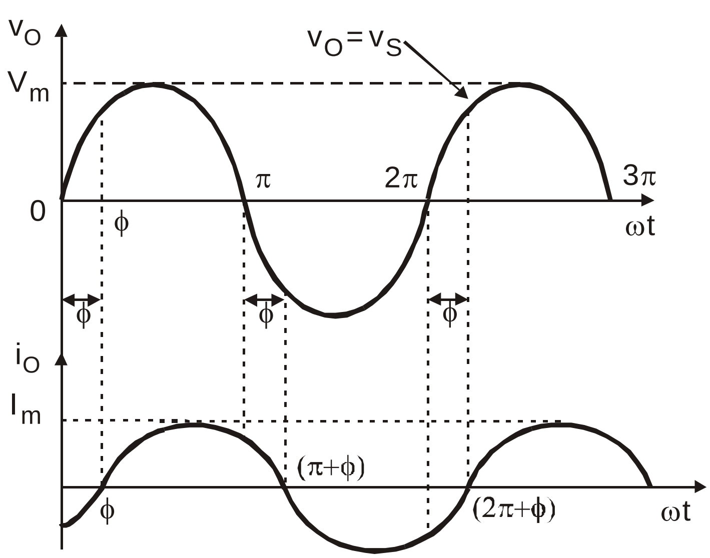


Fig.: Output voltage and output current waveforms for a single phase full wave ac voltage controller with RL load for $\alpha \le \phi$

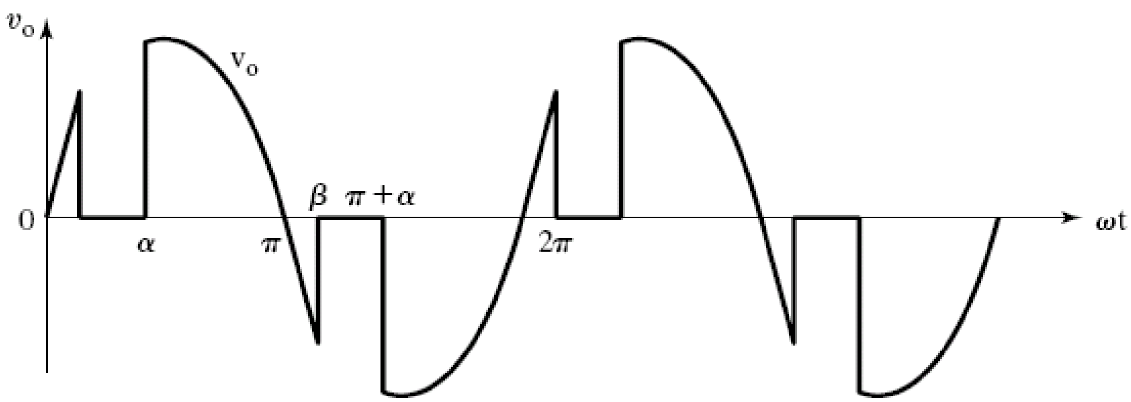
Thus we observe that for trigger angle $\alpha \le \phi$, the load current tends to flow continuously and we have continuous load current operation, without any break in the load current waveform and we obtain output voltage waveform which is a continuous sinusoidal waveform identical to the input supply voltage waveform. We loose the control on the output voltage for $\alpha \le \phi$ as the output voltage becomes equal to the input supply voltage and thus we obtain

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S \quad ; \text{ for } \alpha \le \phi$$

Hence,

RMS output voltage = RMS input supply voltage for $\alpha \le \phi$

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE $V_{O(RMS)}$ OF A SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RL LOAD.



When $\alpha > \varnothing$, the load current and load voltage waveforms become discontinuous as shown in the figure above.

$$V_{O(RMS)} = \left[\frac{1}{\pi}\int_{\alpha}^{\beta} V_m^{\ 2}\sin^2\omega t.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

Output $v_o = V_m \sin\omega t$, for $\omega t = \alpha$ to β, when T_1 is ON.

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{\pi}\int_{\alpha}^{\beta}\frac{\left(1-\cos 2\omega t\right)}{2}d\left(\omega t\right)\right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi}\left\{\int_{\alpha}^{\beta} d\left(\omega t\right) - \int_{\alpha}^{\beta}\cos 2\omega t.d\left(\omega t\right)\right\}\right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi}\left\{\left(\omega t\right)\Big/_{\alpha}^{\beta} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\beta}\right\}\right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi}\left\{\left(\beta-\alpha\right) - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} \left\{ (\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left\{ (\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{1/2}$$

The RMS output voltage across the load can be varied by changing the trigger angle α.

For a purely resistive load $L = 0$, therefore load power factor angle $\phi = 0$.

$$\phi = \tan^{-1}\left(\frac{\omega L}{R} \right) = 0 \quad ;$$

Extinction angle $\beta = \pi \text{ radians} = 180^0$

PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER WITH RESISTIVE LOAD

- **RMS Output Voltage** $V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$; $\frac{V_m}{\sqrt{2}} = V_S$ = RMS input supply voltage.
- $I_{O(RMS)} = \frac{V_{O(RMS)}}{R_L}$ = RMS value of load current.
- $I_S = I_{O(RMS)}$ = RMS value of input supply current.
- **Output load power**

$$P_O = I_{O(RMS)}^2 \times R_L$$

- **Input Power Factor**

$$PF = \frac{P_O}{V_S \times I_S} = \frac{I_{O(RMS)}^2 \times R_L}{V_S \times I_{O(RMS)}} = \frac{I_{O(RMS)} \times R_L}{V_S}$$

$$PF = \frac{V_{O(RMS)}}{V_S} = \sqrt{\frac{1}{\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

- **Average Thyristor Current,**

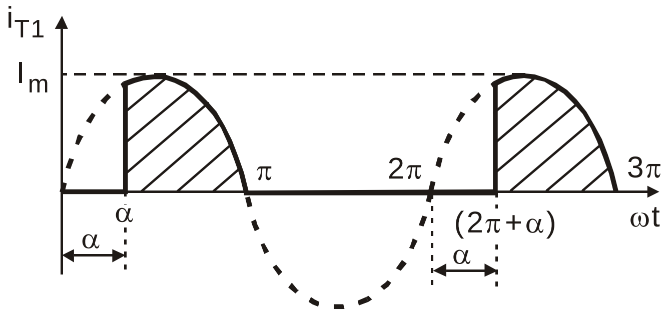


Fig.: Thyristor Current Waveform

$$I_{T(Avg)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} i_T d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t.d(\omega t)$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\int_{\alpha}^{\pi} \sin\omega t.d(\omega t) = \frac{I_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\pi}\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[-\cos\pi + \cos\alpha\right] = \frac{I_m}{2\pi}\left[1+\cos\alpha\right]$$

- **Maximum Average Thyristor Current, for** $\alpha = 0$,

$$I_{T(Avg)} = \frac{I_m}{\pi}$$

- **RMS Thyristor Current**

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t.d(\omega t)\right]}$$

$$I_{T(RMS)} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

- **Maximum RMS Thyristor Current, for** $\alpha = 0$,

$$I_{T(RMS)} = \frac{I_m}{2}$$

In the case of a single phase full wave ac voltage controller circuit using a Triac with resistive load, the average thyristor current $I_{T(Avg)} = 0$. Because the Triac conducts in both the half cycles and the thyristor current is alternating and we obtain a symmetrical thyristor current waveform which gives an average value of zero on integration.

PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER WITH R-L LOAD

The Expression for the Output (Load) Current

The expression for the output (load) current which flows through the thyristor, during $\omega t = \alpha$ to β is given by

$$i_O = i_{T_1} = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \quad ; \quad \text{for } \alpha \le \omega t \le \beta$$

Where,

$V_m = \sqrt{2} V_S$ = Maximum or peak value of input ac supply voltage.

$Z = \sqrt{R^2 + \left(\omega L\right)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (load power factor angle).

α = Thyristor trigger angle = Delay angle.

β = Extinction angle of thyristor, (value of ωt) at which the thyristor (load) current falls to zero.

β is calculated by solving the equation

$$\sin\left(\beta - \phi\right) = \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

Thyristor Conduction Angle $\delta = \left(\beta - \alpha\right)$

Maximum thyristor conduction angle $\delta = \left(\beta - \alpha\right) = \pi$ radians = 180^0 for $\alpha \le \phi$.

RMS Output Voltage

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi}\left[\left(\beta - \alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right]}$$

The Average Thyristor Current

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} i_{T_1} \, d\left(\omega t\right)\right]$$

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] d\left(\omega t\right)\right]$$

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\left[\int_{\alpha}^{\beta}\sin(\omega t-\phi).d(\omega t)-\int_{\alpha}^{\beta}\sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\omega t-\alpha)}d(\omega t)\right]$$

Maximum value of $I_{T(Avg)}$ occur at $\alpha = 0$. The thyristors should be rated for maximum $I_{T(Avg)} = \left(\frac{I_m}{\pi}\right)$, where $I_m = \frac{V_m}{Z}$.

RMS Thyristor Current $I_{T(RMS)}$

$$I_{T(RMS)} = \sqrt{\left[\frac{1}{2\pi}\int_{\alpha}^{\beta} i_{T_1}^2 d(\omega t)\right]}$$

Maximum value of $I_{T(RMS)}$ occurs at $\alpha = 0$. Thyristors should be rated for maximum $I_{T(RMS)} = \left(\frac{I_m}{2}\right)$

When a Triac is used in a single phase full wave ac voltage controller with RL type of load, then $I_{T(Avg)} = 0$ and maximum $I_{T(RMS)} = \frac{I_m}{\sqrt{2}}$