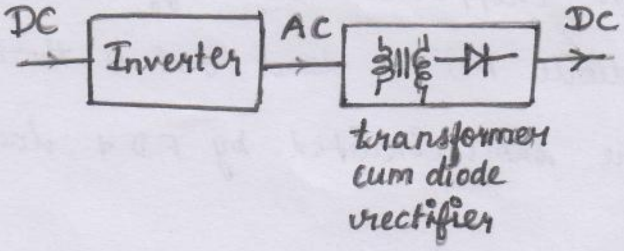
Choppers - Introduction

Ac Link chopper.

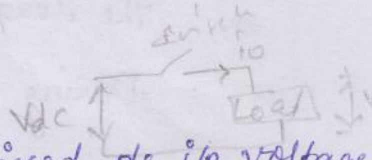


In the ac link chopper, dc is first converted to ac by an inverter (dc to ac converter).

AC is then stepped up or stepped -down by a transformer which is then converted back to dc by a diode rectifier.

As the conversion is in two stages, dc to ac & then ac to dc, ac link chopper is costly, bulky & less efficient.

DC Chopper

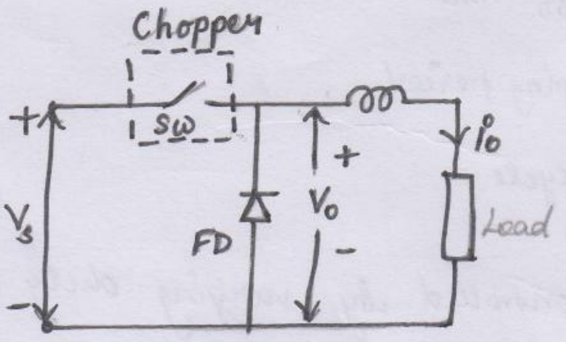


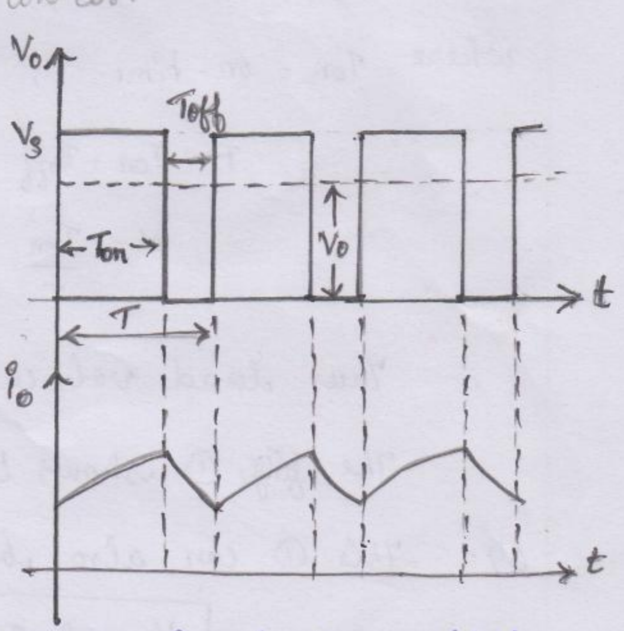
A chopper is a static device that converts fixed dc i/p voltage to a variable dc o/p vol directly.

DC choppers are well used in trolley cars, marine hoists, forklift trucks & mine haulers, battery operated vehicles, traction motor control etc.,

Adv: Greater efficiency, faster response, smooth Ctrl, low maintenance, low cost

Step down Chopper.





Chopper is a high speed on/off semiconductor switch.

For the sake of highlighting the principle of chopper operation, the chopper is used for controlling the on, off periods of this switch is not shown, During the period when chopper is on & load vol is equal to source vol V_s.

During the interval T_{off}, the chopper switches off, load current flows through the freewheeling diode FD. & ~~load vol is therefore~~ As a result, load terminals are short circuited by FD & load voltage is therefore zero during T_{off}.

In this manner, a chopped dc vol is produced at the load terminals.

The load ch as shown in fig is continuous.

During Ton, load ch rise whereas during Toff, load ch decays.

Avg load vol V_0 is given by

$$V_0 = \frac{T_{on}}{T_{on}+T_{off}} V_s$$

$$= \frac{T_{on}}{T} V_s \Rightarrow \boxed{V_0 = \delta V_s} \longrightarrow (1)$$

where T_{on} = on-time ; T_{off} = off-time

$$T = T_{on} + T_{off} = \text{Chopping period}.$$

$$\alpha = \frac{T_{on}}{T} = \text{duty cycle}.$$

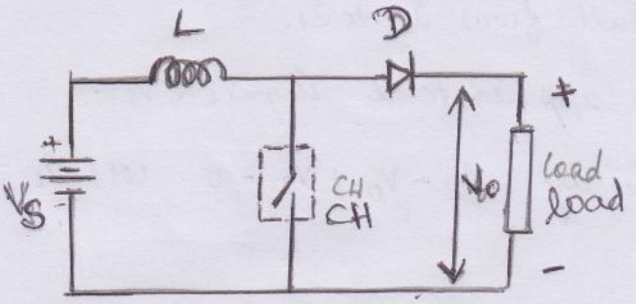
Thus load vol can be controlled by varying duty cycle α.

The ~~fig~~ (1) shows that the load vol is independent of load ch. This (1) can also be written as

$$\boxed{V_0 = f \cdot T_{on} \cdot V_s} \longrightarrow (2)$$

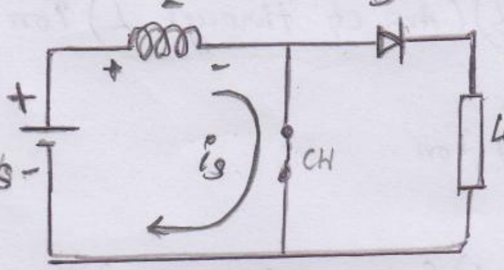
where $(\because f = \frac{1}{T} = \text{Chopping freq})$

Step-up Choppers

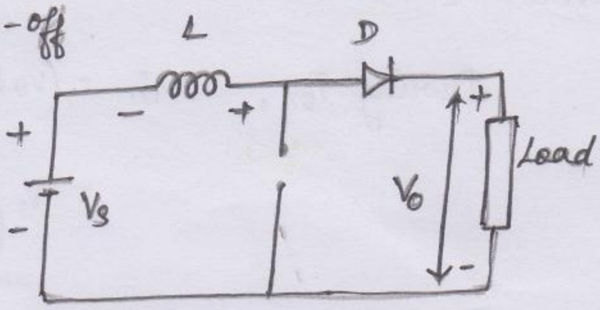


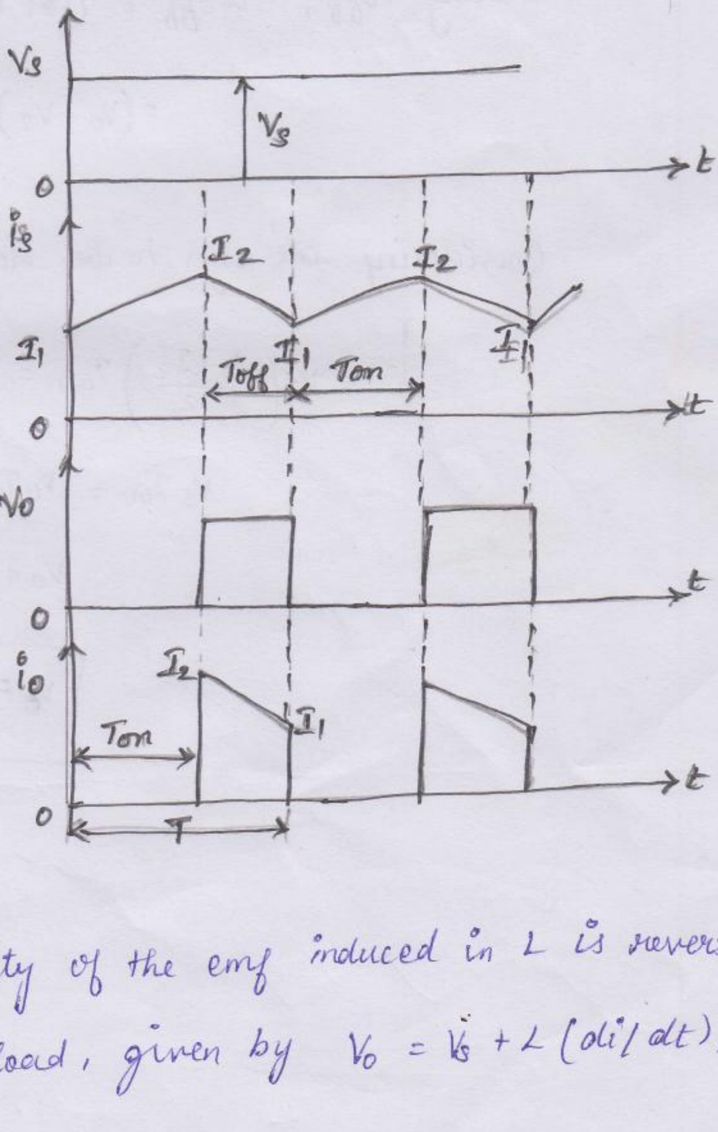
In step-up chopper, a large inductor L in series with source vol V_s is essential.

CH-on



CH-off





When the chopper CH is on, the closed current path is as shown in fig & the inductor stores energy during Ton period.

When CH is off, as the inductor c/t cannot die down instantaneously, this c/t is forced to flow through the diode & load for a time Toff.

As the c/t tends to ↓, polarity of the emf induced in L is reversed.

As a result, vol across the load, given by $V_o = V_s + L\,(di/dt)$, exceeds the source vol V_s.

In this manner, the ckt acts as a step-up chopper & the energy stored in L is released to the load.

When CH is on, ch through L would ↑ from I_1 to I_2.

When CH is off, ch would fall from I_2 to I_1.

With CH-on, source vol is applied to L. ie, $V_L = V_S$.

When CH-off, KVL is given by $V_L - V_0 + V_S = 0$ (or) $V_L = V_0 - V_S$

Here $V_L \rightarrow$ vol across L.

During Ton, W_{in} = (Vol across L)$\underbrace{\text{(Avg ch through L)}}$ Ton

$$= V_S \left(\frac{I_1 + I_2}{2} \right) T_{on}.$$

During Toff, W_{off} = (Vol across L) (Avg ch through L) Toff

$$= (V_0 - V_S) \left(\frac{I_1 + I_2}{2} \right) T_{off}.$$

Considering the sys to be lossless, $W_{in} = W_{off}$.

$$\therefore \quad V_S \left(\frac{I_1 + I_2}{2} \right) T_{on} = (V_0 - V_S) \left(\frac{I_1 + I_2}{2} \right) T_{off}$$

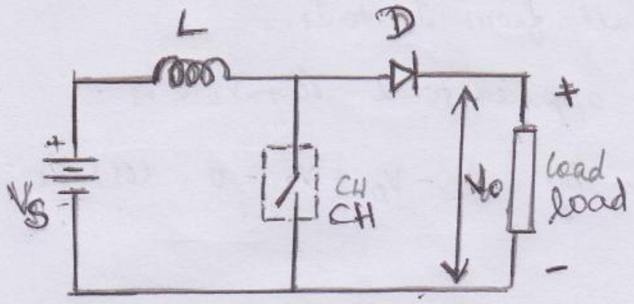
$$V_S T_{on} = V_0 T_{off} - V_S T_{off}$$

$$V_0 = V_S \frac{T}{T_{off}} = V_S \left(\frac{T}{T - T_{on}} \right)$$

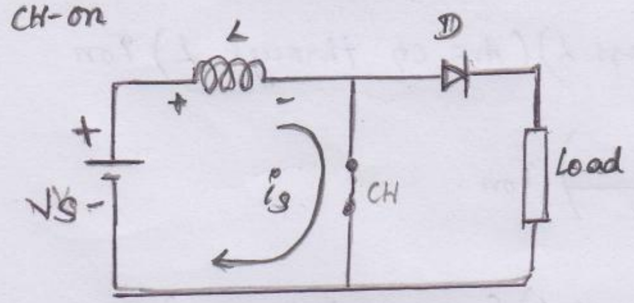
$$\boxed{V_0 = V_S \left(\frac{1}{1 - \alpha} \right)}$$

Step-up Choppers

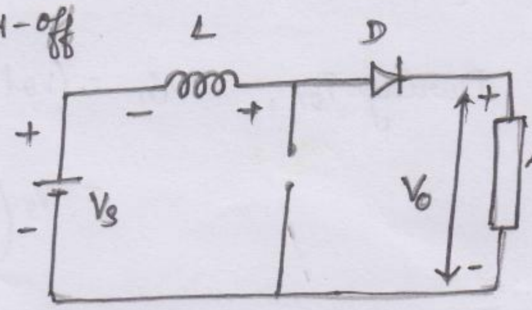
In step-up chopper, a large inductor L in series with source vol V_s is essential.

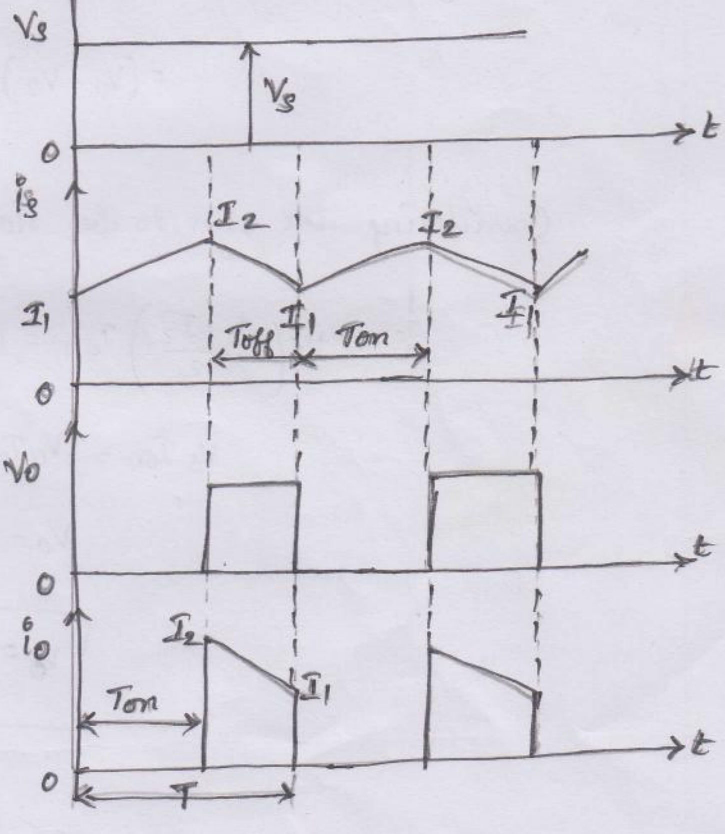


CH-on



CH-off





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When CH is off, as the inductor c/t cannot die down instantaneously, this c/t is forced to flow through the diode & load for a time Toff.

As the c/t tends to ↓, polarity of the emf induced in L is reversed.

As a result, vol across the load, given by $V_o = V_s + L\,(di/dt)$, exceeds the source vol V_s.

In this manner, the ckt acts as a step-up chopper & the energy stored in L is released to the load.

Control strategies of Chopper,

$V_0 = V_{dc}\,\alpha$.

It is seen that, the avg value of o/p vol, V_0 can be controlled by periodic opening & closing of the switches.

There are 2 types of ctrl strategies. 1) Time-ratio ctrl (TRC) & 2) ct limit ctrl.

Time-Ratio ctrl (TRC)

In the time-ratio ctrl, the value of $\frac{T_{on}}{T}$ is varied.

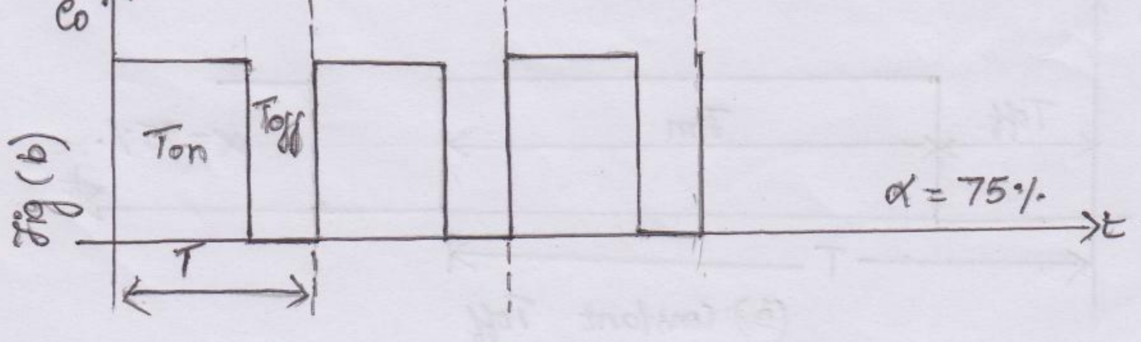
This is effected in 2 ways.

They are variable freq operation & const freq operation.

Const freq ctrl.

In this type of ctrl strategy, the on-time T_{on}, is varied but the chopping freq f ($f = 1/T$, & hence the chopping period T) is kept constant.

Variation of T_{on} means adjustment of pulse width, as such this control strategy is also called as pulse-width modulation ctrl.



The above fig illustrates the principle of pulse-width modulation.

As shown, chopping period T is constant

$T_{on} = \frac{1}{4} T$, so that duty cycle $\alpha = 25\%$ ⟹ from fig (a)

$T_{on} = \frac{3}{4} T$, so that duty cycle $\alpha = 75\%$.

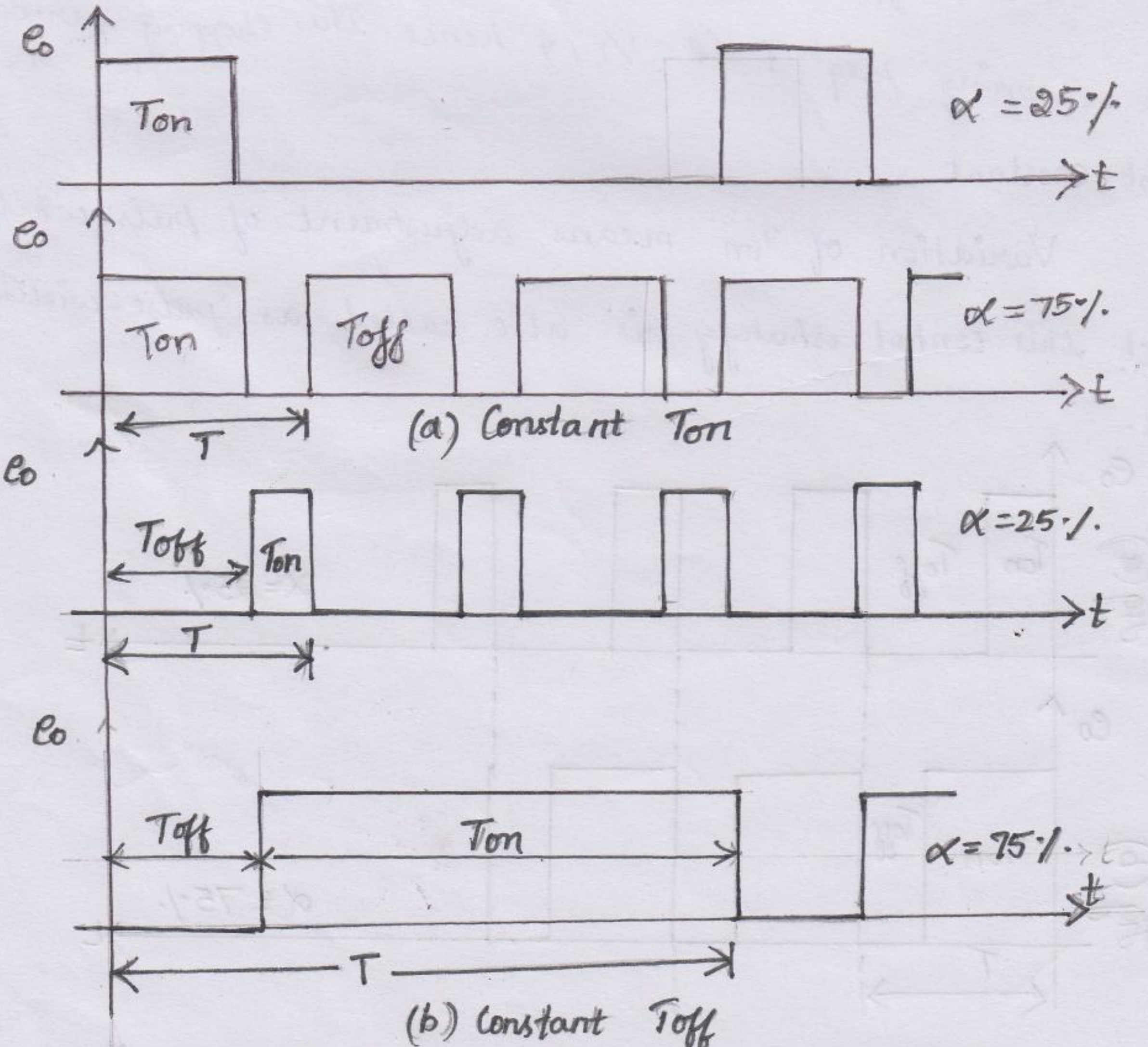
Hence, the o/p vol E_0 can be varied by varying the on-time T_{on}.

Variable freq ctrl.

In this type of ctrl strategy, the chopping frequency f is varied & either

a) On-time T_{on}, is kept constant (or) b) Off time, T_{off} is kept const.

This type of ctrl strategy is also called as freq modulation ctrl.



As shown in fig (a) chopping period T is varied but on-time Ton is kept constant.

The o/p voltage waveforms are shown for two different duty cycles.

In fig (b), chopping period T is varied but Toff is kept constant.

Freq modulation control strategy has the following major disadvantages compared to PWM ctrl.

(i) The chopping freq has to be varied over a wide range for the ctrl of o/p vol in freq modulation. Filter design for such wide freq variation is, therefore, quite difficult.

(ii) For the ctrl of duty cycle, freq variation would be wide. As such, there is a possibility of interference with signalling & telephone lines in freq mody technique.

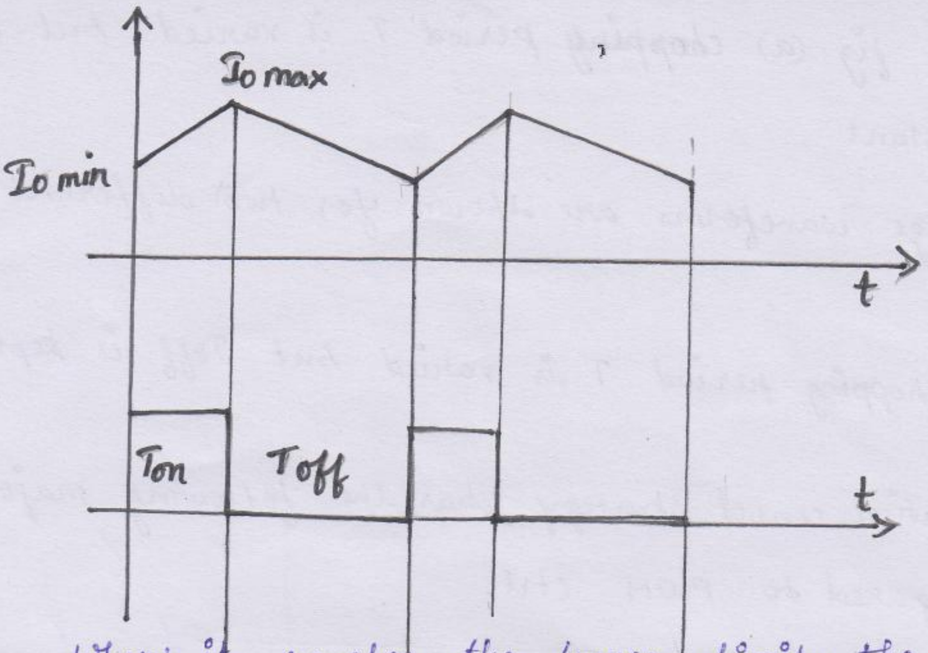
iii) The large off-time in freq mody technique may make the load c/r discontinuous, which is undesirable.

Thus, the constant freq sys (PWM) is preferred scheme for chopper drives.

Current limit control.

In current limit control strategy, the chopper is switched ON and OFF so that the current in the load is maintained b/w two limits.

When the current exceeds upper limit, the chopper is switched OFF, During OFF period, the load c/r freewheels & decreases exponential.



When it reaches the lower limit, the chopper is switch on.

Cr limit ctrl is possible either with const freq or with costant Ton.

The current limit control is used only when the load has energy storage elements.

The reference values are the load current or load-voltage.

The above fig illustrates the principle of cr limit ctrl.

Since, the chopper operates b/w prescribed cr limits, discontinuity cannot occur.

The difference b/w Io max & Io min decides the switching frequency.

The ripple in the load cr can be reduced if the difference b/w the Io max & Io min limits is min.

This in turn ↑s chopper frequency thereby ↑ing the switching losses.

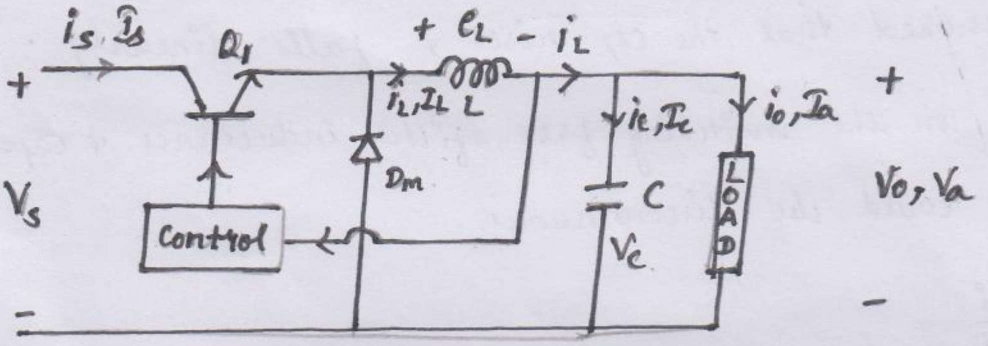
Switching mode regulators

DC converters can be used as switching-mode regulators to convert a dc vol, normally unregulated, to a regulated dc o/p vol.

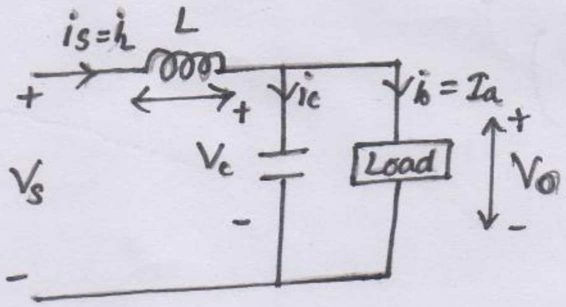
There are 4 basic topologies of switching regulators.

1) Buck regulators
2) Boost regulators
3) Buck-boost regulators
4) Cuk-regulators.

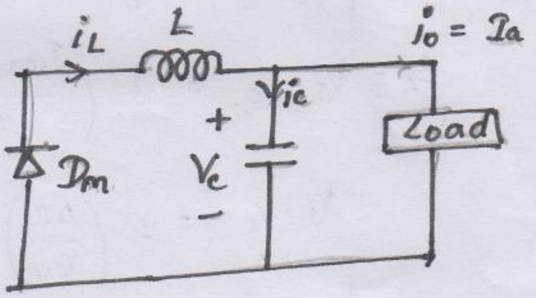
Buck regulators



Mode 1 : Q_1 on



Mode 2 : Q_1-off



In a buck regulator, the avg o/p vol V_a, is less than the i/p vol, V_s & hence the name "buck".

The ckt operation can be divided into two modes.

Mode 1 begins when Q1 is switched on at $t=0$.

The i/p ct which rises, flows through filter inductor L, filter cap C, & load resis R.

Mode 2 begins when Q_1 is switched off at $t = t_1$.

The freewheeling diode D_m, conducts due to energy stored in the inductor; & the inductor c/t continues to flow through L, C, load & diode D_m.

The inductor c/t falls until Q_1 is switched on again in the next cycle.

The waveforms for the vol & ct's are shown in fig for a continuous c/t flow in the inductor L.

It is assumed that the c/t rises & falls linearly.

Depending on the switching freq, filter inductance, & capacitance the inductor c/t could be discontinuous.

$$e_L = L \frac{di}{dt}$$

Voltage across L during ton period of the switch

$$e_L = V_s - V_o = L\left(\frac{I_{max} - I_{min}}{T_{on}}\right)$$

$$V_s - V_o = L\left(\frac{\Delta I}{T_{on}}\right)$$

$$T_{on} = L\left(\frac{\Delta I}{V_s - V_o}\right) \longrightarrow (1)$$

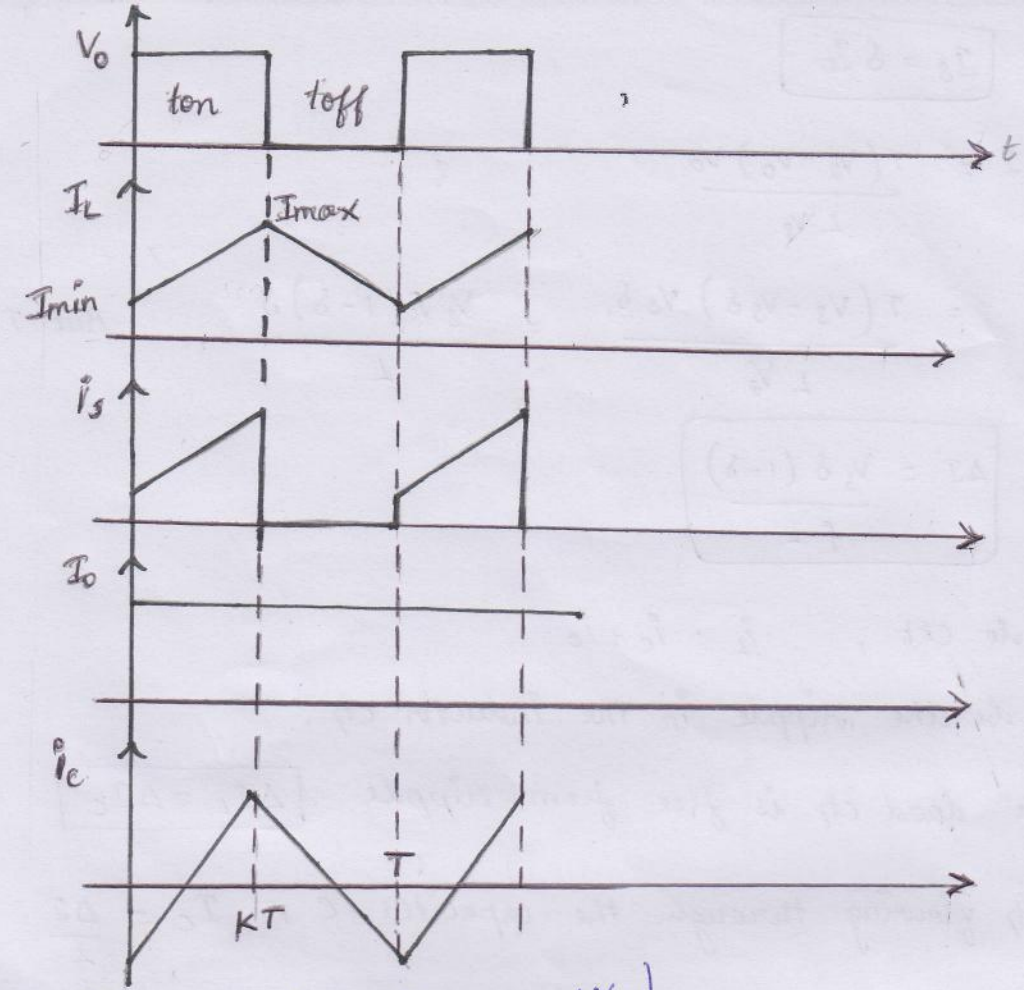
Voltage across L during Toff.

$$e_L = V_o = L\frac{\Delta I}{T_{off}}$$

$$T_{off} = L\left(\frac{\Delta I}{V_o}\right) \longrightarrow (2)$$

from (1) $\Rightarrow$ $$\Delta I = \frac{t_{on}(V_s - V_o)}{L} \longrightarrow (3)$$

from (2) $\Rightarrow$ $$\Delta I = t_{off}\left(\frac{V_o}{L}\right) \longrightarrow (4)$$



③ = ④ $\quad \dfrac{t_{on}(V_s - V_0)}{L} = t_{off}\left(\dfrac{V_0}{L}\right)$

$$t_{on} V_s - t_{on} V_0 = t_{off} V_0$$

$$t_{on} V_s = V_0 (t_{on} + t_{off})$$

$$V_0 = \left(\frac{t_{on}}{t_{on} + t_{off}}\right) V_s \Rightarrow \boxed{V_0 = \delta V_s}$$

Total time period $T = t_{on} + t_{off}$

Add ① & ②

$$T = L\left(\frac{\Delta I}{V_s - V_0}\right) + L\left(\frac{\Delta I}{V_0}\right)$$

$$T = L\Delta I\left(\frac{1}{V_s - V_0} + \frac{1}{V_0}\right)$$

$$\boxed{T = L\Delta I\left[\frac{V_s}{(V_s - V_0)V_0}\right]} \longrightarrow ⑤.$$

Assuming the sys to be loss less,

$$V_s I_s = V_0 I_0.$$

$$V_s I_s = V_s \delta I_0 \qquad (\because V_0 = V_s \delta)$$

$$\boxed{I_s = \delta I_0}$$

from (5)
$$\Delta I = \frac{T(V_s - V_o) V_o}{L V_s}$$

$$= \frac{T(V_s - V_s\delta) V_s \delta}{L V_s} = \frac{V_s T(1-\delta)\delta}{L}$$

But $T = \frac{1}{f}$.

$$\therefore \boxed{\Delta I = \frac{V_s \delta (1-\delta)}{f L}}$$

Apply KCL to ckt, $i_L = i_c + i_o$

Let δI_L be the ripple in the inductor ct.

Assuming the load ct is free from ripple $\boxed{\Delta I_L = \Delta I_C}$

The Avg ct flowing through the capacitor C, $I_C = \frac{\Delta I}{4}$

The vol across the capacitor C

$$V_c = \frac{1}{c}\int_0^{T/2} I_c \, dt + V_c \ (t=0)$$

$$\Delta V_c = V_c - V_c(t=0) = \frac{1}{c}\int_0^{T/2} I_c \, dt.$$

$$\Delta V_c = \frac{1}{c}\frac{\Delta I}{4}\int_0^{T/2} dt.$$

$$= \frac{1}{c}\frac{\Delta I}{4}\left[t\right]_0^{T/2}$$

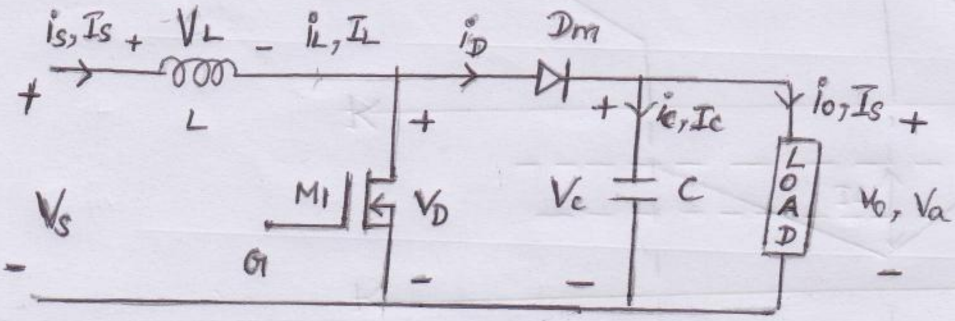
$$\Delta V_c = \frac{1}{c}\frac{\Delta I}{4}\left[T/2\right]$$

$$= \frac{\Delta I}{8C}\left(\frac{1}{f}\right)$$

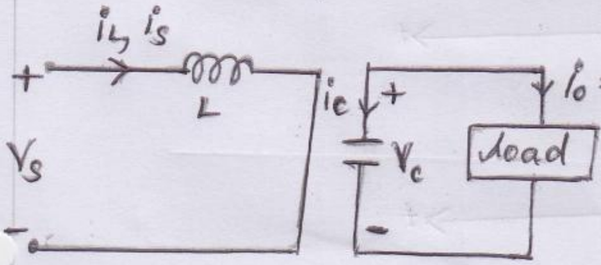
Sub ΔI $\therefore \Delta V_c = \frac{1}{8Cf}\left[\frac{V_s \delta (1-\delta)}{f L}\right]$

$$\boxed{\Delta V_c = \frac{V_s \delta (1-\delta)}{8 c f^2 L}}$$

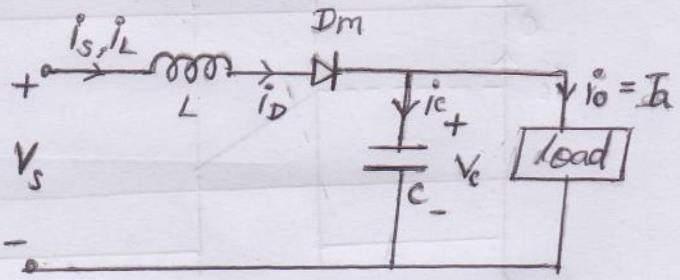
Boost regulators.



Mode 1



Mode 2



In boost regulator, the o/p vol is greater than the i/p vol- hence the name "boost".

The ckt operation can be divided into two modes.

Mode 1 begins when M_1 is switched on at $t = 0$.

The i/p ct, which rises, flows through L & Q_1.

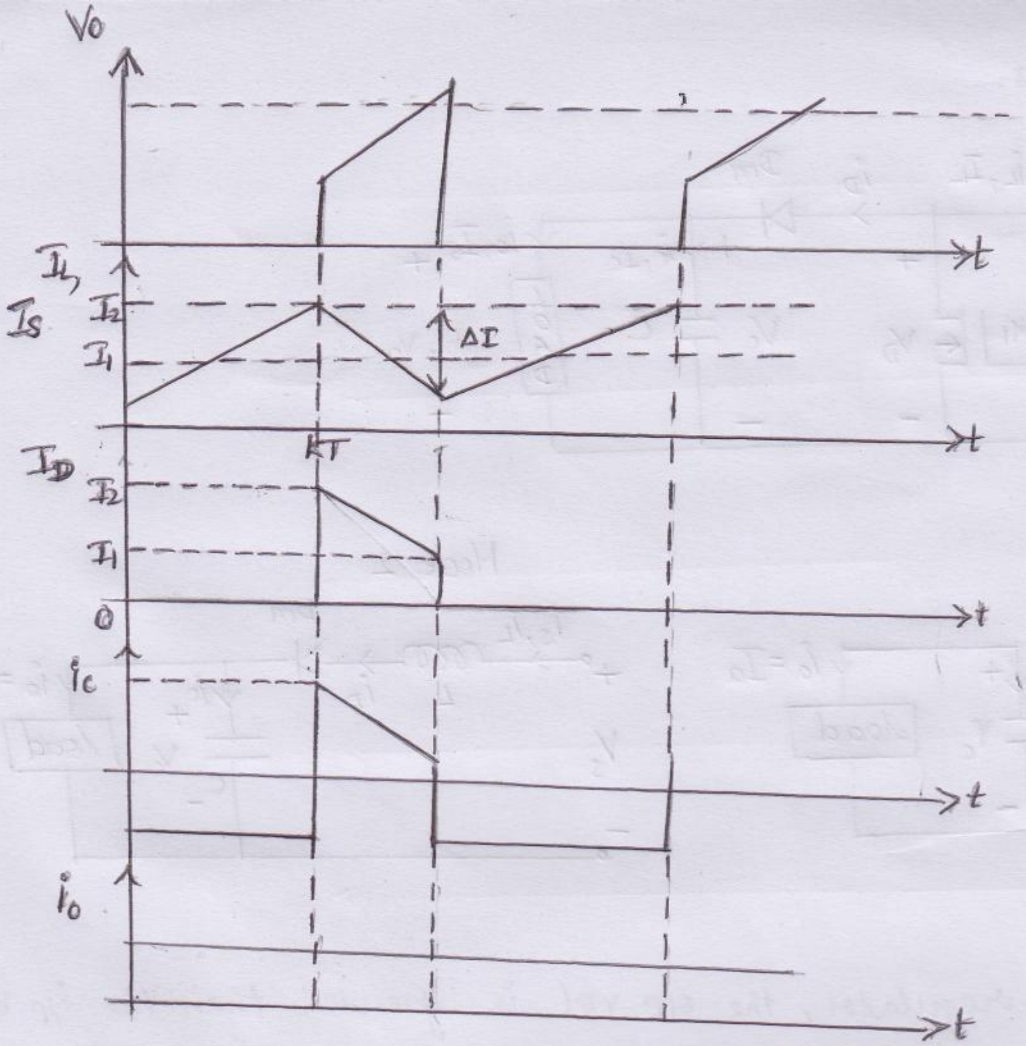
Mode 2 begins when M_1 is switched off at $t = t_1$.

The ct that was flowing through the transistor would now ~~switches off at~~ flow through L, C, load and diode D_m.

The L ct falls until M_1 is turned on again in the next cycle.

The energy stored in L is transferred to the load.

The waveforms for vol & cts are shown in fig for continuous load ct assuming that the ct rises or falls linearly.



Voltage across L $\quad V_L = L\dfrac{di}{dt}$

Vol across L during T_{on} of $T_1 \Rightarrow V_s = L\left(\dfrac{I_{max} - I_{min}}{T_{on}}\right)$

$$= L\frac{\Delta I}{T_{on}}.$$

$$\boxed{T_{on} = \frac{L\,\Delta I}{V_s}} \longrightarrow (1)$$

Vol across L during T_{off} of T_1

$$V_s - V_o = V_L = -L\frac{\Delta I}{T_{off}}.$$

$$T_{off} = \frac{-L\,\Delta I}{V_s - V_o} \longrightarrow (2).$$

From (1) $\quad \Delta I = \dfrac{T_{on}\,V_s}{L} \longrightarrow (3)$

From (2) $\quad \Delta I = -T_{off}\dfrac{(V_s - V_o)}{L} \longrightarrow (4).$

(1) = (2) $\quad T_{on} \frac{V_s}{\cancel{L}} = \frac{-T_{off}(V_s - V_0)}{\cancel{L}}$

$$T_{on} V_s = -T_{off} V_s + T_{off} V_0$$

$$V_s (T_{on} + T_{off}) = V_0 T_{off}$$

$$V_0 = V_s \left(\frac{T_{on} + T_{off}}{T_{off}} \right) = V_s \left(\frac{T}{T_{off}} \right)$$

$$= V_s \left(\frac{T}{T - T_{on}} \right)$$

$$= V_s \left(\frac{1}{\frac{T}{T} - \frac{T_{on}}{T}} \right)$$

$$\boxed{V_0 = V_s \left(\frac{1}{1-\delta} \right)}$$

from (1) & (2).

Total time period $T = T_{on} + T_{off}$

$$T = L\left(\frac{\Delta I}{V_s}\right) + -L\left(\frac{\Delta I}{V_s - V_0}\right)$$

$$= L \Delta I \left(\frac{1}{V_s} - \frac{1}{V_s - V_0} \right)$$

$$= L \Delta I \left[\frac{\cancel{V_s} - V_0 - \cancel{V_s}}{V_s (V_s - V_0)} \right] = L \Delta I \left(\frac{-V_0}{V_s (V_s - V_0)} \right)$$

$$T = \frac{1}{f} = \frac{L \Delta I V_0}{V_s (V_0 - V_s)}$$

Sub $V_0 = \frac{V_s}{1-\delta}$ $\quad \therefore T = \frac{L \Delta I \left(\frac{V_s}{1-\delta}\right)}{V_s \left[\left(\frac{V_s}{1-\delta}\right) - V_s\right]} = \frac{L \Delta I \left[\frac{V_s}{1-\delta}\right]}{V_s^2 \left[\frac{1}{1-\delta} - 1\right]}$

$$= \frac{L \Delta I}{(1-\delta) V_s \left[\frac{1-1+\delta}{1-\delta}\right]} = \frac{L \Delta I}{(\cancel{1-\delta}) V_s \left(\frac{\delta}{\cancel{1-\delta}}\right)}$$

$$\therefore \quad T = \frac{1}{f} = \frac{L\,\Delta I}{V_s\,\delta},$$

$$\boxed{\Delta I = \frac{V_s\,\delta}{L f}}$$

Assuming the system to be loss less $\quad V_s I_s = V_0 I_0$

$$\cancel{V_s} I_s = \left(\frac{\cancel{V_s}}{1-\delta}\right) I_0$$

$$\boxed{I_s = \frac{I_0}{1-\delta}}$$

Capacitor vol

$$V_c = \frac{1}{C}\int_0^{t_{on}} i_c\,dt + V_c(t=0)$$

$\because$ the capacitor is initially in charged condn.

$$V_c - V_c(t=0) = \frac{1}{C}\int_0^{t_{on}} i_c\,dt$$

$$\Delta V_c = \frac{1}{C}\int_0^{t_{on}} i_c\,dt$$

$$= \frac{1}{C}\int_0^{t_{on}} I_0\,dt = \frac{I_0}{C}\Big[t\Big]_0^{t_{on}}$$

During Ton, capacitor is supplying ch to load. $\therefore \boxed{i_0 = i_c}$

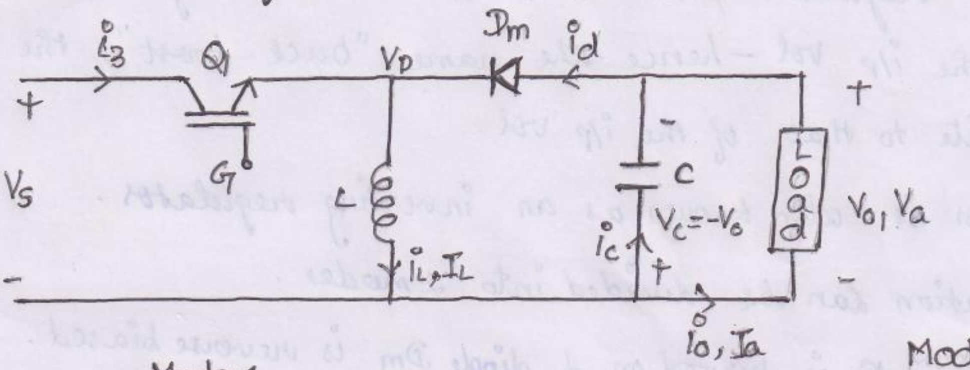
$$\Delta V_c = \frac{I_0}{C}\Big[t\Big]_0^{t_{on}} = \frac{I_0}{C}\left[t_{on}\right]$$

substitute ton $\Rightarrow \Delta V_c = \frac{I_0}{C}\left[\frac{L\,\Delta I}{V_s}\right]$

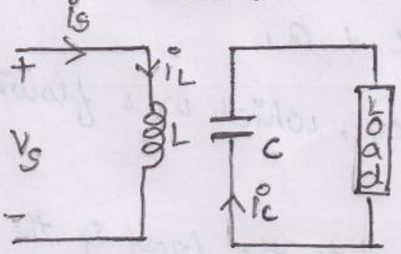
sub ΔI $\Rightarrow \quad \Delta V_c = \frac{I_0}{C}\,\frac{L}{V_s}\left[\frac{V_s\,\delta}{L f}\right]$

$$\boxed{\Delta V_c = \frac{I_0\,\delta}{C f}}$$

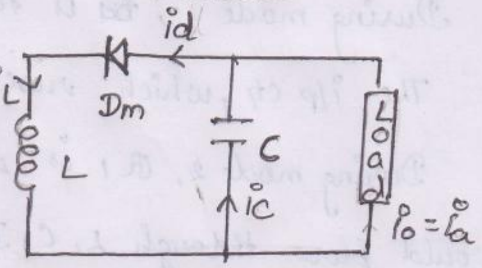
Buck-boost Regulators.

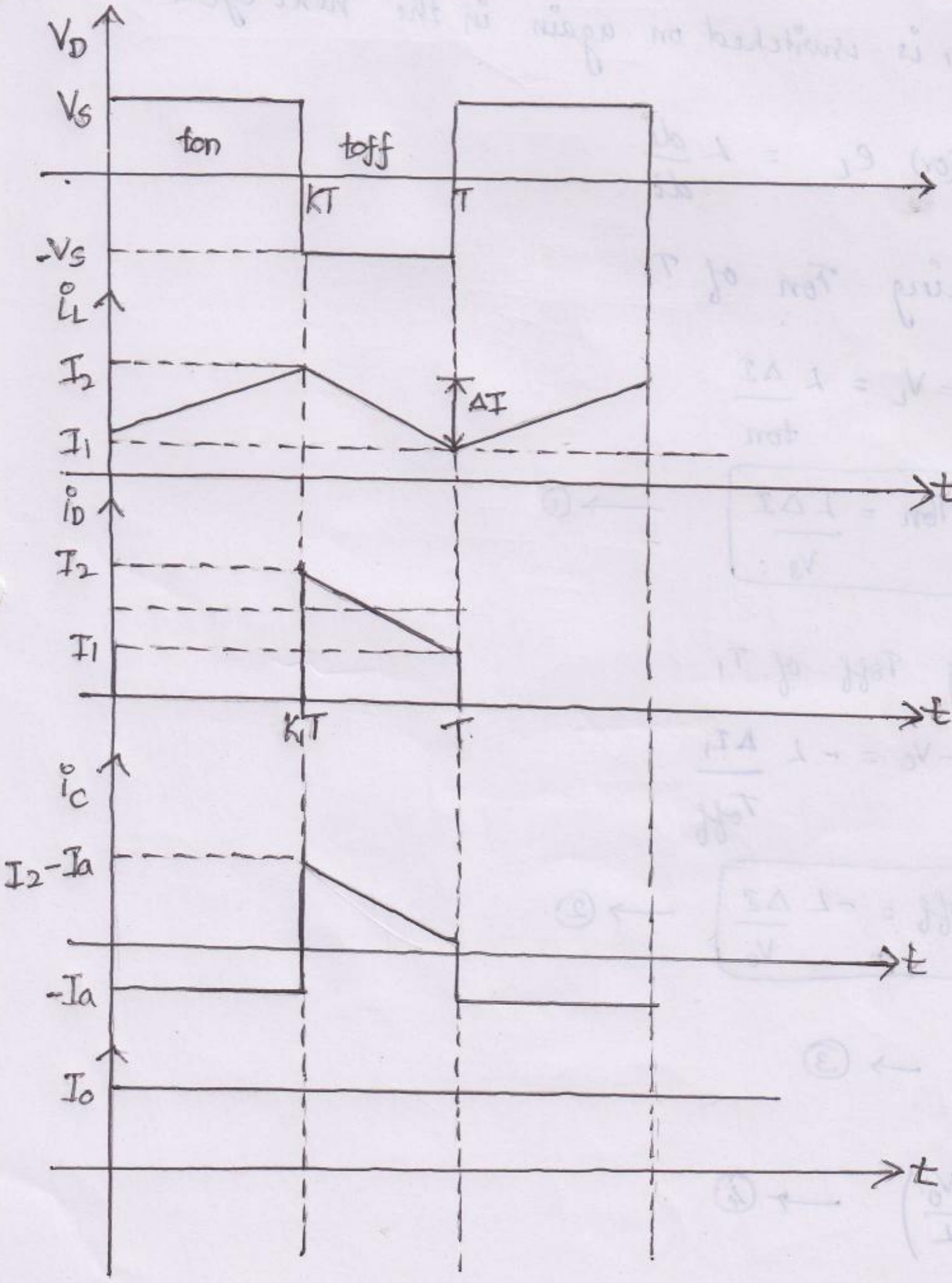


Mode 1



Mode 2





A buck-boost regulator provides an o/p vol that may be less than or greater than the i/p vol - hence the name "buck-boost"; the o/p vol polarity is opposite to that of the i/p vol.

This regulator is also known as an inverting regulator.

The ckt operation can be divided into 2 modes.

During mode 1, Q1 is turned on & diode Dm is reverse biased.

The i/p ct, which rises, flows through L & Q1.

During mode 2, Q1 is switched off & the ct, which was flowing through L, would flow through L, C, Dm & the load.

The energy stored in L would be transferred to the load & the inductor ct would fall until Q1 is switched on again in the next cycle.

Vol across L $\quad V_L$ (or) $e_L = L\frac{di}{dt}$.

Vol across L during Ton of T_1

$$V_s = V_L = L\frac{\Delta I}{t_{on}}$$

$$\boxed{T_{on} = \frac{L\Delta I}{V_s}} \longrightarrow (1)$$

Vol across L during Toff of T_1

$$V_o = -V_c = -L\frac{\Delta I_1}{T_{off}}$$

$$\boxed{T_{off} = -L\frac{\Delta I}{V_o}} \longrightarrow (2)$$

$$(1) \Rightarrow \quad \Delta I = \frac{V_s}{L} t_{on} \longrightarrow (3)$$

$$(2) \Rightarrow \quad \Delta I = T_{off}\left(\frac{-V_o}{L}\right) \longrightarrow (4)$$

③ = ④

$$\frac{V_s}{L} t_{on} = T_{off}\left(\frac{-V_0}{L}\right)$$

$$V_0 = -V_s\left(\frac{T_{on}}{T_{off}}\right)$$

$$= -V_s\left(\frac{t_{on}}{T - t_{on}}\right)$$

$$= -V_s\left(\frac{1}{\frac{T}{t_{on}} - 1}\right) = -V_s\left(\frac{1}{\frac{1}{\delta} - 1}\right)$$

$$\boxed{V_0 = -V_s\left(\frac{\delta}{1-\delta}\right)}$$

Total time period $T = T_{on} + T_{off}$.

$$= L\frac{\Delta I}{V_s} + \frac{-L\Delta I}{V_0}$$

$$= L\Delta I\left(\frac{1}{V_s} - \frac{1}{V_0}\right)$$

$$\frac{1}{f} = T = L\Delta I\left[\frac{V_0 - V_s}{V_0 V_s}\right]$$

put $V_0 = \dfrac{-V_s\delta}{1-\delta}$

$$\therefore T = \frac{1}{f} = L\Delta I\left(\frac{\frac{-V_s\delta}{1-\delta} - V_s}{\left(\frac{-V_s\delta}{1-\delta}\right)V_s}\right)$$

$$= L\Delta I\left(\frac{-V_s\left[\left(\frac{\delta}{1-\delta}\right)+1\right]}{\frac{-V_s^2\delta}{1-\delta}}\right)$$

$$= L\Delta I \left[\frac{1 + \frac{\delta}{1-\delta}}{\frac{\cancel{V_s}\,\delta}{1-\delta}}\right]$$

$$= L\Delta I \left[\frac{\frac{1-\cancel{\delta}+\cancel{\delta}}{\cancel{1-\delta}}}{\frac{V_s\,\delta}{\cancel{1-\delta}}}\right]$$

$$\boxed{T = L\Delta I \left[\frac{1}{V_s\,\delta}\right]}$$

Sub $T = \frac{1}{f}$ $\Rightarrow$ $\frac{1}{f} = L\Delta I \left(\frac{1}{V_s\,\delta}\right)$

$$\boxed{\Delta I = \frac{V_s\,\delta}{Lf}}$$

Assuming the system to be loss less

$$V_s I_s = -V_o I_o$$

$$\cancel{V_s} I_s = \cancel{+}\left[\frac{\cancel{+V_s}\,\delta}{1-\delta}\right] I_o.$$

$$\boxed{I_s = \left(\frac{\delta}{1-\delta}\right) I_o}$$

Capacitor voltage

$$V_c = \frac{1}{c}\int_0^{t_{on}} i_c\,dt + V_c(t=0)$$

$$V_c - V_c(t=0) = \frac{1}{c}\int_0^{t_{on}} i_c\,dt$$

$$\Delta V_c = \frac{I_c}{c}\left[t_{on}\right]$$

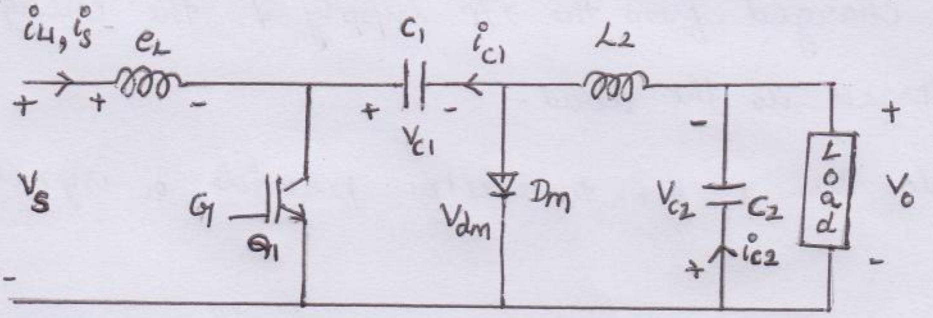
substitute t_{on} $\quad \Delta V_c = \frac{I_c}{c}\left[\frac{L\,\Delta I}{V_s}\right]$

sub ΔI $\quad \Delta V_c = \frac{I_c}{c}\left[\frac{L}{V_s}\left(\frac{V_s\,\delta}{Lf}\right)\right]$

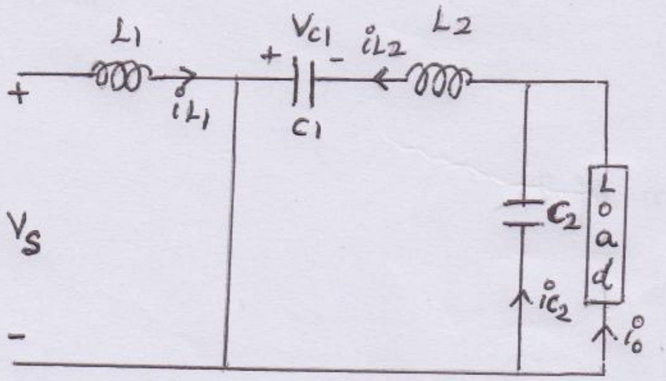
$$\Delta V_c = \frac{I_c}{c}\left(\frac{\delta}{f}\right) \qquad \text{Put } I_c = -I_o$$

$$\therefore \boxed{\Delta V_c = -\frac{I_o\,\delta}{cf}}$$

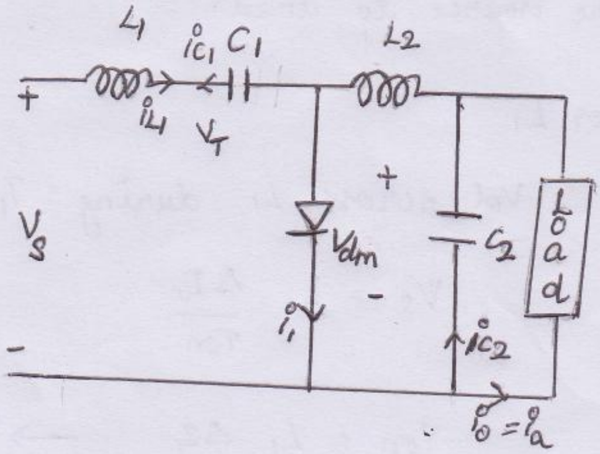
Cuk Regulator.



Mode 1 :



Mode 2 :



The ckt arrangement of the cuk regulator using a power BJT is shown in fig.

Illr to the buck-boost regulator, the cuk regulator provides an o/p vol that is less than or greater than the i/p vol, but the o/p vol polarity is opposite to that of the i/p vol.

When the i/p vol is turned on & Q_1 is switched off, diode D_m is frwd biased & C_1 is charged through L_1, D_m, & the i/p supply V_s.

The ckt operation can be divided into 2 modes.

Mode 1 begins when Q_1 is turned on at $t = 0$.

The Ct through L_1 rises.

At the same time, the vol of C_1 reverse biases diode D_m turns it off.

The capacitor C_1 discharges its energy to the ckt formed by C_1, C_2, the load & L_2.

Mode 2 begins when Q_1 is turned off at $t = t_1$.

The C_1 is charged from the i/p supply & the energy stored in L_2 is transferred to the load.

The diode D_m & Q_1 transistor provide a synchronous switching action.

The capacitor C_1 is the medium for transferring energy from the source to load.

For L_1

Vol across L_1 during T_1 on of T_1

$$V_S = L_1 \frac{\Delta I_1}{T_{on}}.$$

$$T_{on} = L_1 \frac{\Delta I_1}{V_S} \longrightarrow (1)$$

During T_{off} Ct through L falls from I_{max1} to I_{min1} & voltage across L during T_{off} is $V_S - V_{C1}$

$$V_s - V_{C1} = -L_1 \frac{\Delta I_1}{T_{off}}$$

$$T_{off} = -\left(\frac{L_1 \Delta I_1}{V_s - V_{C1}}\right) \rightarrow (2).$$

$(1) \Rightarrow \quad \Delta I_1 = \dfrac{T_{on} V_s}{L_1} \longrightarrow (a)$

$(2) \Rightarrow \quad \Delta I_1 = \dfrac{T_{off}(V_{C1} - V_s)}{L_1} \longrightarrow (b)$

$(a) = (b) \Rightarrow$ Avg voltage across capacitor C_1

$$\Delta I_1 = V_s t_{on} = (V_{C1} - V_s) t_{off}$$

$$V_s t_{on} = V_{C1} t_{off} - V_s t_{off}$$

$$V_s(t_{on} + t_{off}) = V_{C1} t_{off}$$

$$V_{C1} = V_s\left(\frac{T_{on} + T_{off}}{T_{off}}\right)$$

$$= V_s\left(\frac{T}{T - T_{on}}\right)$$

$$= V_s\left(\frac{1}{\frac{T}{T} - \frac{T_{on}}{T}}\right)$$

$$= V_s\left(\frac{1}{1 - \frac{T_{on}}{T}}\right)$$

$$\boxed{V_{C1} = V_s\left(\frac{1}{1-\delta}\right)} \longrightarrow (3).$$

During Ton of T_1, ct through L_2 rises from I_{min2} to I_{max2}

i.e. $\Delta I_2 = I_{max2} - I_{min2}$.

Voltage across L_2 during on period of T_1

Note. $\boxed{-V_{C2} = V_0}$

$$V_{C1} - V_{C2} = L_2 \frac{\Delta I_2}{t_{on}}.$$

$$T_{on} = \frac{\Delta I_2 L_2}{V_{C1} + V_0} \rightarrow (4).$$

During Toff ct through the L falls from I_{max2} to I_{min2}

$$V_0 = -L_2 \frac{\Delta I_2}{T_{off}}$$

$$T_{off} = \frac{-L_2 \Delta I_2}{V_0} \rightarrow (5).$$

$(4) \Rightarrow \quad \Delta I_2 = \frac{T_{on}(V_{C1} + V_0)}{L_2}$

$(5) \Rightarrow \quad \Delta I_2 = \frac{-T_{off} V_0}{L_2}$

$(4) = (5) \quad \frac{T_{on}(V_{C1} + V_0)}{L_2} = \frac{-T_{off} V_0}{L_2}$

$$T_{on} V_{C1} + T_{on} V_0 = -T_{off} V_0$$

$$V_0(T_{on} + T_{off}) = -T_{on} V_{C1}$$

$$V_{C1} = \frac{-V_0 (T_{on} + T_{off})}{T_{on}}.$$

$$V_{C1} = -V_o\left(\frac{T}{T_{on}}\right),$$

$$= -V_o\left(\frac{1}{\frac{T_{on}}{T}}\right)$$

$$= -V_o/\delta$$

$$\therefore \boxed{V_{C1} = \frac{-V_o}{\delta}} \longrightarrow (6)$$

from (3) & (6)

$$V_s\left(\frac{1}{1-\delta}\right) = -\frac{V_o}{\delta}$$

$$\boxed{V_o = \frac{-V_s\,\delta}{1-\delta}} \longrightarrow (7)$$

Considering the sys to be loss less

$$V_s I_s = -V_o I_o.$$

sub V_o

$$V_s I_s = -\left(\frac{-V_s\,\delta}{1-\delta}\right) I_o$$

$$\boxed{I_s = \frac{I_o\,\delta}{1-\delta}} \longrightarrow (8)$$

Total time period for L_1 $\quad T = t_{on} + t_{off}$

$$= \frac{L_1\,\Delta I_1}{V_s} + \frac{-L_1\,\Delta I_1}{V_s - V_{C1}}.$$

$$= L_1\,\Delta I_1\left(\frac{1}{V_s} - \frac{1}{V_s - V_{C1}}\right)$$

$$= L_1\,\Delta I_1\left(\frac{V_s - V_{C1} - V_s}{V_s\,(V_s - V_{C1})}\right)$$

$$\boxed{\frac{1}{f} = T = L_1 \Delta I_1 \left(\frac{-V_{C1}}{V_S (V_S - V_{C1})} \right)}$$

$$\Rightarrow \Delta I_1 = \frac{-V_S (V_S - V_{C1})}{f V_{C1} L_1}$$

$$\Delta I_1 = \frac{V_S (V_{C1} - V_S)}{f V_{C1} L_1}$$

but $V_{C1} = \frac{V_S}{1-\delta}$ $\therefore$ sub V_{C1} $\Rightarrow$ $\Delta I_1 = \frac{V_S \left(\frac{V_S}{1-\delta} - V_S \right)}{f \left(\frac{V_S}{1-\delta} \right) L_1}$

$$\Delta I_1 = \frac{V_S \delta}{f L_1}$$

peak to peak ripple c/t in L_1 $\Delta I_1 = \frac{V_S}{f L_1} \longrightarrow (10)$.

Total time for L_2 $T = T_{on} + T_{off}$

$$= \frac{\Delta I_2 L_2}{V_{C1} + V_0} - \frac{L_2 \Delta I_2}{V_0}$$

$$= L_2 \Delta I_2 \left[\frac{1}{V_{C1} + V_0} - \frac{1}{V_0} \right]$$

$$= L_2 \Delta I_2 \left(\frac{V_0 - V_{C1} - V_0}{V_0 (V_{C1} + V_0)} \right)$$

$$\boxed{\frac{1}{f} = T = \frac{-L_2 \Delta I_2 V_{C1}}{V_0 (V_{C1} + V_0)}} \longrightarrow (11).$$

peak to peak ripple c/t in L_2

(11) $\Rightarrow$ $$\Delta I_2 = \frac{-V_0(V_{C1}+V_0)}{L_2 V_{C1} f}$$

sub (3) & (4)

$$\Delta I_2 = \frac{-\left[\frac{V_s}{1-\delta} + \left(\frac{-V_s\delta}{1-\delta}\right)\right]\left[\frac{-V_s\delta}{1-\delta}\right]}{L_2\left[\frac{V_s}{1-\delta}\right]f}$$

$$= \frac{-1}{L_2 f}\left(\frac{\left(\frac{V_s}{1-\delta} - \frac{V_s\delta}{1-\delta}\right)\left(\frac{-V_s\delta}{1-\delta}\right)}{\frac{V_s}{1-\delta}}\right)$$

$$= \frac{-1}{L_2 f}\left(\frac{\left(\frac{-V_s\delta}{1-\delta}\right) V_s\left(\frac{1-\delta}{1-\delta}\right)}{\frac{V_s}{1-\delta}}\right)$$

$$= \frac{-1}{L_2 f}\left[-V_s\delta\right]$$

$$\boxed{\Delta I_2 = \frac{V_s\delta}{L_2 f}} \longrightarrow (12)$$

When T_1 is off, C_1 is charged by i/p c/t for $T = T_{off}$.

Avg charging c/t $I_C = I_s$

peak to peak ripple voltage of C_1

$$\Delta V_{C1} = \frac{1}{C_1}\int_0^{T_{off}} I_C\, dt = \frac{1}{C_1}\int_0^{T_{off}} I_s\, dt = \frac{I}{C_1}\left[T_{off}\right]$$

Sub ② $\therefore \Delta V_{C1} = \frac{I_s}{C_1}\left[\frac{-L_1 \Delta I_1}{V_s - V_{C1}}\right]$

Sub the value of V_{C1} ③.

$$\Delta V_{C1} = \frac{-I_s L_1 \Delta I_1}{C_1\left[V_s - \frac{V_s}{1-\delta}\right]} = \frac{-I_s L_1 \Delta I_1}{C_1 V_s\left[\frac{1-\delta-1}{1-\delta}\right]} = \frac{-I_s L_1 \Delta I_1}{-C_1 V_s\left(\frac{\delta}{1-\delta}\right)}$$

Sub ΔI_1 ⑩

$$\Delta V_{C1} = \frac{I_s L_1 (1-\delta)}{C_1 V_s \delta}\left[\frac{V_s \delta}{L_1 f}\right]$$

$$\boxed{\Delta V_{C1} = \frac{I_s (1-\delta)}{C_1 f}} \longrightarrow ⑬.$$

If we assume that load ripple c_t. So I_o is negligible.

$\therefore \Delta I_{L2} = \Delta I_{C2} + \cancel{I_o}^{0} \Rightarrow \boxed{\Delta I_{L2} = \Delta I_{C2}}$

Average charging c_t of capacitor C_2 which flows for time $T/2$ becomes $I_{C2} = \frac{\Delta I_2}{4}$

Peak to peak ripple voltage across C_2

$$\Delta V_{C2} = \frac{1}{C_2}\int_0^{T/2} I_{C2}\, dt = \frac{1}{C_2}\int_0^{T/2} \frac{\Delta I_2}{4}\, dt = \frac{\Delta I_2}{4 C_2}\left[\frac{T}{2}\right]$$

$$= \frac{\Delta I_2}{8 C_2}\left[\frac{1}{f}\right]$$

But $\Delta I_2 = \frac{V_s \delta}{L_2 f}$ $\therefore \Delta V_{C2} = \frac{1}{8 f C_2}\left(\frac{V_s \delta}{L_2 f}\right)$

$$\boxed{\Delta V_{C2} = \frac{V_s \delta}{8 f^2 L_2 C_2}} \longrightarrow ⑭.$$