

THREE PHASE CONTROLLED RECTIFIERS

INTRODUCTION TO 3-PHASE CONTROLLED RECTIFIERS

Single phase half controlled bridge converters & fully controlled bridge converters are used extensively in industrial applications up to about 15kW of output power. The single phase controlled rectifiers provide a maximum dc output of $V_{dc(max)} = \frac{2V_m}{\pi}$.

The output ripple frequency is equal to the twice the ac supply frequency. The single phase full wave controlled rectifiers provide two output pulses during every input supply cycle and hence are referred to as two pulse converters.

Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load.

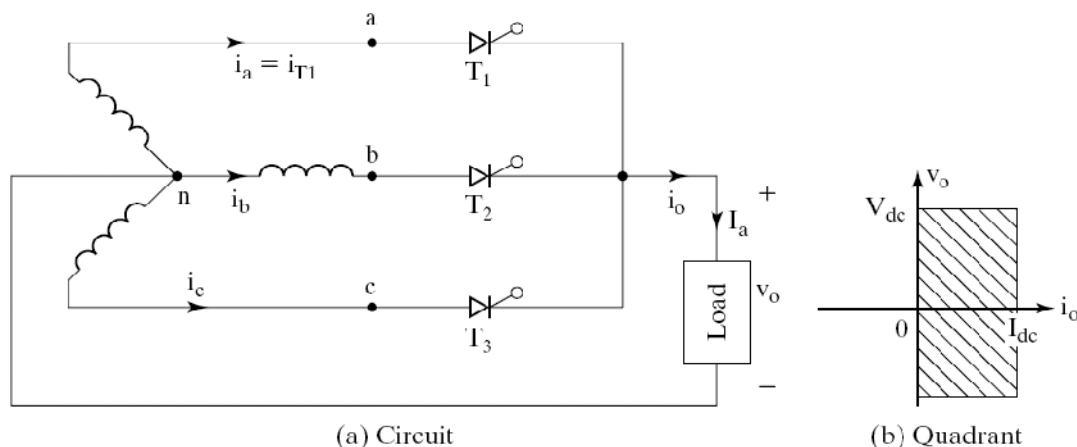
Features of 3-phase controlled rectifiers are

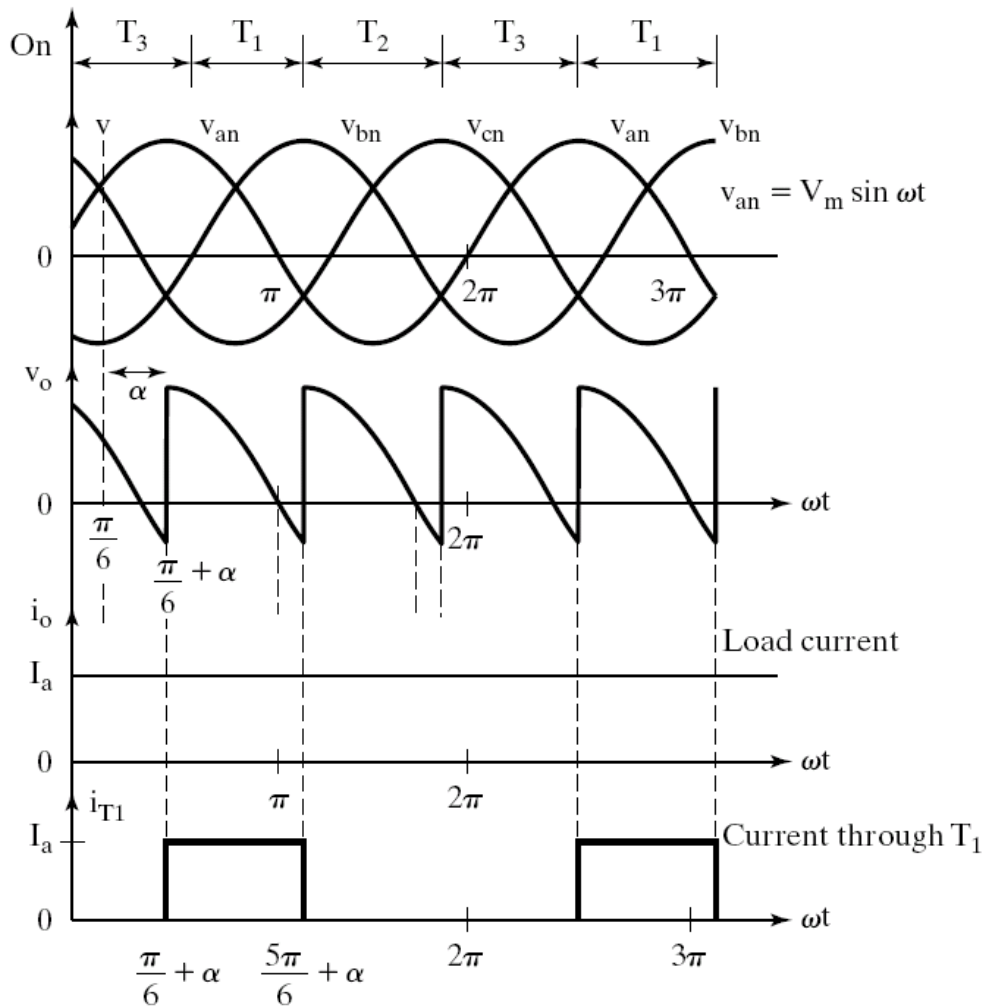
- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage and higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current

Three phase controlled rectifiers are extensively used in high power variable speed industrial dc drives.

3-PHASE HALF WAVE CONVERTER

Three single phase half-wave converters are connected together to form a three phase half-wave converter as shown in the figure.





(c) For inductive load

THEE PHASE SUPPLY VOLTAGE EQUATIONS

We define three line neutral voltages (3 phase voltages) as follows

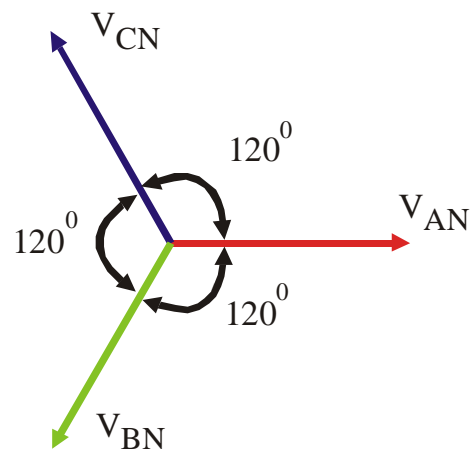
$$v_{RN} = v_{an} = V_m \sin \omega t; \quad V_m = \text{Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right)$$

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - 120^\circ \right)$$

$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right)$$

$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + 120^\circ \right)$$



Vector diagram of 3-phase supply voltages

$$v_{BN} = v_{cn} = V_m \sin(\omega t - 240^\circ)$$

The 3-phase half wave converter combines three single phase half wave controlled rectifiers in one single circuit feeding a common load. The thyristor T_1 in series with one of the supply phase windings 'a-n' acts as one half wave controlled rectifier. The second thyristor T_2 in series with the supply phase winding 'b-n' acts as the second half wave controlled rectifier. The third thyristor T_3 in series with the supply phase winding 'c-n' acts as the third half wave controlled rectifier.

The 3-phase input supply is applied through the star connected supply transformer as shown in the figure. The common neutral point of the supply is connected to one end of the load while the other end of the load connected to the common cathode point.

When the thyristor T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right) = (30^\circ + \alpha)$, the phase voltage v_{an} appears across the load when T_1 conducts. The load current flows through the supply phase winding 'a-n' and through thyristor T_1 as long as T_1 conducts.

When thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right) = (150^\circ + \alpha)$, T_1 becomes reverse biased and turns-off. The load current flows through the thyristor T_2 and through the supply phase winding 'b-n'. When T_2 conducts the phase voltage v_{bn} appears across the load until the thyristor T_3 is triggered.

When the thyristor T_3 is triggered at $\omega t = \left(\frac{3\pi}{2} + \alpha\right) = (270^\circ + \alpha)$, T_2 is reversed biased and hence T_2 turns-off. The phase voltage v_{cn} appears across the load when T_3 conducts.

When T_1 is triggered again at the beginning of the next input cycle the thyristor T_3 turns off as it is reverse biased naturally as soon as T_1 is triggered. The figure shows the 3-phase input supply voltages, the output voltage which appears across the load, and the load current assuming a constant and ripple free load current for a highly inductive load and the current through the thyristor T_1 .

For a purely resistive load where the load inductance 'L = 0' and the trigger angle $\alpha > \left(\frac{\pi}{6}\right)$, the load current appears as discontinuous load current and each thyristor is naturally commutated when the polarity of the corresponding phase supply voltage reverses. The frequency of output ripple frequency for a 3-phase half wave converter is $3f_s$, where f_s is the input supply frequency.

The 3-phase half wave converter is not normally used in practical converter systems because of the disadvantage that the supply current waveforms contain dc components (i.e., the supply current waveforms have an average or dc value).

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

The reference phase voltage is $v_{RN} = v_{an} = V_m \sin \omega t$. The trigger angle α is measured from the cross over points of the 3-phase supply voltage waveforms. When the phase supply voltage v_{an} begins its positive half cycle at $\omega t = 0$, the first cross over point appears at

$$\omega t = \left(\frac{\pi}{6}\right) \text{ radians} = 30^\circ.$$

The trigger angle α for the thyristor T_1 is measured from the cross over point at $\omega t = 30^\circ$. The thyristor T_1 is forward biased during the period $\omega t = 30^\circ$ to 150° , when the phase supply voltage v_{an} has a higher amplitude than the other phase supply voltages. Hence T_1 can be triggered between 30° to 150° . When the thyristor T_1 is triggered at a trigger angle α , the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} v_o \cdot d(\omega t) \right]$$

Output voltage $v_o = v_{an} = V_m \sin \omega t$ for $\omega t = (30^\circ + \alpha)$ to $(150^\circ + \alpha)$

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} V_m \sin \omega t \cdot d(\omega t) \right]$$

As the output load voltage waveform has three output pulses during the input cycle of 2π radians

$$V_{dc} = \frac{3V_m}{2\pi} \left[\int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} \sin \omega t \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\frac{-\cos \omega t}{\frac{\pi}{6} + \alpha} \right]_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha}$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos \left(\frac{5\pi}{6} + \alpha \right) + \cos \left(\frac{\pi}{6} + \alpha \right) \right]$$

Note from the trigonometric relationship

$$\cos(A + B) = (\cos A \cdot \cos B - \sin A \cdot \sin B)$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos \left(\frac{5\pi}{6} \right) \cos(\alpha) + \sin \left(\frac{5\pi}{6} \right) \sin(\alpha) + \cos \left(\frac{\pi}{6} \right) \cdot \cos(\alpha) - \sin \left(\frac{\pi}{6} \right) \sin(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos(150^\circ) \cos(\alpha) + \sin(150^\circ) \sin(\alpha) + \cos(30^\circ) \cdot \cos(\alpha) - \sin(30^\circ) \sin(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos(180^\circ - 30^\circ) \cos(\alpha) + \sin(180^\circ - 30^\circ) \sin(\alpha) + \cos(30^\circ) \cdot \cos(\alpha) - \sin(30^\circ) \sin(\alpha) \right]$$

Note: $\cos(180^\circ - 30^\circ) = -\cos(30^\circ)$

$$\sin(180^\circ - 30^\circ) = \sin(30^\circ)$$

Therefore

$$V_{dc} = \frac{3V_m}{2\pi} \left[+\cos(30^\circ) \cos(\alpha) + \sin(30^\circ) \sin(\alpha) + \cos(30^\circ) \cdot \cos(\alpha) - \sin(30^\circ) \sin(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[2 \cos(30^\circ) \cos(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[2 \times \frac{\sqrt{3}}{2} \cos(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\sqrt{3} \cos(\alpha) \right] = \frac{3\sqrt{3}V_m}{2\pi} \cos(\alpha)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi} \cos(\alpha)$$

Where

$V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage for a 3-phase star connected transformer.

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3} V_m}{2\pi} \text{ Where } V_m \text{ is the peak phase voltage.}$$

And the normalized average output voltage is $V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos \alpha$

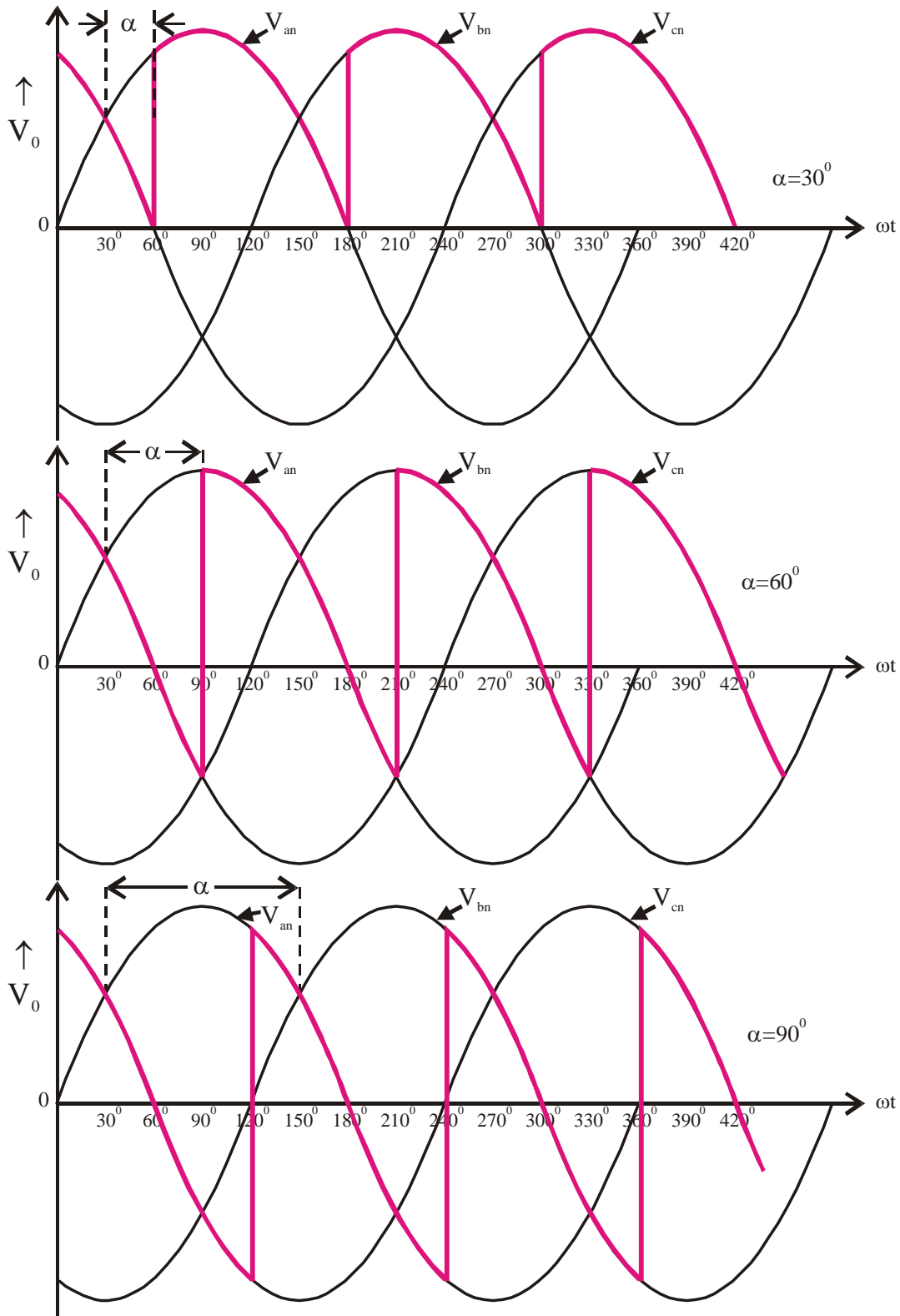
TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

The rms value of output voltage is found by using the equation

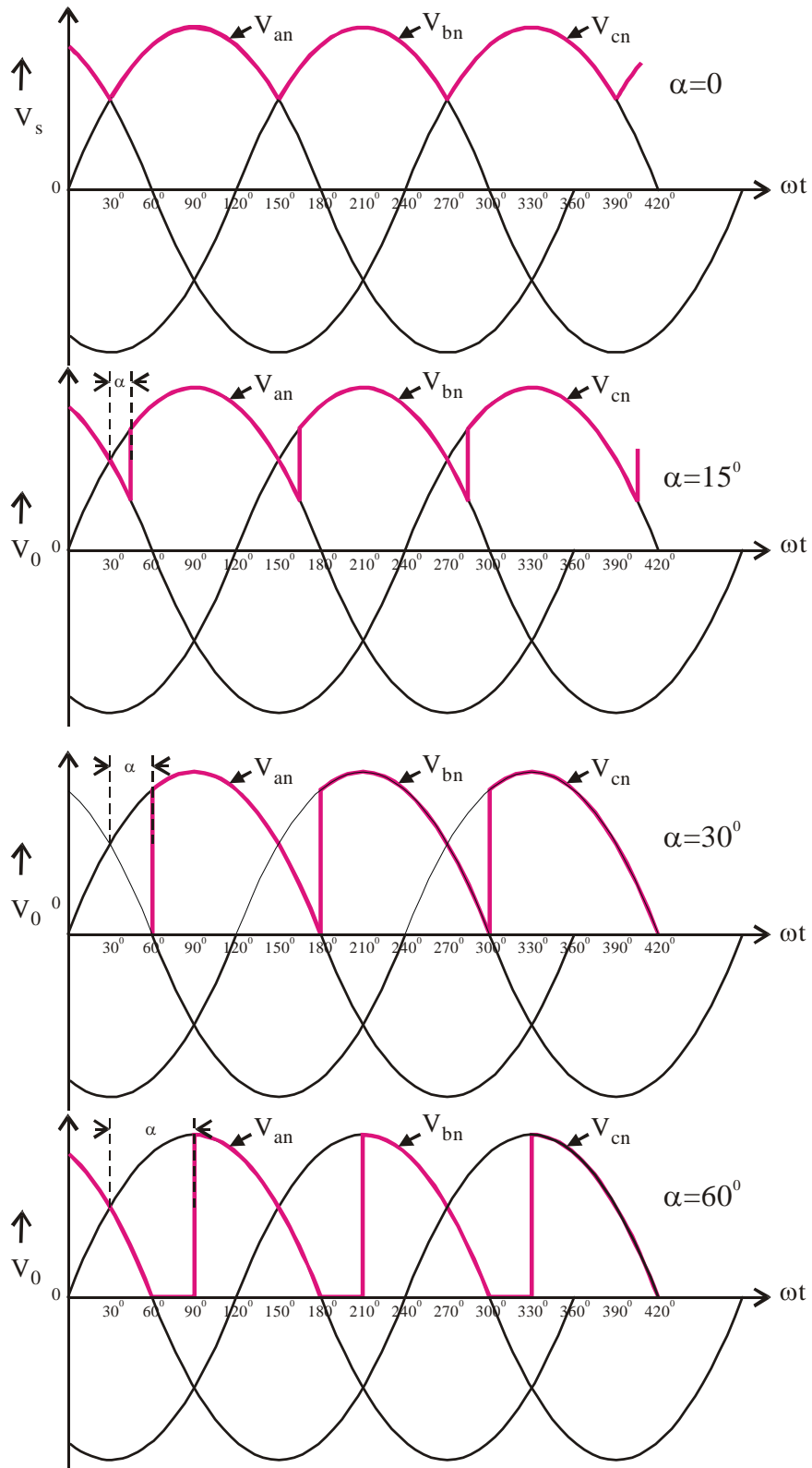
$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{\frac{1}{2}}$$

and we obtain $V_{O(RMS)} = \sqrt{3}V_m \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$

3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH RL LOAD



3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH R LOAD



TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF A 3 PHASE HALF WAVE CONVERTER WITH RESISTIVE LOAD OR RL LOAD WITH FWD.

In the case of a three-phase half wave controlled rectifier with resistive load, the thyristor T_1 is triggered at $\omega t = (30^\circ + \alpha)$ and T_1 conducts up to $\omega t = 180^\circ = \pi$ radians. When the phase supply voltage v_{an} decreases to zero at $\omega t = \pi$, the load current falls to zero and the thyristor T_1 turns off. Thus T_1 conducts from $\omega t = (30^\circ + \alpha)$ to (180°) .

Hence the average dc output voltage for a 3-pulse converter (3-phase half wave controlled rectifier) is calculated by using the equation

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\alpha+30^\circ}^{180^\circ} v_o \cdot d(\omega t) \right]$$

$$v_o = v_{an} = V_m \sin \omega t; \text{ for } \omega t = (\alpha + 30^\circ) \text{ to } (180^\circ)$$

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\alpha+30^\circ}^{180^\circ} V_m \sin \omega t \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\int_{\alpha+30^\circ}^{180^\circ} \sin \omega t \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos \omega t \Big|_{\alpha+30^\circ}^{180^\circ} \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos 180^\circ + \cos(\alpha + 30^\circ) \right]$$

Since $\cos 180^\circ = -1$,

We get
$$V_{dc} = \frac{3V_m}{2\pi} \left[1 + \cos(\alpha + 30^\circ) \right]$$

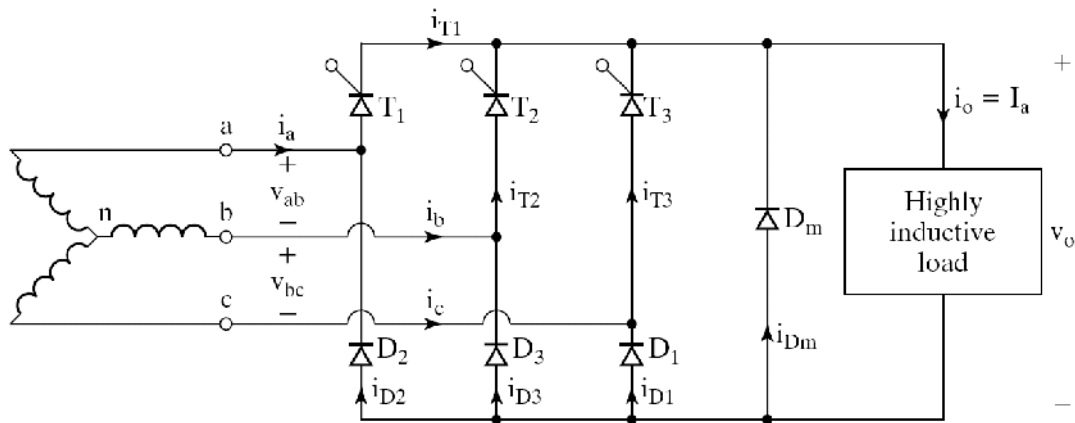
THREE PHASE SEMICONVERTERS

3-phase semi-converters are three phase half controlled bridge controlled rectifiers which employ three thyristors and three diodes connected in the form of a bridge configuration. Three thyristors are controlled switches which are turned on at appropriate times by applying

appropriate gating signals. The three diodes conduct when they are forward biased by the corresponding phase supply voltages.

3-phase semi-converters are used in industrial power applications up to about 120kW output power level, where single quadrant operation is required. The power factor of 3-phase semi-converter decreases as the trigger angle α increases. The power factor of a 3-phase semi-converter is better than three phase half wave converter.

The figure shows a 3-phase semi-converter with a highly inductive load and the load current is assumed to be a constant and continuous load current with negligible ripple.



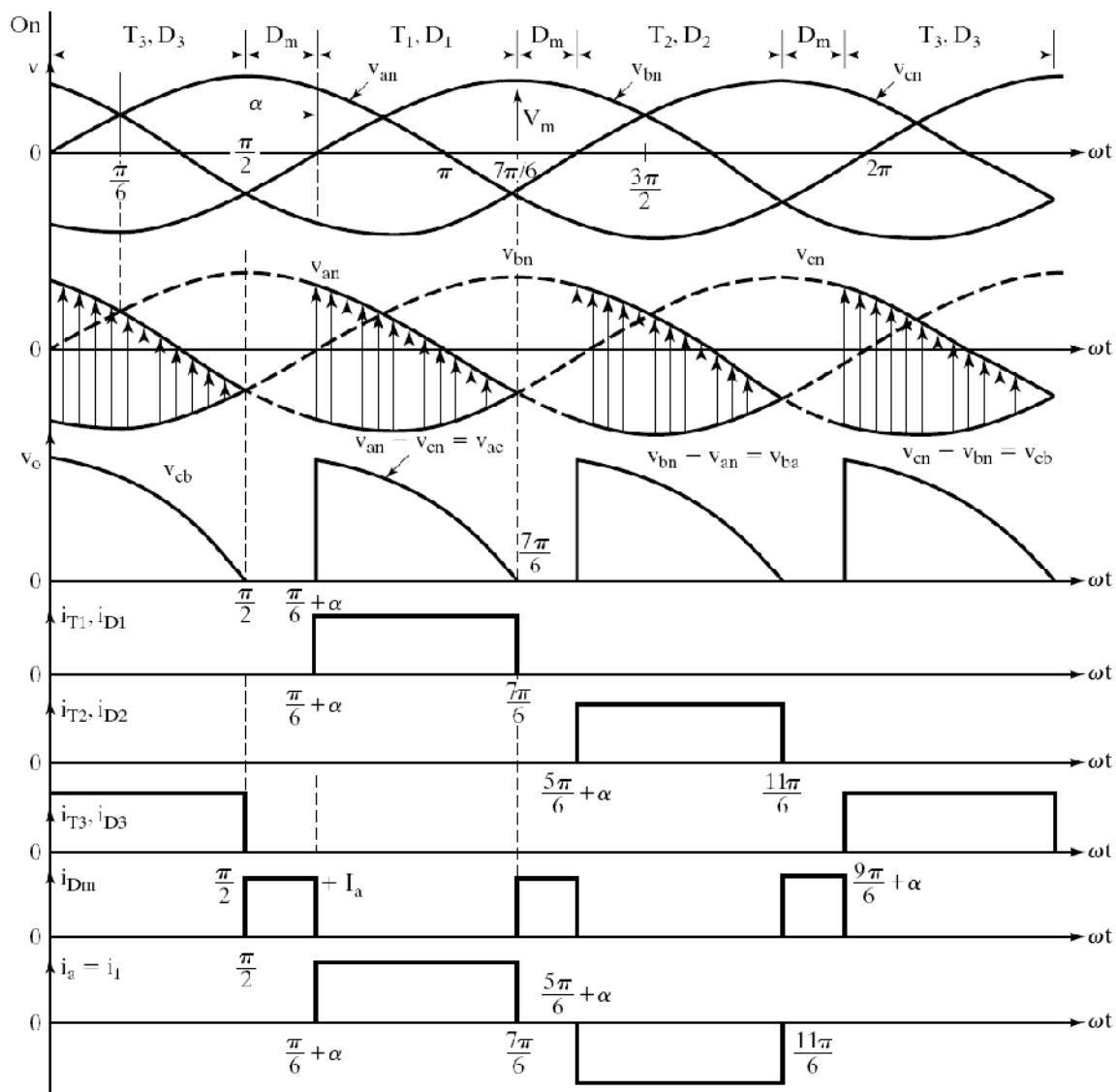
Thyristor T_1 is forward biased when the phase supply voltage v_{an} is positive and greater than the other phase voltages v_{bn} and v_{cn} . The diode D_1 is forward biased when the phase supply voltage v_{cn} is more negative than the other phase supply voltages.

Thyristor T_2 is forward biased when the phase supply voltage v_{bn} is positive and greater than the other phase voltages. Diode D_2 is forward biased when the phase supply voltage v_{an} is more negative than the other phase supply voltages.

Thyristor T_3 is forward biased when the phase supply voltage v_{cn} is positive and greater than the other phase voltages. Diode D_3 is forward biased when the phase supply voltage v_{bn} is more negative than the other phase supply voltages.

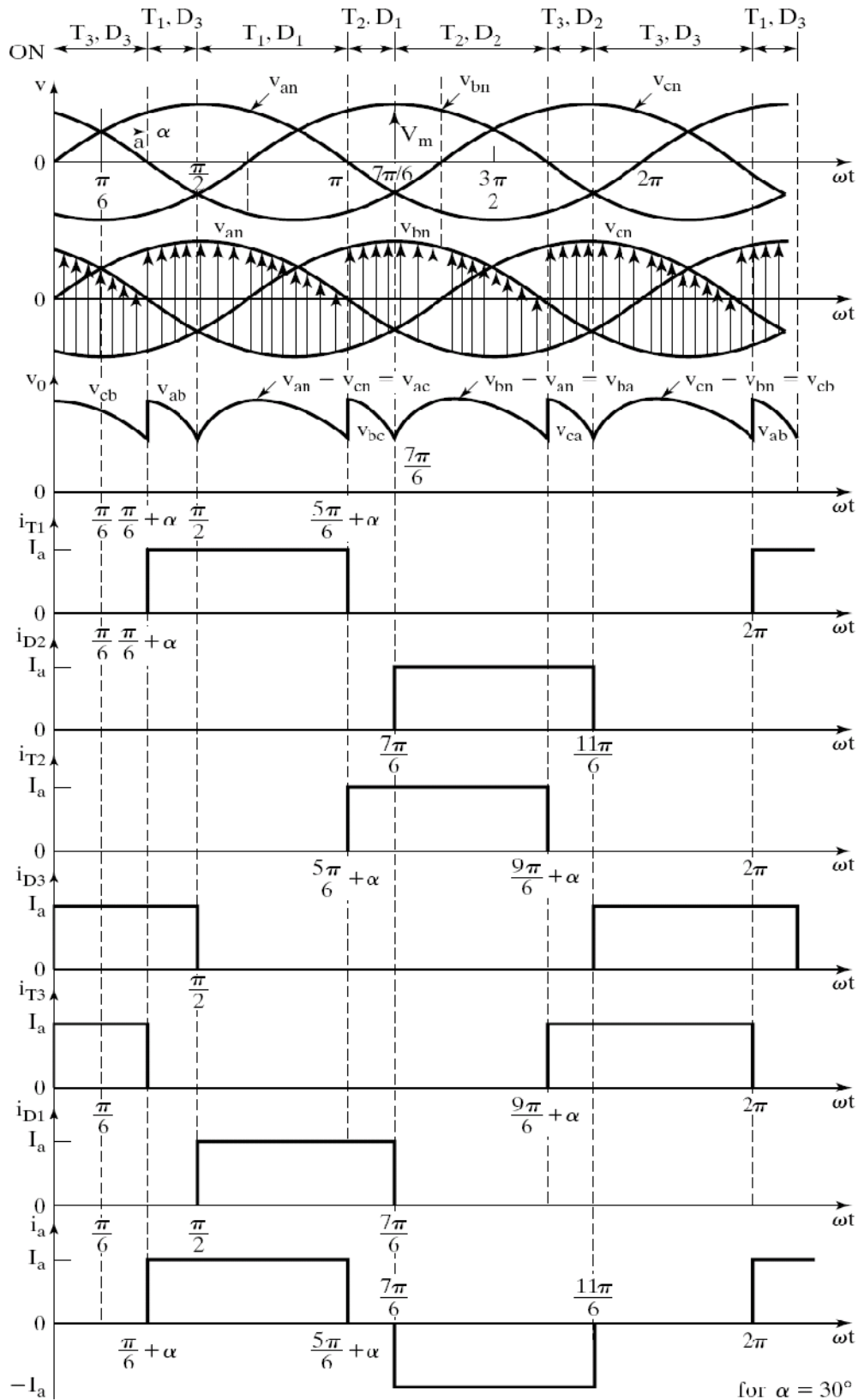
The figure shows the waveforms for the three phase input supply voltages, the output voltage, the thyristor and diode current waveforms, the current through the free wheeling diode D_m and the supply current i_a . The frequency of the output supply waveform is $3f_s$, where f_s is the input ac supply frequency. The trigger angle α can be varied from 0° to 180° .

During the time period $\left(\frac{\pi}{6}\right) \leq \omega t \leq \left(\frac{7\pi}{6}\right)$ i.e., for $30^\circ \leq \omega t \leq 210^\circ$, thyristor T_1 is forward biased. If T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right)$, T_1 and D_1 conduct together and the line to line supply voltage v_{ac} appears across the load. At $\omega t = \left(\frac{7\pi}{6}\right)$, v_{ac} starts to become negative and the free wheeling diode D_m turns on and conducts. The load current continues to flow through the free wheeling diode D_m and thyristor T_1 and diode D_1 are turned off.



(b) Waveforms for $\alpha = 90^\circ$

If the free wheeling diode D_m is not connected across the load, then T_1 would continue to conduct until the thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right)$ and the free wheeling action is accomplished through T_1 and D_2 , when D_2 turns on as soon as v_{an} becomes more negative at $\omega t = \left(\frac{7\pi}{6}\right)$. If the trigger angle $\alpha \leq \left(\frac{\pi}{3}\right)$ each thyristor conducts for $\frac{2\pi}{3}$ radians (120°) and the free wheeling diode D_m does not conduct. The waveforms for a 3-phase semi-converter with $\alpha \leq \left(\frac{\pi}{3}\right)$ is shown in figure



We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t; \quad V_m = \text{Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right), \quad v_{YN} = v_{bn} = V_m \sin \left(\omega t - 120^\circ \right), \quad v_{BN} = v_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right)$$

$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + 120^\circ \right), \quad v_{BN} = v_{cn} = V_m \sin \left(\omega t - 240^\circ \right)$$

The corresponding line-to-line voltages are

$$v_{RB} = v_{ac} = (v_{an} - v_{cn}) = \sqrt{3}V_m \sin \left(\omega t - \frac{\pi}{6} \right), \quad v_{YR} = v_{ba} = (v_{bn} - v_{an}) = \sqrt{3}V_m \sin \left(\omega t - \frac{5\pi}{6} \right)$$

$$v_{BY} = v_{cb} = (v_{cn} - v_{bn}) = \sqrt{3}V_m \sin \left(\omega t + \frac{\pi}{2} \right), \quad v_{RY} = v_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin \left(\omega t + \frac{\pi}{6} \right)$$

Where V_m is the peak phase voltage of a star (Y) connected source.

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF THREE PHASE SEMICONVERTER FOR $\alpha > \left(\frac{\pi}{3} \right)$ AND DISCONTINUOUS OUTPUT VOLTAGE

For $\alpha \geq \frac{\pi}{3}$ and discontinuous output voltage: the average output voltage is found from

$$V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} v_{ac} \cdot d(\omega t), \quad V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3} V_m \sin \left(\omega t - \frac{\pi}{6} \right) d(\omega t)$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos \alpha)$$

$$V_{dc} = \frac{3V_{mL}}{2\pi} (1 + \cos \alpha)$$

The maximum average output voltage that occurs at a delay angle of $\alpha = 0$ is

$$V_{dm} = \frac{3\sqrt{3}V_m}{\pi}$$

The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1 + \cos \alpha)$$

The rms output voltage is found from

$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} 3V_m^2 \sin^2 \left(\omega t - \frac{\pi}{6} \right) d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \sqrt{3}V_m \left[\frac{3}{4\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right) \right]^{\frac{1}{2}}$$

For $\alpha \leq \frac{\pi}{3}$, and continuous output voltage

Output voltage $v_o = v_{ab} = \sqrt{3}V_m \sin \left(\omega t + \frac{\pi}{6} \right)$; for $\omega t = \left(\frac{\pi}{6} + \alpha \right)$ to $\left(\frac{\pi}{2} \right)$

Output voltage $v_o = v_{ac} = \sqrt{3}V_m \sin \left(\omega t - \frac{\pi}{6} \right)$; for $\omega t = \left(\frac{\pi}{2} \right)$ to $\left(\frac{5\pi}{6} + \alpha \right)$

The average or dc output voltage is calculated by using the equation

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\pi/6+\alpha}^{\pi/2} v_{ab} \cdot d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac} \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos \alpha)$$

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1 + \cos \alpha)$$

The RMS value of the output voltage is calculated by using the equation

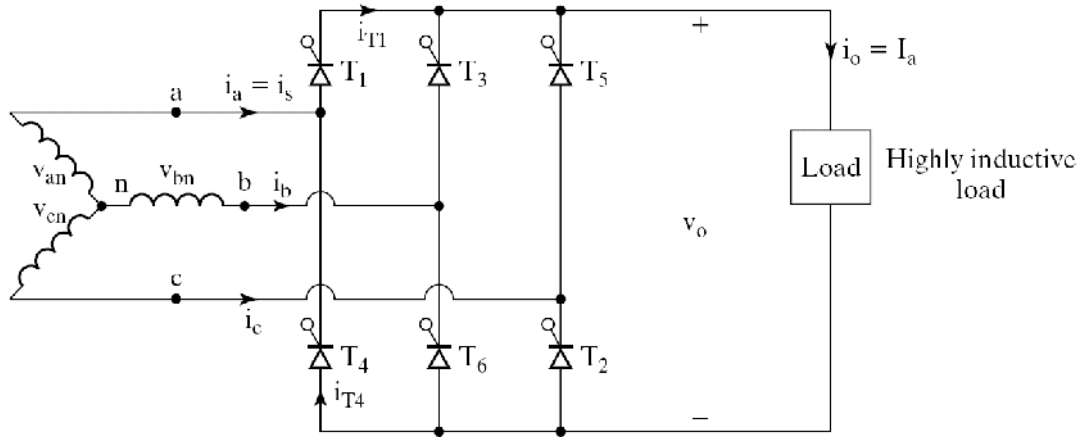
$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{\pi/2} v_{ab}^2 \cdot d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}^2 \cdot d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \sqrt{3}V_m \left[\frac{3}{4\pi} \left(\frac{2\pi}{3} + \sqrt{3} \cos^2 \alpha \right) \right]^{\frac{1}{2}}$$

THREE PHASE FULL CONVERTER

Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration. All the six thyristors are controlled switches which are turned on at a appropriate times by applying suitable gate trigger signals. The three phase full converter is extensively used in industrial power applications upto about 120kW output power level, where two quadrant operation is required. The figure shows a

three phase full converter with highly inductive load. This circuit is also known as three phase full wave bridge or as a six pulse converter. The thyristors are triggered at an interval of $\left(\frac{\pi}{3}\right)$ radians (i.e. at an interval of 60°). The frequency of output ripple voltage is $6f_s$ and the filtering requirement is less than that of three phase semi and half wave converters.



At $\omega t = \left(\frac{\pi}{6} + \alpha\right)$, thyristor T_6 is already conducting when the thyristor T_1 is turned on by applying the gating signal to the gate of T_1 . During the time period $\omega t = \left(\frac{\pi}{6} + \alpha\right)$ to $\left(\frac{\pi}{2} + \alpha\right)$, thyristors T_1 and T_6 conduct together and the line to line supply voltage v_{ab} appears across the load. At $\omega t = \left(\frac{\pi}{2} + \alpha\right)$, the thyristor T_2 is triggered and T_6 is reverse biased immediately and T_6 turns off due to natural commutation. During the time period $\omega t = \left(\frac{\pi}{2} + \alpha\right)$ to $\left(\frac{5\pi}{6} + \alpha\right)$, thyristor T_1 and T_2 conduct together and the line to line supply voltage v_{ac} appears across the load. The thyristors are numbered in the circuit diagram corresponding to the order in which they are triggered. The trigger sequence (firing sequence) of the thyristors is 12, 23, 34, 45, 56, 61, 12, 23, and so on. The figure shows the waveforms of three phase input supply voltages, output voltage, the thyristor current through T_1 and T_4 , the supply current through the line 'a'.

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \quad V_m = \text{Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right) = V_m \sin (\omega t - 120^\circ),$$

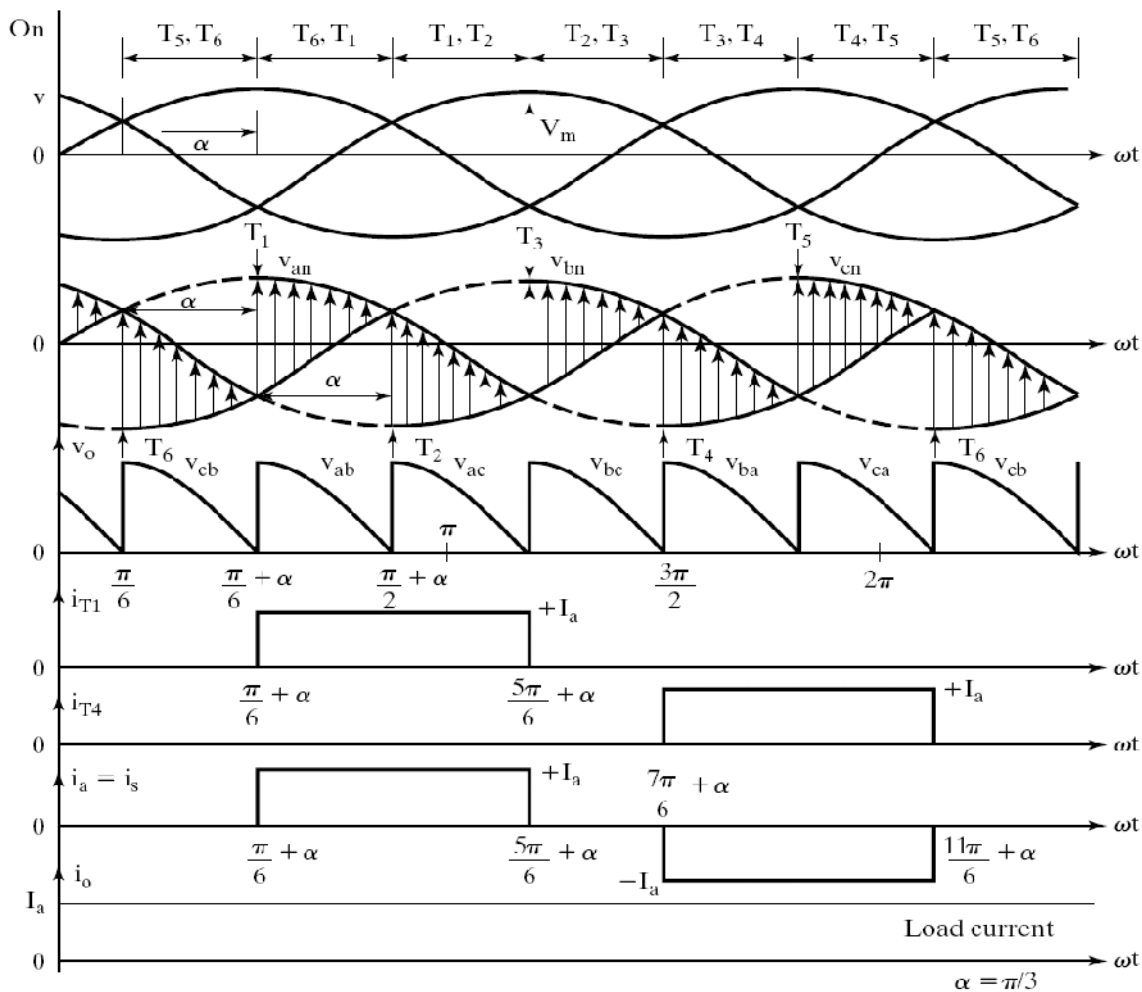
$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right) = V_m \sin (\omega t + 120^\circ) = V_m \sin (\omega t - 240^\circ)$$

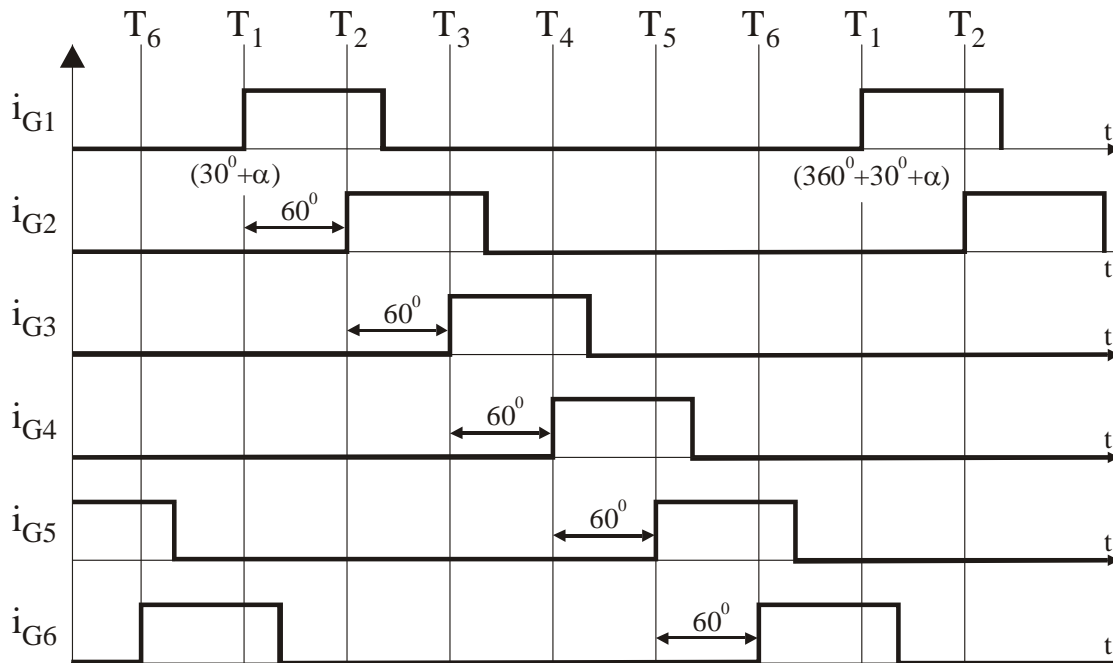
Where V_m is the peak phase voltage of a star (Y) connected source.

The corresponding line-to-line voltages are

$$v_{RY} = v_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin \left(\omega t + \frac{\pi}{6} \right), \quad v_{YB} = v_{bc} = (v_{bn} - v_{cn}) = \sqrt{3}V_m \sin \left(\omega t - \frac{\pi}{2} \right)$$

$$v_{BR} = v_{ca} = (v_{cn} - v_{an}) = \sqrt{3}V_m \sin \left(\omega t + \frac{\pi}{2} \right)$$





Gating (Control) Signals of 3-phase full converter

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF THREE PHASE FULL CONVERTER WITH HIGHLY INDUCTIVE LOAD ASSUMING CONTINUOUS AND CONSTANT LOAD CURRENT

The output load voltage consists of 6 voltage pulses over a period of 2π radians, hence the average output voltage is calculated as

$$V_{O(dc)} = V_{dc} = \frac{6}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} v_o \cdot d\omega t \quad ;$$

$$v_o = v_{ab} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) \cdot d\omega t$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha = \frac{3V_{mL}}{\pi} \cos \alpha$$

Where $V_{mL} = \sqrt{3}V_m = \text{Max. line-to-line supply voltage}$

The maximum average dc output voltage is obtained for a delay angle $\alpha = 0$,

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi} = \frac{3V_{mL}}{\pi}$$

The normalized average dc output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos \alpha$$

The rms value of the output voltage is found from

$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}+\alpha} v_o^2 \cdot d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}+\alpha} v_{ab}^2 \cdot d(\omega t) \right]^{\frac{1}{2}}$$

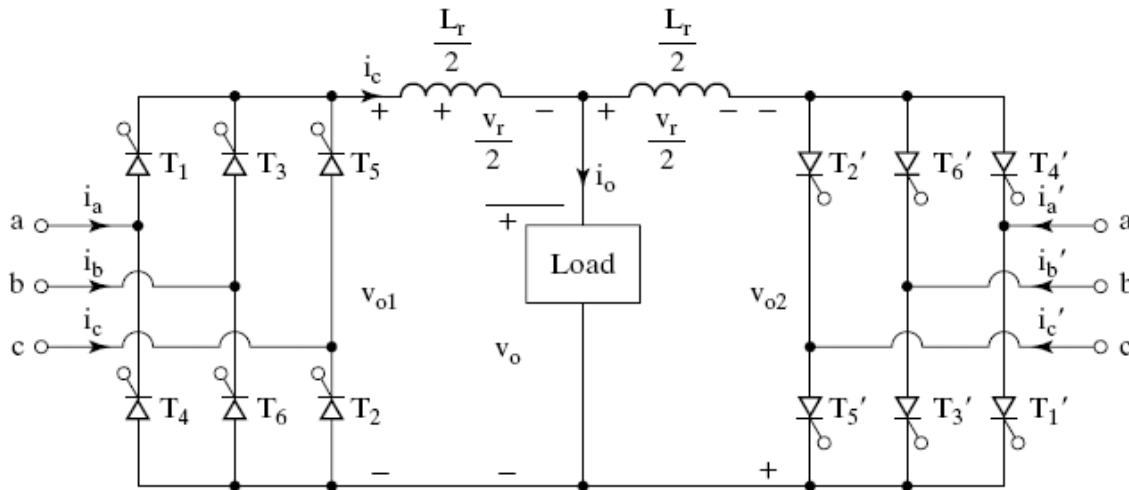
$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}+\alpha} 3V_m^2 \sin^2 \left(\omega t + \frac{\pi}{6} \right) \cdot d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3}V_m \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha \right)^{\frac{1}{2}}$$

THREE PHASE DUAL CONVERTERS

In many variable speed drives, the four quadrant operation is generally required and three phase dual converters are extensively used in applications up to the 2000 kW level. Figure shows three phase dual converters where two three phase full converters are connected back to back across a common load. We have seen that due to the instantaneous voltage differences between the output voltages of converters, a circulating current flows through the converters. The circulating current is normally limited by circulating reactor, L_r . The two converters are controlled in such a way that if α_1 is the delay angle of converter 1, the delay angle of converter 2 is $\alpha_2 = (\pi - \alpha_1)$.

The operation of a three phase dual converter is similar that of a single phase dual converter system. The main difference being that a three phase dual converter gives much higher dc output voltage and higher dc output power than a single phase dual converter system. But the drawback is that the three phase dual converter is more expensive and the design of control circuit is more complex.



The figure below shows the waveforms for the input supply voltages, output voltages of converter1 and converter2, and the voltage across current limiting reactor (inductor) L_r . The operation of each converter is identical to that of a three phase full converter.

During the interval $\left(\frac{\pi}{6} + \alpha_1\right)$ to $\left(\frac{\pi}{2} + \alpha_1\right)$, the line to line voltage v_{ab} appears across the output of converter 1 and v_{bc} appears across the output of converter 2

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \quad V_m = \text{Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right) = V_m \sin (\omega t - 120^\circ)$$

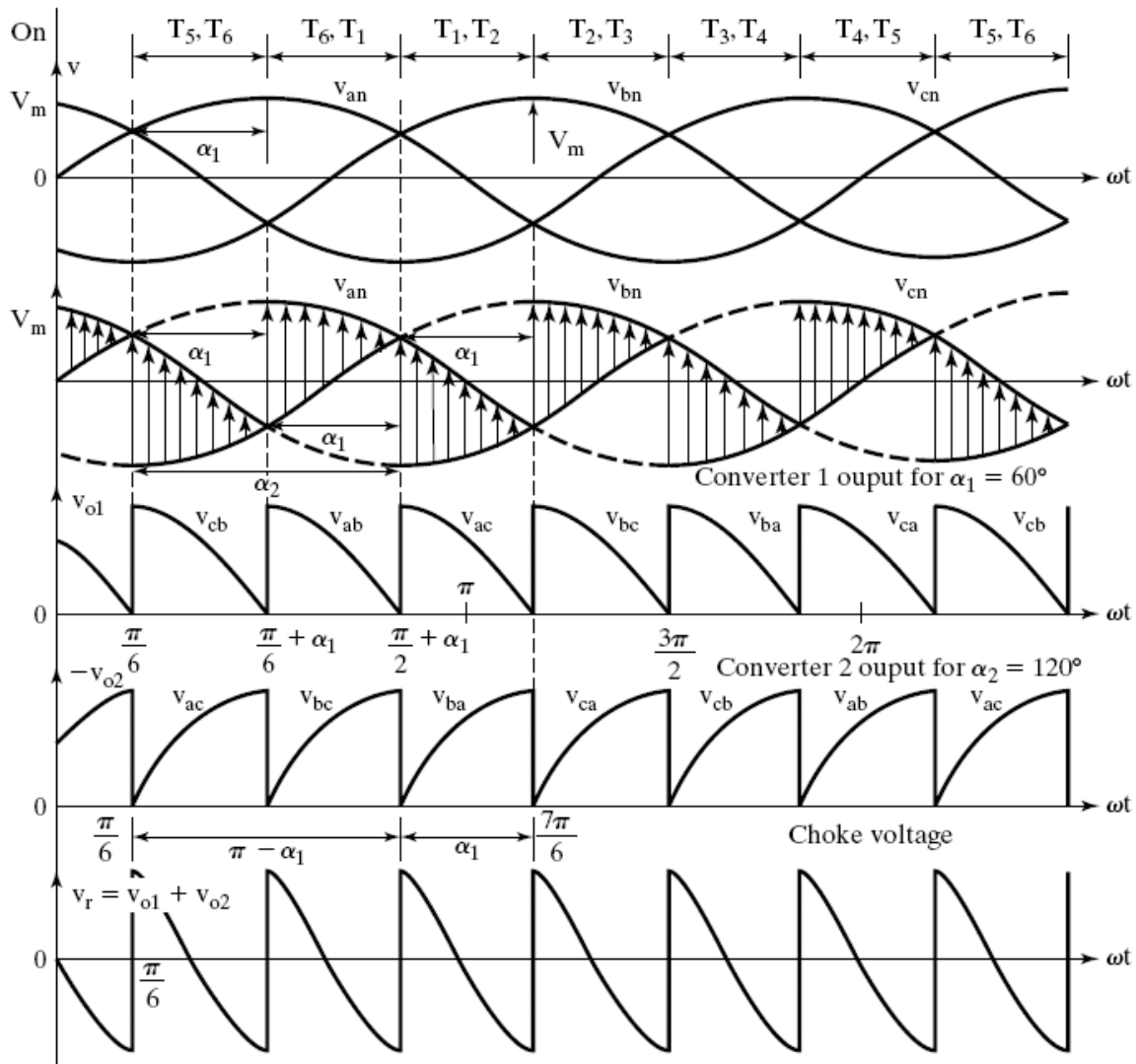
$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right) = V_m \sin (\omega t + 120^\circ) = V_m \sin (\omega t - 240^\circ)$$

The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = (v_{bn} - v_{cn}) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = (v_{cn} - v_{an}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$



TO OBTAIN AN EXPRESSION FOR THE CIRCULATING CURRENT

If v_{o1} and v_{o2} are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval

$$\left(\frac{\pi}{6} + \alpha_1\right) \leq \omega t \leq \left(\frac{\pi}{2} + \alpha_1\right) \text{ is}$$

$$v_r = (v_{o1} + v_{o2}) = (v_{ab} - v_{bc})$$

$$v_r = \sqrt{3}V_m \left[\sin\left(\omega t + \frac{\pi}{6}\right) - \sin\left(\omega t - \frac{\pi}{2}\right) \right]$$

$$v_r = 3V_m \cos\left(\omega t - \frac{\pi}{6}\right)$$

The circulating current can be calculated by using the equation

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6} + \alpha_1}^{\omega t} v_r \cdot d(\omega t)$$

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6} + \alpha_1}^{\omega t} 3V_m \cos\left(\omega t - \frac{\pi}{6}\right) \cdot d(\omega t)$$

$$i_r(t) = \frac{3V_m}{\omega L_r} \left[\sin\left(\omega t - \frac{\pi}{6}\right) - \sin \alpha_1 \right]$$

$$i_{r(\max)} = \frac{3V_m}{\omega L_r} = \text{maximum value of the circulating current.}$$

There are two different modes of operation of a three phase dual converter system.

- Circulating current free (non circulating) mode of operation
- Circulating current mode of operation

CIRCULATING CURRENT FREE (NON-CIRCULATING) MODE OF OPERATION

In this mode of operation only one converter is switched on at a time when the converter number 1 is switched on and the gate signals are applied to the thyristors the average output voltage and the average load current are controlled by adjusting the trigger angle α_1 and the gating signals of converter 1 thyristors.

The load current flows in the downward direction giving a positive average load current when the converter 1 is switched on. For $\alpha_1 < 90^\circ$ the converter 1 operates in the rectification mode V_{dc} is positive, I_{dc} is positive and hence the average load power P_{dc} is positive.

The converter 1 converts the input ac supply and feeds a dc power to the load. Power flows from the ac supply to the load during the rectification mode. When the trigger angle α_1 is increased above 90° , V_{dc} becomes negative where as I_{dc} is positive because the thyristors of converter 1 conduct in only one direction and reversal of load current through thyristors of converter 1 is not possible.

For $\alpha_1 > 90^\circ$ converter 1 operates in the inversion mode & the load energy is supplied back to the ac supply. The thyristors are switched-off when the load current decreases to zero & after a short delay time of about 10 to 20 milliseconds, the converter 2 can be switched on by releasing the gate control signals to the thyristors of converter 2.

We obtain a reverse or negative load current when the converter 2 is switched ON. The average or dc output voltage and the average load current are controlled by adjusting the trigger angle α_2 of the gate trigger pulses supplied to the thyristors of converter 2. When α_2 is less than 90° , converter 2 operates in the rectification mode and converts the input ac supply in to dc output power which is fed to the load.

When α_2 is less than 90° for converter 2, V_{dc} is negative & I_{dc} is negative, converter 2 operates as a controlled rectifier & power flows from the ac source to the load circuit. When α_2 is increased above 90° , the converter 2 operates in the inversion mode with V_{dc} positive and I_{dc} negative and hence P_{dc} is negative, which means that power flows from the load circuit to the input ac supply. The power flow from the load circuit to the input ac source is possible if the load circuit has a dc source of appropriate polarity. When the load current falls to zero the thyristors of converter 2 turn-off and the converter 2 can be turned off.

CIRCULATING CURRENT MODE OF OPERATION

Both the converters are switched on at the same time in the mode of operation. One converter operates in the rectification mode while the other operates in the inversion mode. Trigger angles α_1 & α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^\circ$

When $\alpha_1 < 90^\circ$, converter 1 operates as a controlled rectifier. When α_2 is made greater than 90° , converter 2 operates in the inversion mode. V_{dc} , I_{dc} , P_{dc} are positive.

When $\alpha_2 < 90^\circ$, converter 2 operates as a controlled rectifier. When α_1 is made greater than 90° , converter 1 operates as an Inverter. V_{dc} and I_{dc} are negative while P_{dc} is positive.