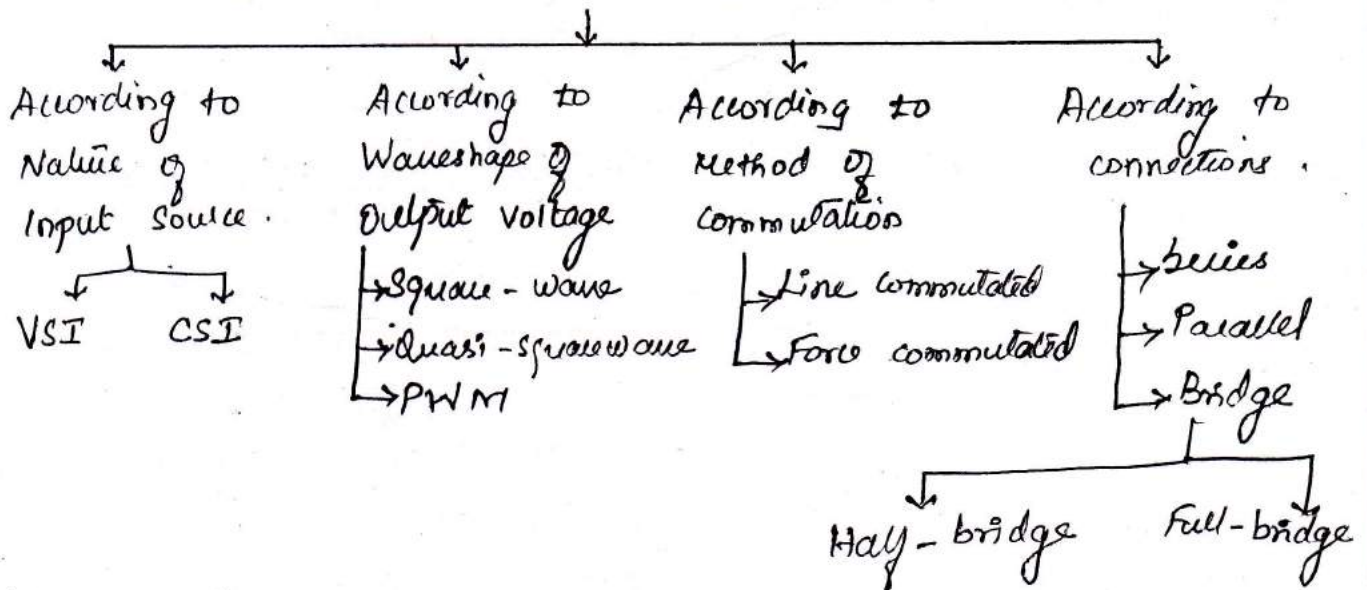


INTRODUCTION.

The d.c to a.c power converters are known as inverters. An inverter is a circuit which converts a d.c. power into an a.c power at desired output voltage and frequency.

APPLICATIONS OF INVERTERS.

1. Variable speed a.c motor drives.
2. Induction heating.
3. Aircraft power supplies.
4. Uninterruptible power supplies (UPS)
5. High Voltage d.c transmission lines.
6. Battery - vehicle drives
7. Regulated voltage and frequency power supplies.

CLASSIFICATION OF INVERTERS.VSI - Voltage Source Inverters.

The input to the inverter is provided by a ripple free dc voltage source.

CSI - Current Source Inverters.

The voltage is first converted into a current source and then used to supply the power to the inverter.

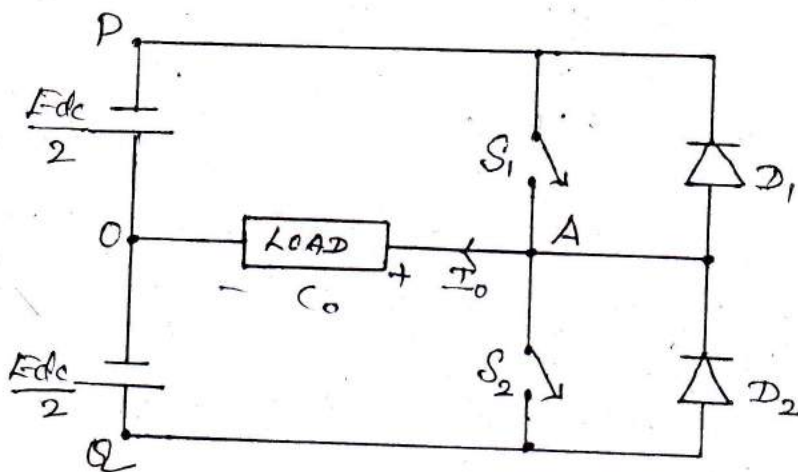
Line Commutated Inverters.

In case of a.c. circuits, a.c. line voltage available across the device. When the current in the SCR goes through a natural zero, the device is turned-off. This process is known as natural commutation process and the inverters based on this principle are known as line commutated inverters.

Forced Commutated Inverters.

In case of d.c. circuits, since the supply voltage does not go through the zero point, some external source is required to commutate the device. This process is known as the forced commutation process and the inverters based on this principle are called as forced commutated inverters.

SINGLE PHASE HALF-BRIDGE VOLTAGE SOURCE INVERTERS.



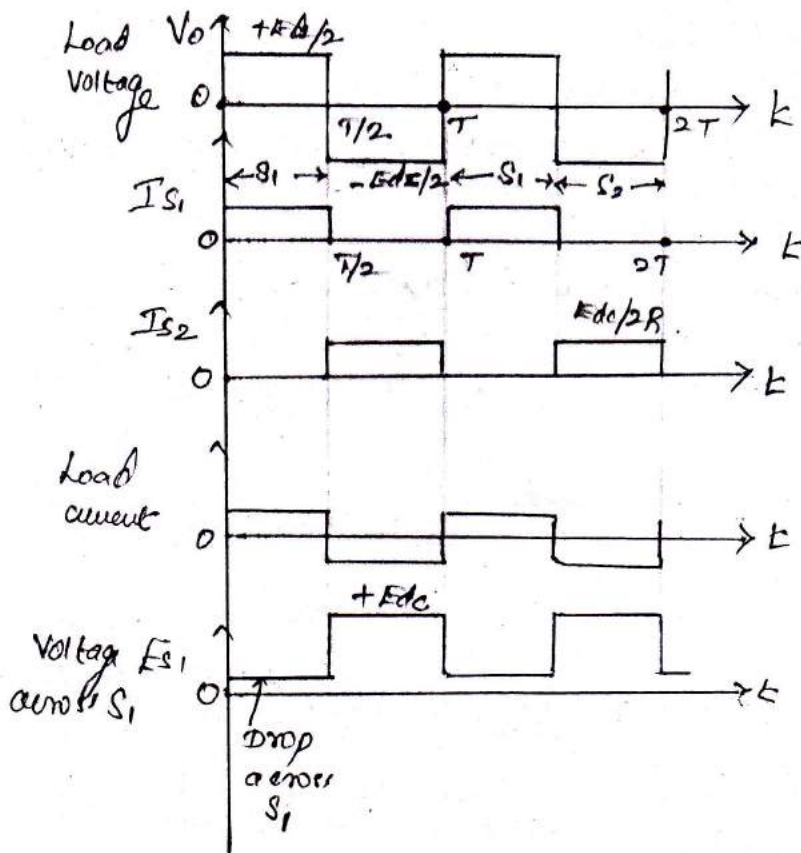
Half-bridge Inverter.

Switches S_1 and S_2 are the gate commutated devices such as power BJT, MOSFET, GTO, IGBT, MCT etc. When closed, these switches conduct and current flows in the direction of arrow.

Operation with Resistive load.

The operation of the circuit can be divided into two periods.

- i) Period-I, where switch S_1 is conducting from $0 \leq t \leq \frac{T}{2}$
 - and ii) Period-II, where switch S_2 is conducting from $\frac{T}{2} \leq t \leq T$
- where $T = \frac{1}{f}$ and f is the frequency of the output voltage



Voltage and Current Waveforms.

i) RMS Output voltage

The average value of the output voltage is given by

$$F_{o(av)} = \frac{1}{2\pi} \int_0^{2\pi} e_o(\omega t) d\omega t.$$

Now, rms value of the output voltage is given by

$$\begin{aligned} F_{o(rms)} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} e_o^2(\omega t) d\omega t.} \\ &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} e_o^2(\omega t) d\omega t} \\ &= \sqrt{\frac{2}{\pi} \int_0^{\pi/2} \left(\frac{E_{dc}}{2}\right)^2 d\omega t} \\ &= \frac{E_{dc}}{2}. \end{aligned}$$

Rms value of a square-wave is equal to its peak-value.

ii) Instantaneous Output-voltage.

The Fourier-series can be found out by using the following equation

$$e_o(\omega t) = \sum_{n=1,2,3,\dots}^{\infty} C_n \sin(n\omega t + \phi_n)$$

where $C_n = \sqrt{a_n^2 + b_n^2}$ and $\phi_n = \tan^{-1}(a_n/b_n)$

and $a_n = \frac{1}{\pi} \int_0^{2\pi} e_o(\omega t) \cos(n\omega t) d\omega t = 0$ due to square wave symmetry

and $b_n = \frac{1}{\pi} \int_0^{2\pi} e_o(\omega t) \cdot \sin(n\omega t) d\omega t.$

Due to quarter-wave symmetry, $b_n = 0$, for all even 'n'

$$\therefore b_n = \frac{4}{\pi} \int_0^{\pi/2} \frac{E_{dc}}{2} \sin(n\omega t) d\omega t, \text{ for all odd 'n'}$$

$$b_n = \frac{2E_{dc}}{n\pi}, \text{ for odd value of } n.$$

$$\therefore C_n = \sqrt{a_n^2 + b_n^2} = \frac{2E_{dc}}{n\pi} \text{ and } \phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right) = 0.$$

Therefore, the instantaneous output voltage of a half-bridge inverter can be expressed in fourier-series form as

$$e_o(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2}{\pi} \frac{E_{dc}}{n} \sin(n\omega t) \\ = 0, \text{ for } n=2,4,\dots \text{ (even values of } n)$$

The n^{th} harmonic component is given by

$$e_o(n) = \frac{C_n}{\sqrt{2}} = \frac{2E_{dc}}{n\pi\sqrt{2}} = \frac{\sqrt{2}}{n} \frac{E_{dc}}{\pi} \text{ for } n=1,3,5,\dots$$

The value of fundamental component is obtained by substituting $n=1$ in above equation,

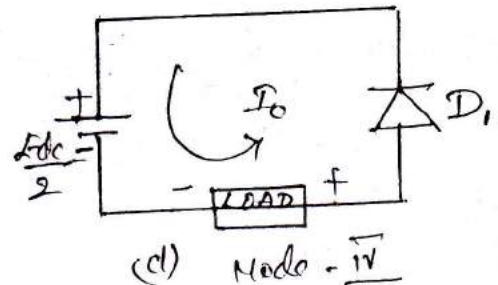
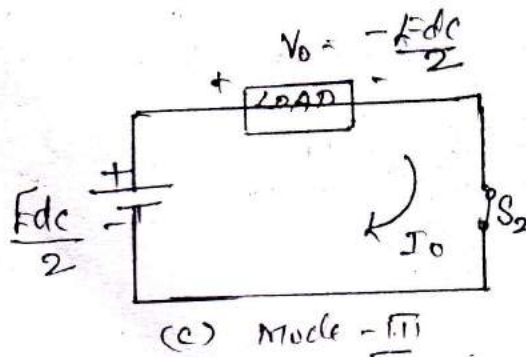
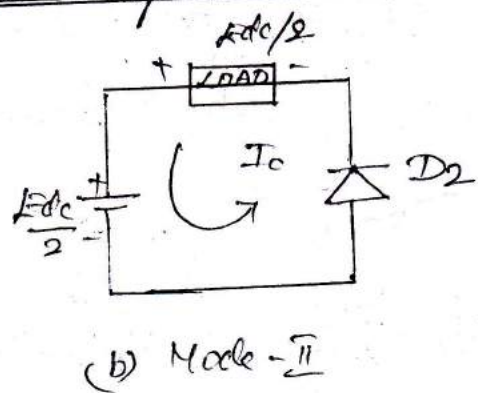
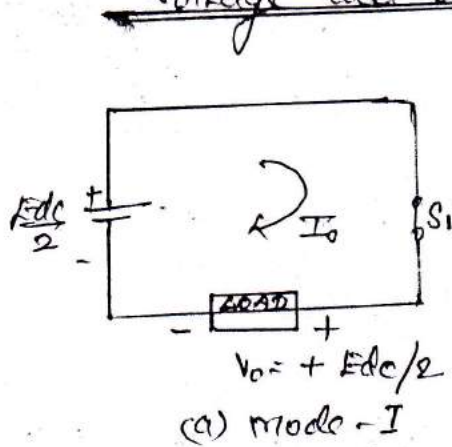
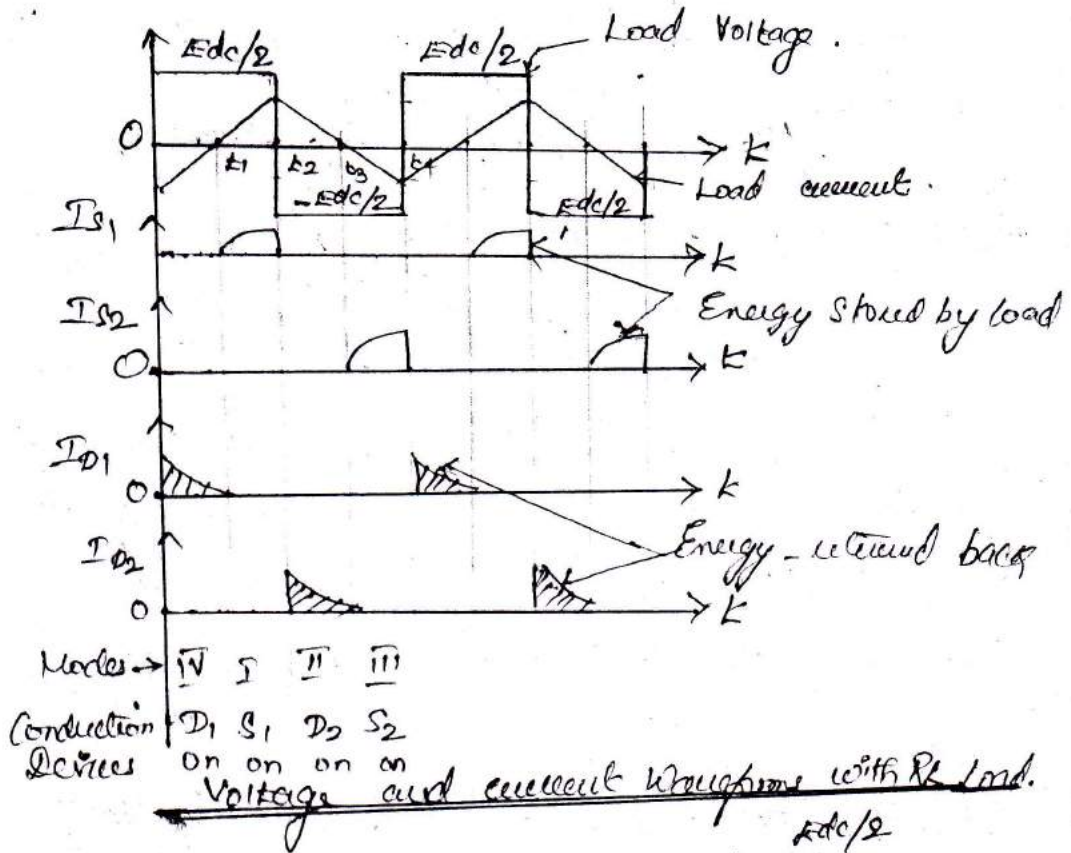
$$\therefore E_1(\text{max}) = \frac{\sqrt{2}}{\pi} E_{dc} = 0.45 E_{dc}.$$

Operation with RL Load.

With an inductive load, the output voltage waveform is similar to that with a resistive load, however the load-current cannot change immediately with the output voltage.

The operation of half-bridge inverter with RL load is divided into four distinct modes.

- (i) Mode 1 ($t_1 < t < t_2$)
- (ii) Mode 2 ($t_2 < t < t_3$)
- (iii) Mode 3 ($t_3 < t < t_4$)
- (iv) Mode 4 ($t_4 < t < t_1$)



Mode - II ($t_2 < t < t_3$).

Both the switches S_1 and S_2 are turned-off at instant t_2 . Due to inductive nature of the load, the load current does not reduce to zero instantaneously.

There is a self-induced voltage across the load which maintains the flow of current in the same direction. The polarity of this voltage is exactly opposite to that in mode-1. The output voltage becomes $-E_{dc}$, but the load current continues to flow in the same direction, through D_3 and D_4 .

Thus, in this mode, the stored energy in the load inductance is returned back to the source.

Load current decreases exponentially and goes to 0 at instant t_3 when all the energy stored in the load is returned back to supply. D_3 and D_4 are turned-off at t_3 .

Mode - III ($t_3 < t < t_4$)

Switches S_3 and S_4 are turned-ON simultaneously at instant t_3 .

Load voltage remains negative ($-E_{dc}$) but the direction of load current will reverse. The current increases exponentially in the other direction and the load again stores the energy.

Mode - IV ($t_0 < t < t_1$).

Switches S_2 and S_4 are turned-off at instant t_0 . The load inductance tries to maintain the load current in the same direction by inducing the positive-load voltage.

This will forward-bias the diodes D_1 and D_2 . The load energy is returned back to the input dc supply. The load voltage becomes $E_0 = +E_{dc}$ but the load current remains negative and

decreases exponentially towards 0. At t_1 , the load current goes to zero and switches S_1 and S_2 can be turned-on again. The conduction period with a very highly inductive load, will be $T/4$ or 90° for all the switches as well as the diodes. The conduction period of switches will increase towards $\frac{T}{2}$ or 180° with increase in the load power factor.

(i) RMS output voltage

$$V_{rms} = \left[\frac{2}{T/2} \int_0^{T/2} E^2 dt \right]^{1/2}$$

$$= E_{dc}$$

(ii) The instantaneous output voltage can be expressed in four series as

$$e_o(\omega t) = \sum_{n=1,3,5,\dots,\infty} \frac{4E_{dc}}{n\pi} \sin n\omega t$$

The output voltage waveform contains only the odd harmonic components.

(iii) For $n=1$, the rms value of the fundamental component

$$E_1(rms) = \frac{4E_{dc}}{\sqrt{2} \cdot \pi} = 0.9 E_{dc}$$

(iv) For RL Load, the equation for the instantaneous current i_o ,

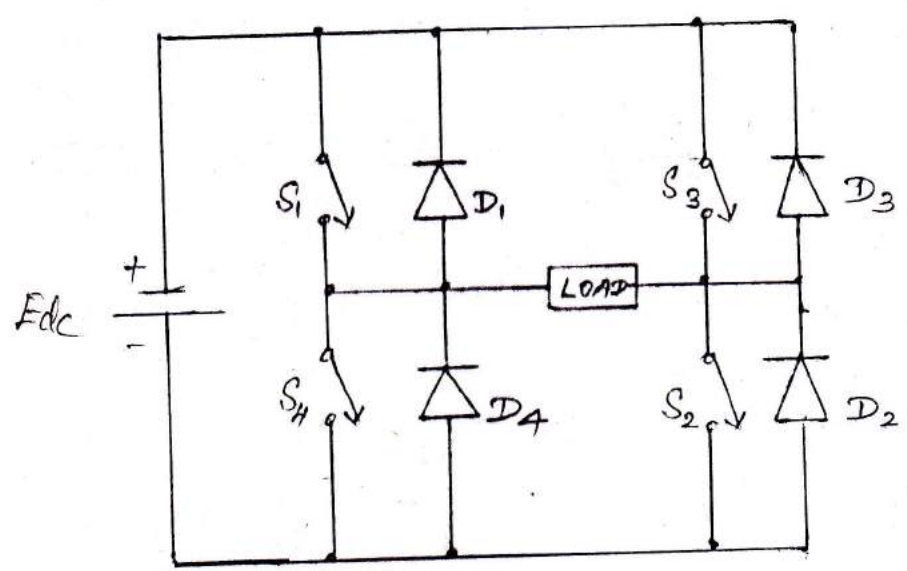
$$i_o(\omega t) = \sum_{n=1,3,5,\dots,\infty} \frac{4E_{dc}}{\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n)$$

where $Z_n = \sqrt{R^2 + (n\omega L)^2}$ is the impedance offered by the load to the n^{th} harmonic component and $\frac{4E_{dc}}{\sqrt{R^2 + (n\omega L)^2}}$ is the peak amplitude of n^{th} harmonic voltage and $\theta_n = \tan^{-1}(n\omega L/R)$.

SINGLE PHASE FULL BRIDGE INVERTER.

The inverter uses two pairs of controlled switches (S_1, S_2 and S_3, S_4) and two pairs of diodes (D_1, D_2 and D_3, D_4).

In order to develop a positive voltage ($+E_o$) across the load, switches S_1 and S_2 are turned-on simultaneously whereas to have a negative voltage ($-E_o$) across the load, we need to turn-on the switches S_3 and S_4 . Diodes D_1, D_2, D_3 and D_4 are known as the feedback diodes.



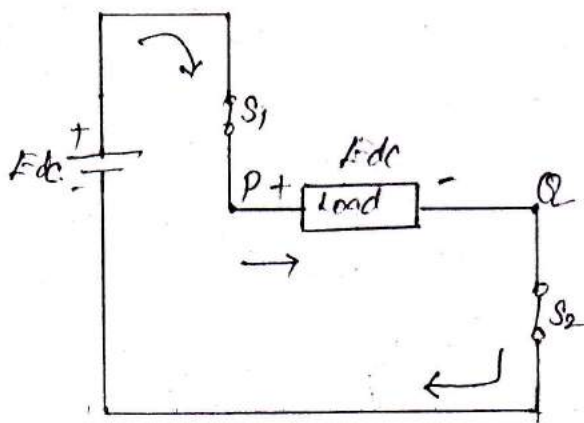
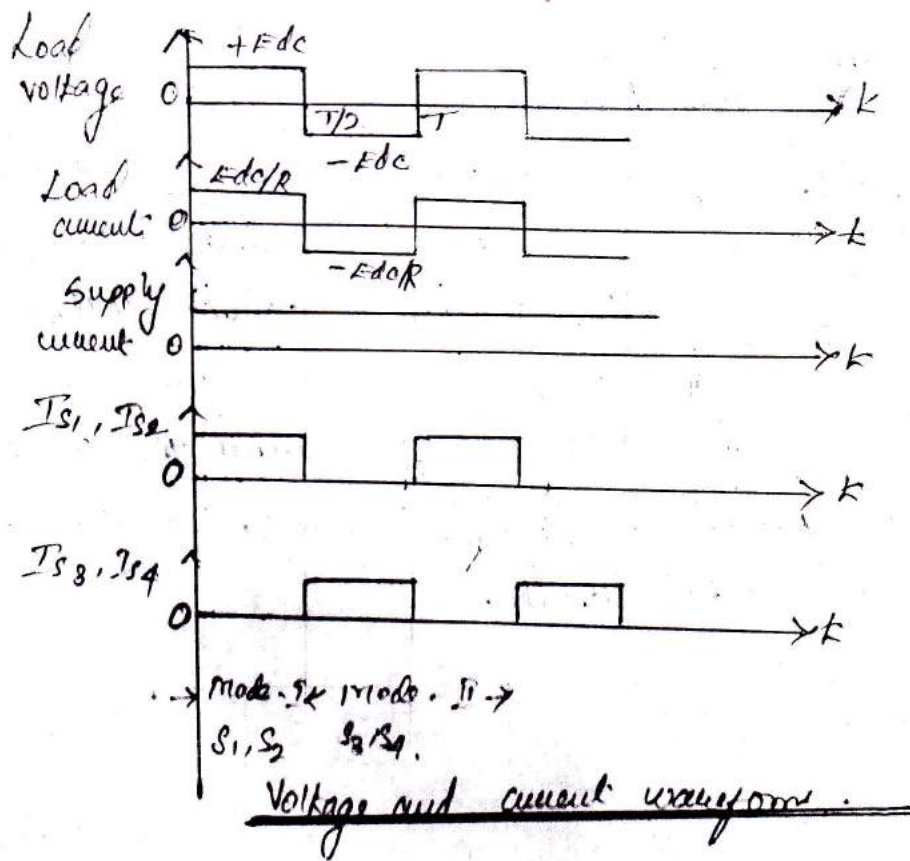
1 ϕ full bridge inverter.

Operation with Resistive Load.

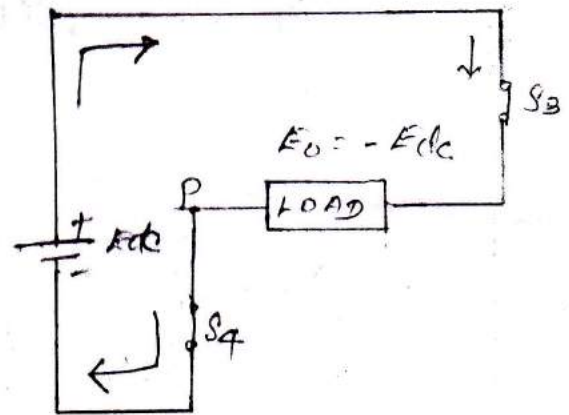
The bridge-inverter operates in two-modes in one-cycle of the output.

mode - I ($0 < t < T/2$).

In this mode, switches S_1 and S_2 conduct simultaneously. The load voltage is $+E_{dc}$ and load current flows from P to Q. At $t = T/2$, S_1 and S_2 are turned-off and S_3 and S_4 are turned-on.



(a) Mode - I



(b) Mode - II

Equivalent circuit.

Mode - II ($T/2 < t < T$).

At $t = T/2$, switches S_3 and S_4 are turned - ON and S_1 and S_2 are turned - OFF. The load voltage is $-E_{dc}$ and load current flows from Q to P . As the load is resistive, it

Does not store any energy.

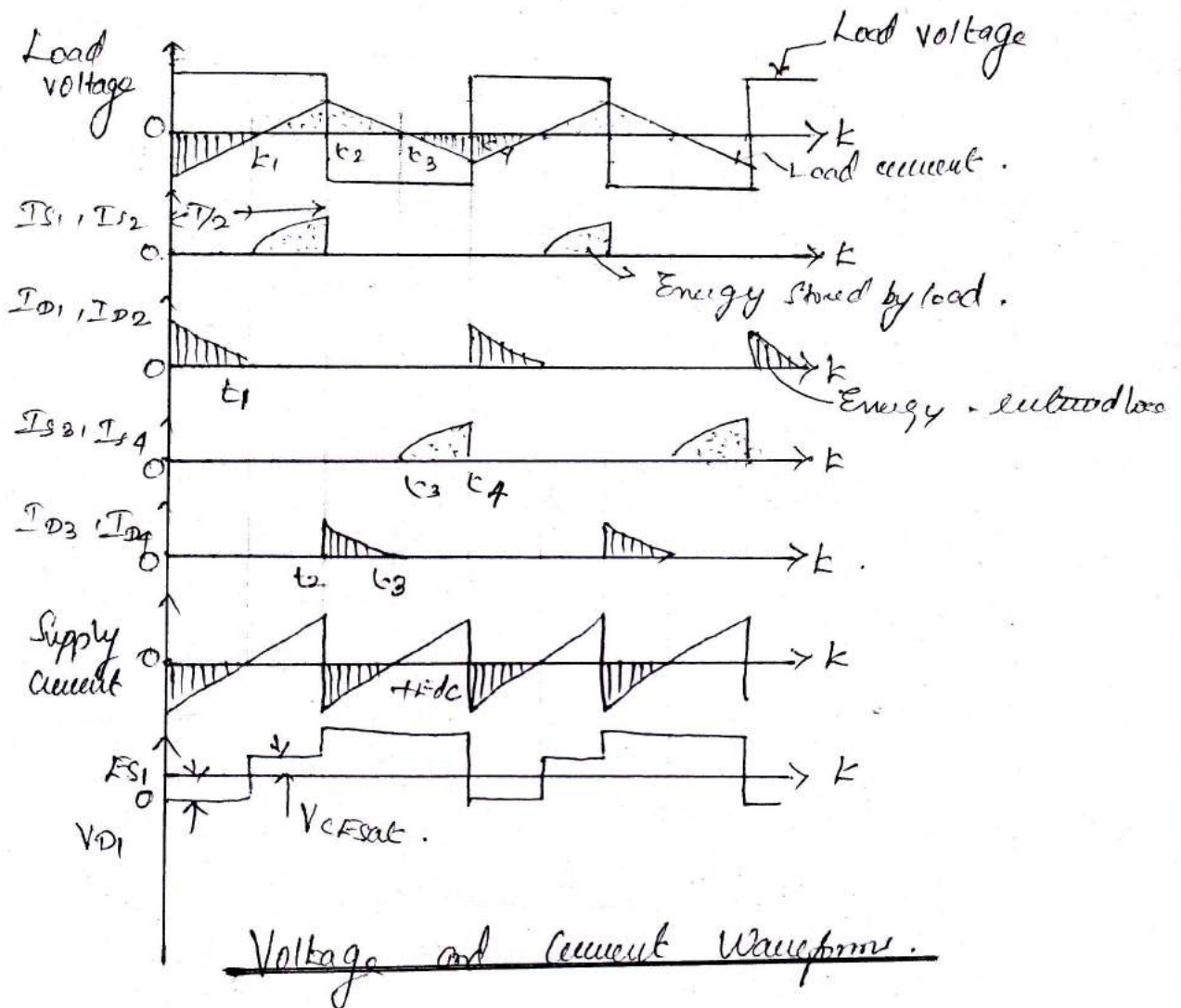
- (i) RMS output voltage, $E_o(\text{rms}) = E_{dc}$.
- (ii) Fourier series, $E_o(\omega t) = \sum_{n=1,3,5,\dots}^{\infty} (4 E_{dc} / n\pi) \cdot \sin(n\omega t)$.
- (iii) Fundamental output voltage, $E_o(\text{fund}) = \frac{2\sqrt{2}}{\pi} \cdot E_{dc}$.
- (iv) n th harmonic voltage $E_o(n) = \frac{E_o(\text{fund})}{n}$.
- (v) Transistor (switch) ratings, n

$$V_{CE0} \geq E_{dc}, \quad I_{T(\text{av})} = \frac{E_{dc}}{R}$$

$$I_{T(\text{rms})} = \frac{E_{dc}}{\sqrt{2} \cdot R}, \quad I_{T(\text{peak})} = \frac{E_{dc}}{R}$$

Operation with R_L Load.

The operation of the circuit is explained in four modes.

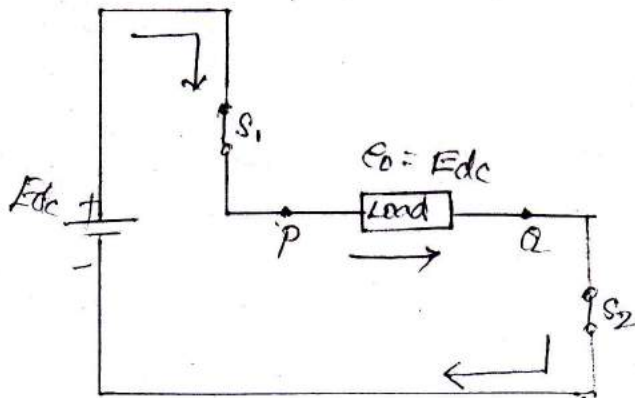


Voltage and Current Waveforms.

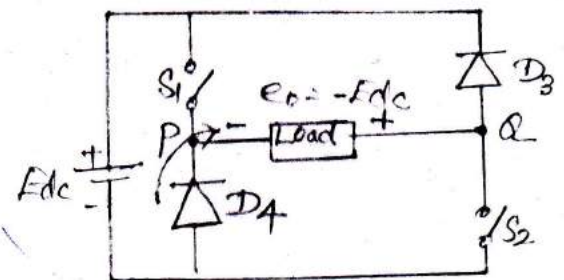
Mode - 1 ($t_1 < t < t_2$)

At instant t_1 , the switch S_1 and S_2 are turned - on. Point P gets connected to positive point of dc source E_{dc} through S_1 and point Q gets connected to negative point of input supply.

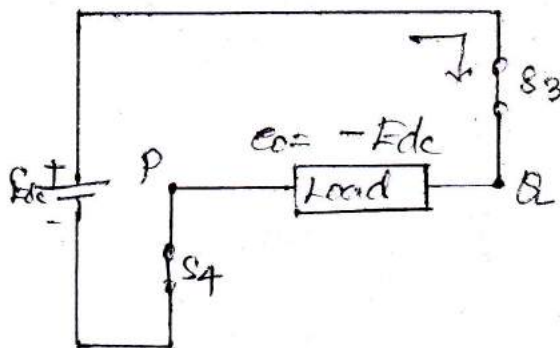
The output voltage, $e_o = +E_{dc}$. The instantaneous current through S_1 and S_2 is equal to the instantaneous load current. During this interval, energy is stored in inductive load



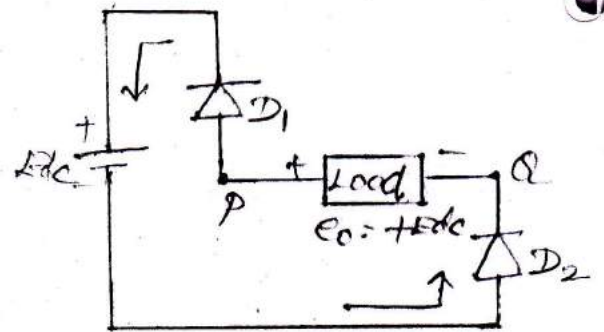
(a) Mode I ($t_1 < t < t_2$)



(b) Mode II ($t_2 < t < t_3$)



(c) Mode - III ($t_3 < t < t_4$)



(d) Mode - IV ($t_4 < t < t_1$)

Equivalent Circuits.

THREE PHASE INVERTERS.

Three-phase inverters are used for high-power applications such as ac motor drives, induction heating, uninterruptible power supplies.

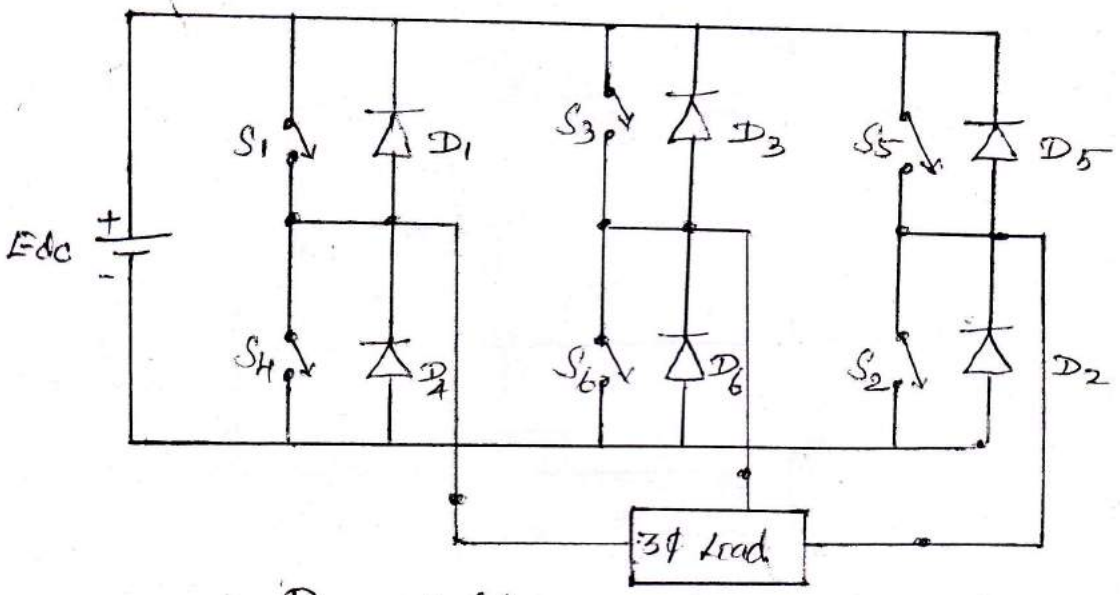
A three phase inverter circuit changes dc input voltage to a three-phase variable frequency, variable voltage output.

The circuit consists of six power-switches with six associated freewheeling diodes. The switches are opened and closed periodically in the proper sequence to produce the desired output waveform. The rate of switching determines the output frequency of the inverter.

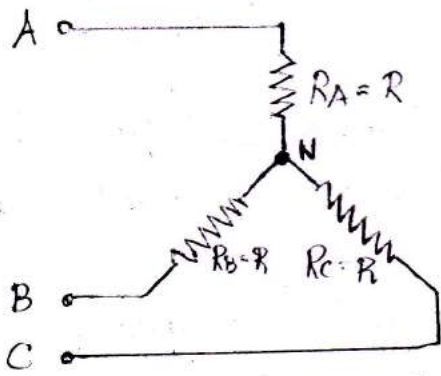
Basically, there are two possible schemes of gating the devices. In one scheme, each device (switch) conducts for 180° and in the other scheme, each device conducts for 120° .

180° - Conduction Mode with Resistive Load.

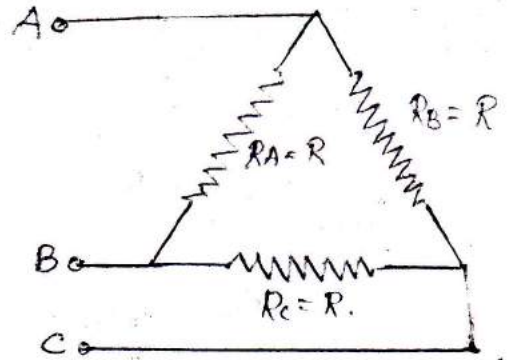
In this control scheme, each switch conducts for a period of 180° or half-cycle electrical. Switches are triggered in sequence of their numbers with an interval of 60° .



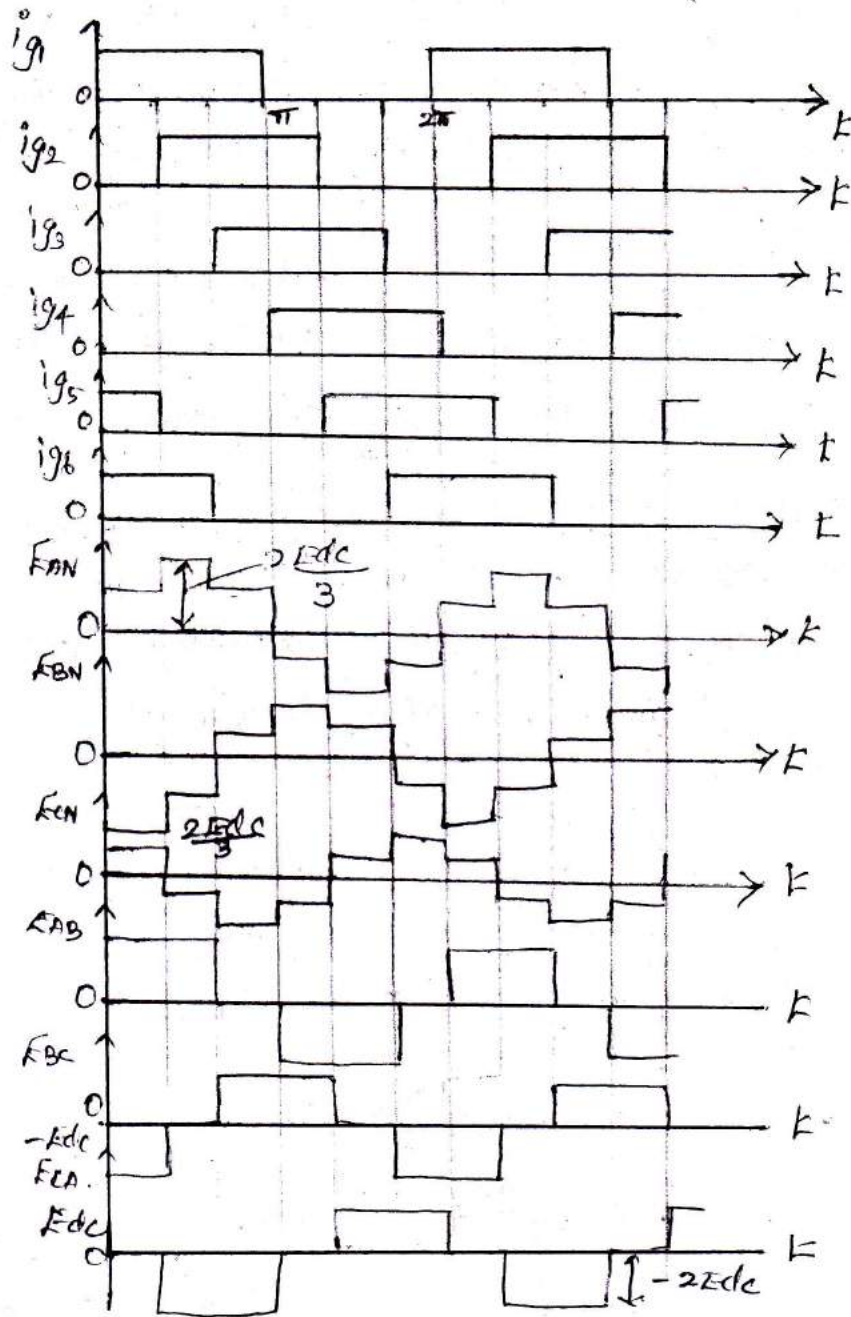
Power circuit.



Star-connected load.



Delta-connected load



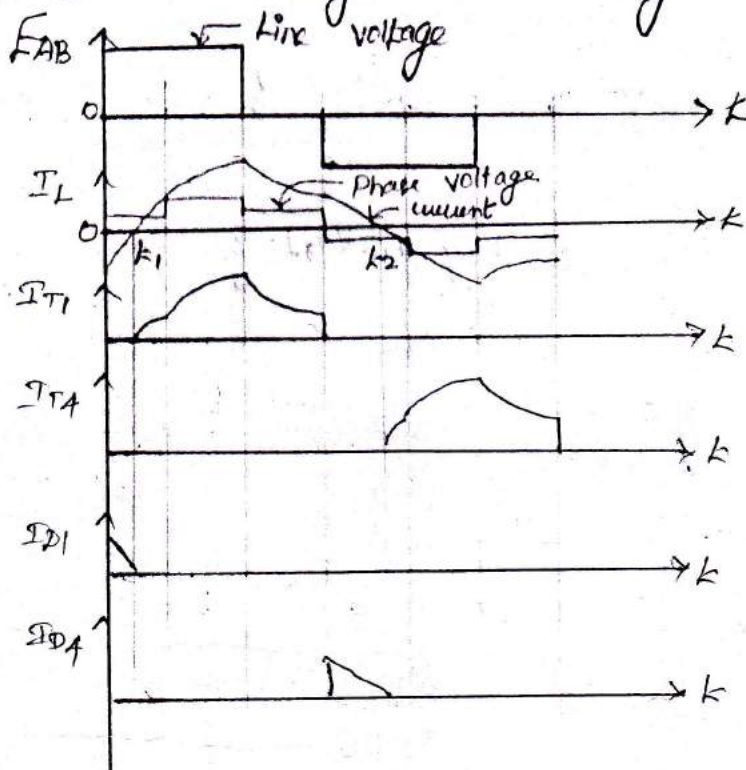
Voltage waveforms for 180° conduction.

Operation Table.

Sl. No	Interval	Device conducting	Incoming device.	Outgoing device.
1.	I	5, 6, 1	1	4
2.	II	6, 1, 2	2	5
3.	III	1, 2, 3	3	6
4.	IV	2, 3, 4	4	1
5.	V	3, 4, 5	5	2
6.	VI	4, 5, 6	6	3

180° Conduction Mode with RL Load.

If the load is inductive, then the current in each arm of the load will be delayed to its voltage.



Waveform for 180° firing with an RL load.

When switch S_1 is triggered, S_4 is turned-off but, because the load current cannot reverse, the only path for this current is through diode D_1 .

Hence the load phase is connected to the positive end of the dc source but, until the load current reverses at t_1 , switch S_1 will not take up conduction.

(i) Phase Voltage, $E_{AN} = \frac{E_{AB}}{\sqrt{3}}$ with a delay of 30° .

(ii) Line current, I_L , for an RL Load is given by

$$I_L = \sum_{n=1,3,5}^{\infty} \left[\frac{A E_{dc}}{\sqrt{3} \cdot n\pi \sqrt{R^2 + (n\omega L)^2}} \cdot \frac{\cos n\pi}{6} \right] \sin(n\omega t - \theta_n)$$

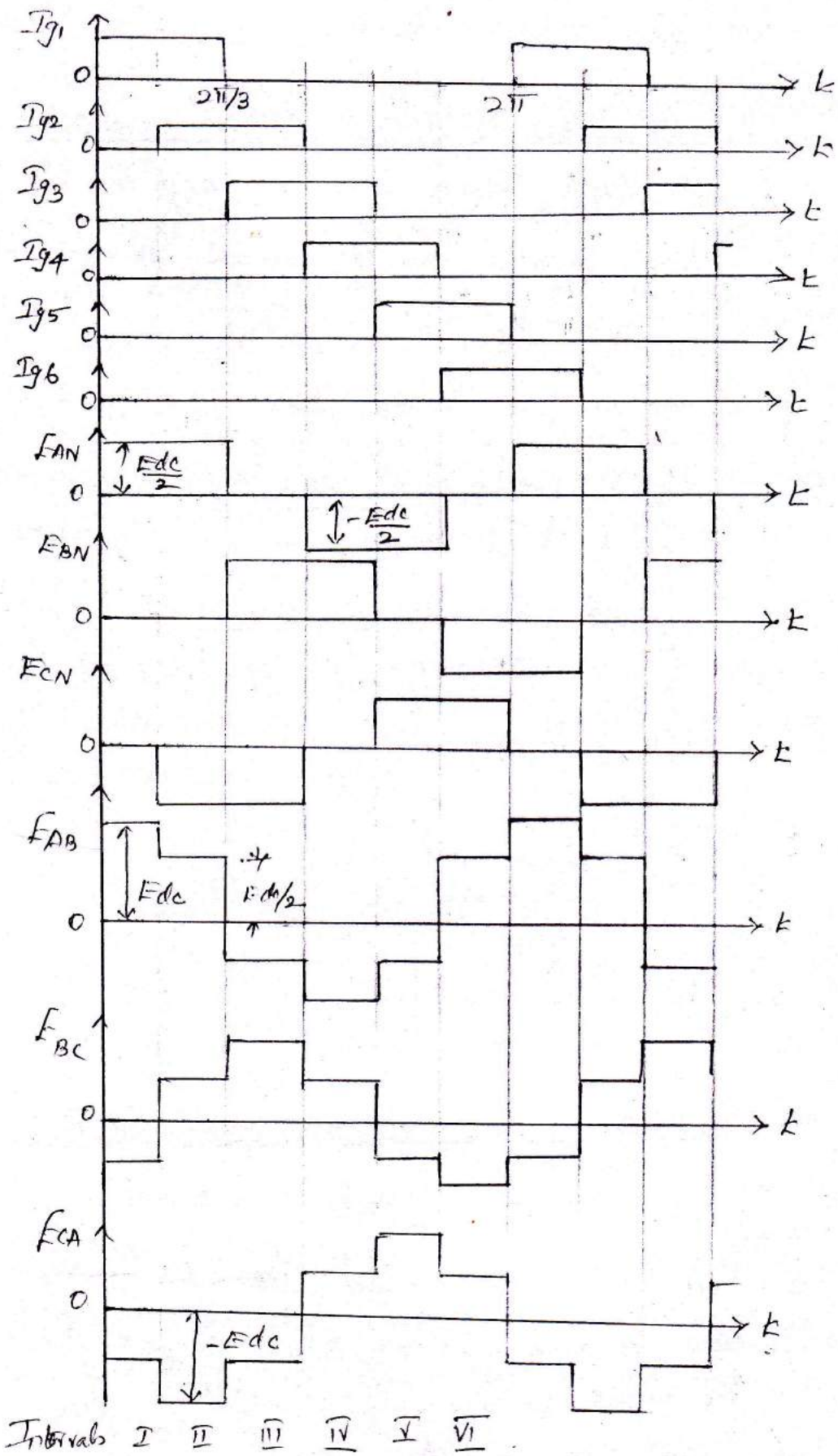
where $\theta_n = \tan^{-1} \left(\frac{n\omega L}{R} \right)$

120° Conduction Mode with Resistive Load.

In this type of conduction mode, each switch conducts for 120° . At any instant of time, only two switches remain on.

Operation Table.

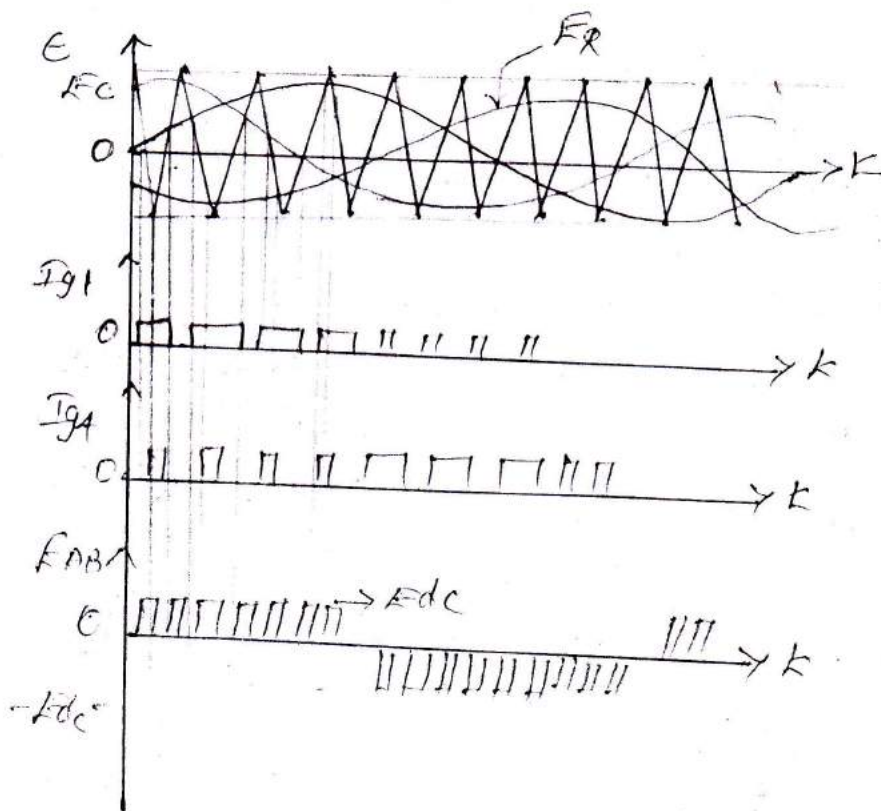
St. NO	Interval	conducting devices	Incoming device	Outgoing device.
1.	I	S_6, S_1	S_1	S_5
2.	II	S_1, S_2	S_2	S_6
3.	III	S_2, S_3	S_3	S_1
4.	IV	S_3, S_4	S_4	S_2
5.	V	S_4, S_5	S_5	S_3
6.	VI	S_5, S_6	S_6	S_4



Grating signals and Voltage waveform for 120° conduction.

VOLTAGE CONTROL OF THREE-PHASE INVERTERS.

A three phase inverter may be considered as three single-phase inverters and the output of each single-phase inverter is shifted by 120° .



Sinusoidal pulse-width modulation for 3 ϕ Inverters

A carrier wave is compared with the reference signal corresponding to a phase to generate the gating signals for that phase. The output voltage, is generated by eliminating the condition that two switching devices in the same leg cannot conduct at the same time.

VOLTAGE CONTROL OF SINGLE-PHASE INVERTERS.

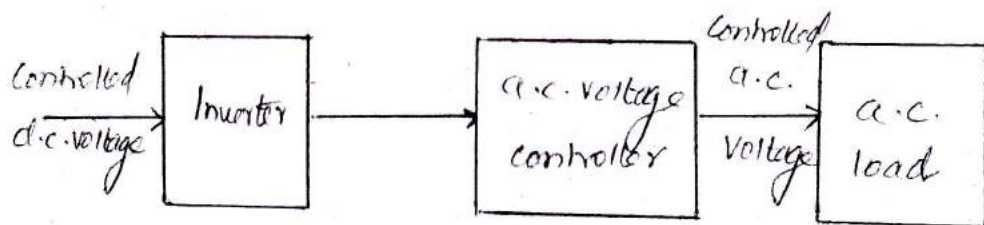
The various methods for the control of output voltage of inverters are

- i) External control of a.c. output voltage.
- ii) External control of d.c. input voltage.
- iii) Internal control of inverter.

The first two methods require the use of peripheral components, whereas the third method requires no peripheral components.

i) External control of a.c. Output Voltage.

In this type of control, an a.c. voltage controller is inserted between the output terminals of inverter and the load terminals.

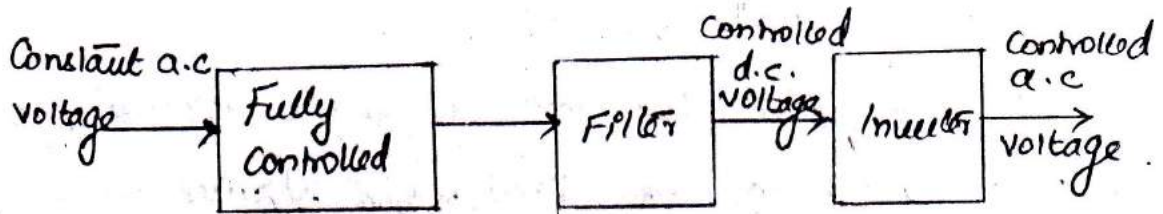


Through the firing angle control of a.c. voltage controller, the voltage input to the a.c. load is regulated. This method gives rise to higher harmonic content in the output voltage, particularly when the output voltage from the a.c. voltage controller is at low level.

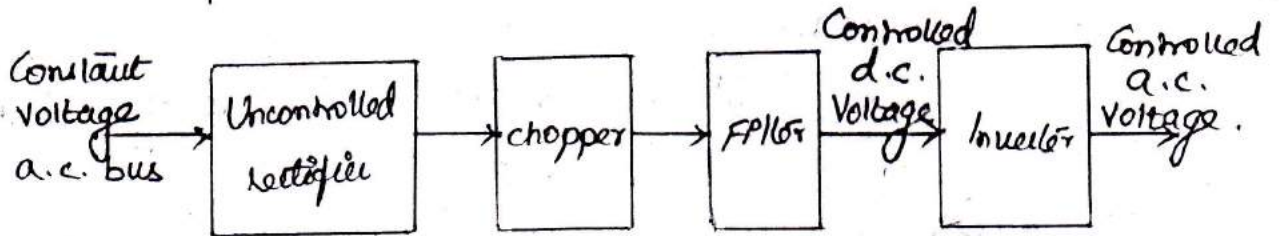
Therefore this method is rarely employed except for low power applications.

(ii) External Control of D.C. Input Voltage.

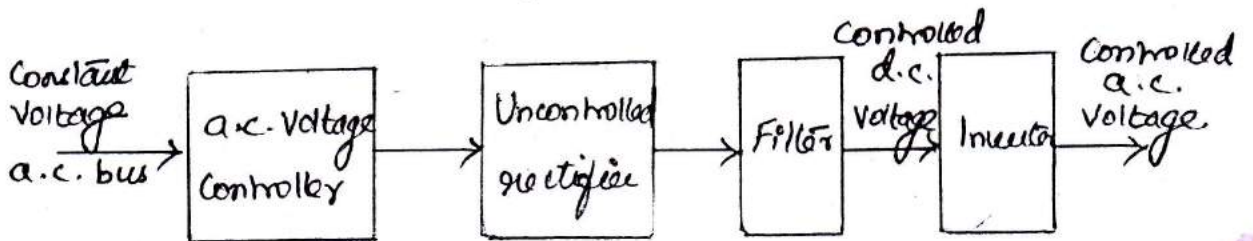
When the available voltage source is a.c., then, d.c. voltage input to the inverter is controlled through a fully-controlled rectifier.



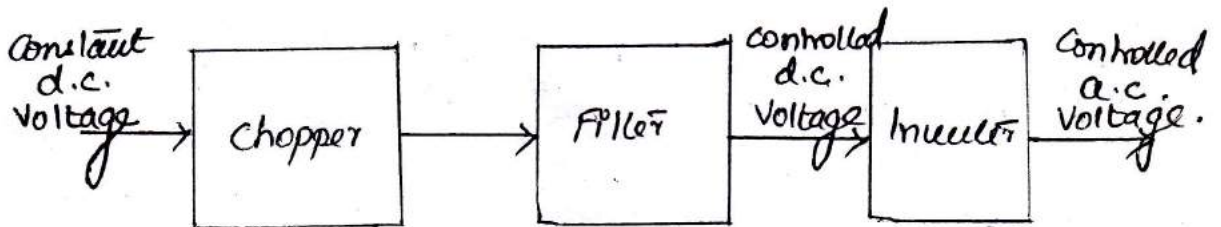
(a)



(b)



(c)



(d)

Voltage control by controlling d.c. i/p voltage.

(iii) Internal control of Inverter.

Inverter output voltage can also be adjusted by exercising a control within the inverter itself. The two possible ways are

1. Series inverter control and
2. Pulse-width modulation control.

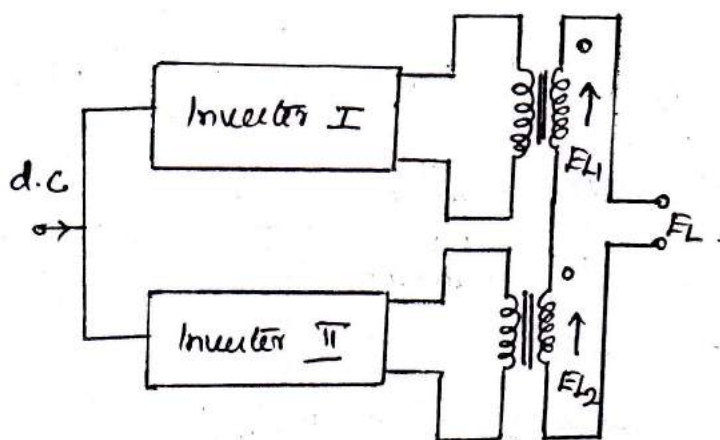
1. Series Inverter control.

This method of voltage control involves the use of two or more inverters in series.

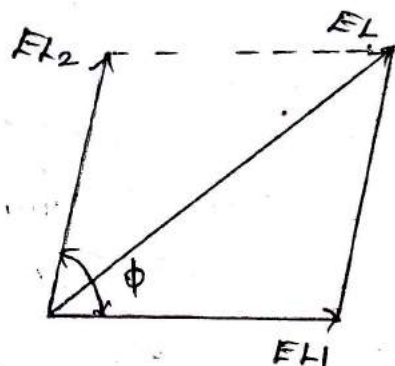
The inverter output is fed to two transformers whose secondaries are connected in series. Phasor sum of the two voltages E_{L1} , E_{L2} gives the resultant voltage E_L .

The voltage E_L is given by

$$E_L = \left[E_{L1}^2 + E_{L2}^2 + 2E_{L1}E_{L2} \cos \theta \right]^{1/2}$$



(a)



(b)

Internal control of inverter by series connection.

2. Pulse - width Modulation Control.

The most efficient method of controlling the output voltage is to incorporate pulse width modulation control (PWM control) within the inverter.

In this method, a fixed d.c. input voltage is supplied to the inverter and a controlled a.c. output voltage is obtained by adjusting the ON and OFF periods of the inverter devices.

Advantages.

1. The output voltage control can be obtained without any additional components.

2. With this type of control, lower order harmonics can be eliminated or minimised along with its output voltage control. The filtering requirements are minimised as higher order harmonics can be filtered easily.

PWM TECHNIQUES.

The commonly used PWM control techniques are

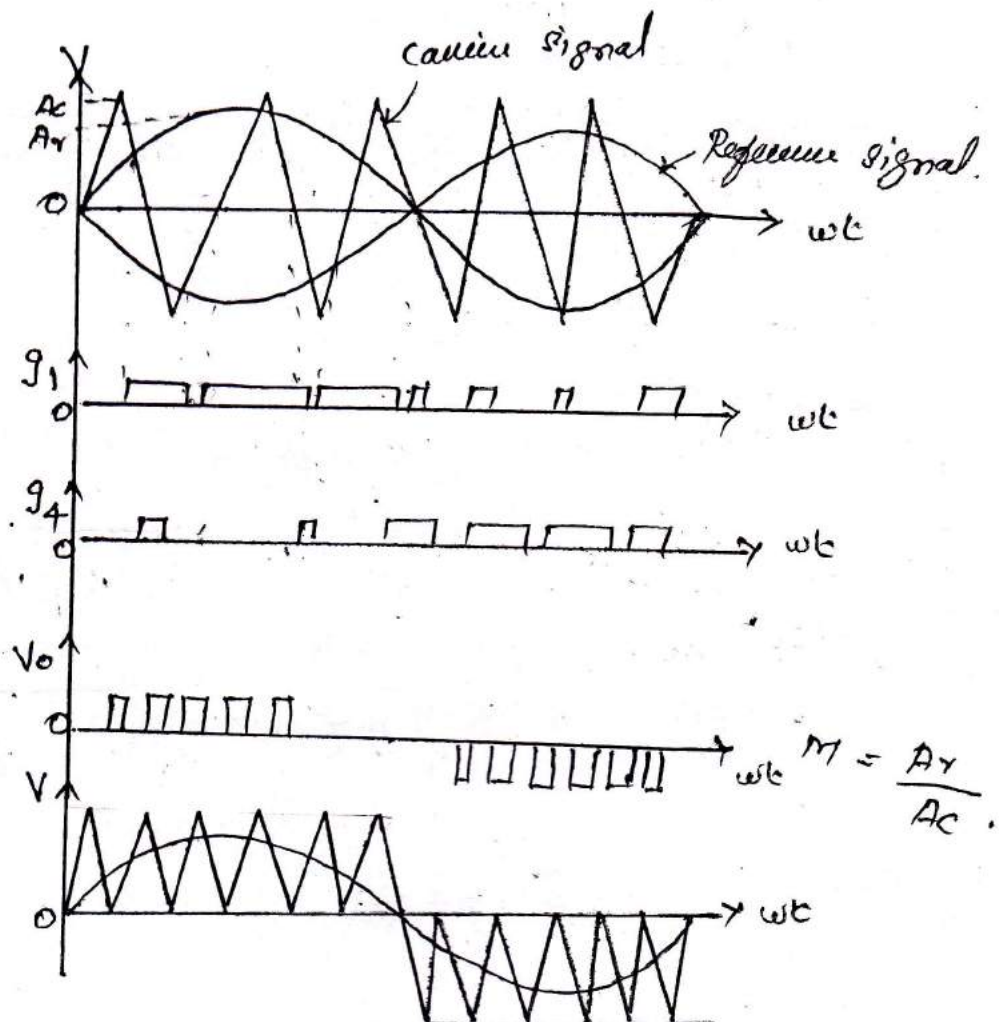
- (i) Single - pulse width modulation
- (ii) Multiple - pulse width modulation
- (iii) Sinusoidal - pulse width modulation
- (iv) Modified sinusoidal pulse - width modulation.
- (v) Phase - displacement control.

i) Sinusoidal pulse-width modulation.

The width of each pulse is varied in proportion to the amplitude of a sine wave evaluated at the centre of the same pulse.

The DF and LOH are reduced significantly. The gating signals are generated by comparing a sinusoidal reference signal with a triangular carrier wave of frequency f_c . This sinusoidal pulse-width modulation (SPWM) is commonly used in industrial applications.

The frequency of reference signal f_r determines the inverter output frequency f_o and its peak amplitude A_r controls the modulation index M and then in turn the rms output voltage V_o .



$V_r = V_r \sin \omega t$, is reference signal.

$V_o = V_s (g_1 - g_4)$, is output voltage.

\therefore The rms output voltage is

$$V_o = V_s \left(\sum_{m=1}^{2p} \frac{\delta_m}{\pi} \right)^2$$

Fourier coefficient of output voltage is

$$B_n = \sum_{m=1}^{2p} \frac{4V_s}{n\pi} \sin \frac{n\delta_m}{4} \left[\sin n \left(d_m + \frac{3\delta_m}{4} \right) - \sin n \left(\pi + d_m + \frac{\delta_m}{4} \right) \right]$$

for $n = 1, 3, 5, \dots$

The m^{th} time t_m and angle d_m of intersection can be determined from

$$t_m = \frac{d_m}{\omega} = t_x + m \frac{T_s}{2}$$

where t_x can be solved from

$$1 - \frac{2t}{T_s} = m \sin \left[\omega \left(t_x + \frac{mT_s}{2} \right) \right] \text{ for } m = 1, 3, \dots, 2p$$

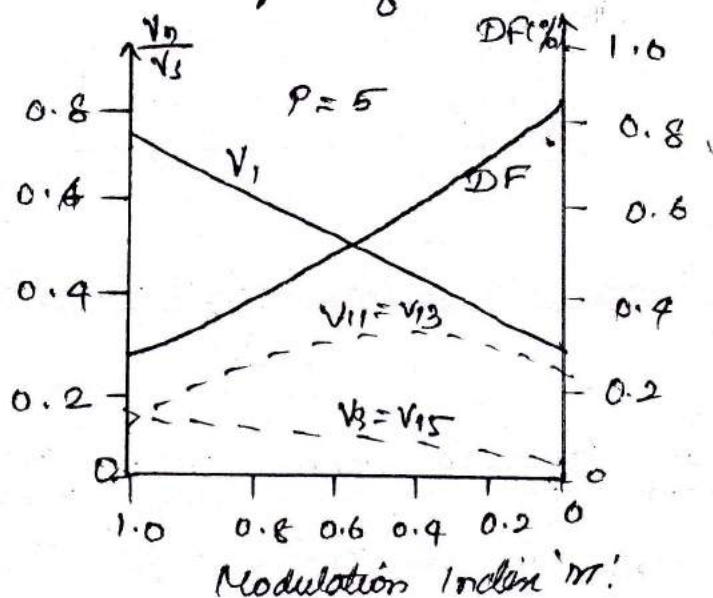
$$\frac{2t}{T_s} = m \sin \left[\omega \left(t_x + \frac{mT_s}{2} \right) \right] \text{ for } m = 2, 4, \dots, 2p$$

where $T_s = T/2(p+1)$.

The width of the m th pulse d_m (or pulse angle δ_m) can be found from

$$d_m = \frac{\delta_m}{\omega} = t_{m+1} - t_m.$$

Harmonic profile of sinusoidal pulse-width modulation.



The output voltage of an inverter contains harmonics. The PWM pushes the harmonics into a high-frequency range around the switching frequency f_c and its multiples, i.e. around harmonics $m_f, 2m_f, 3m_f, \dots$

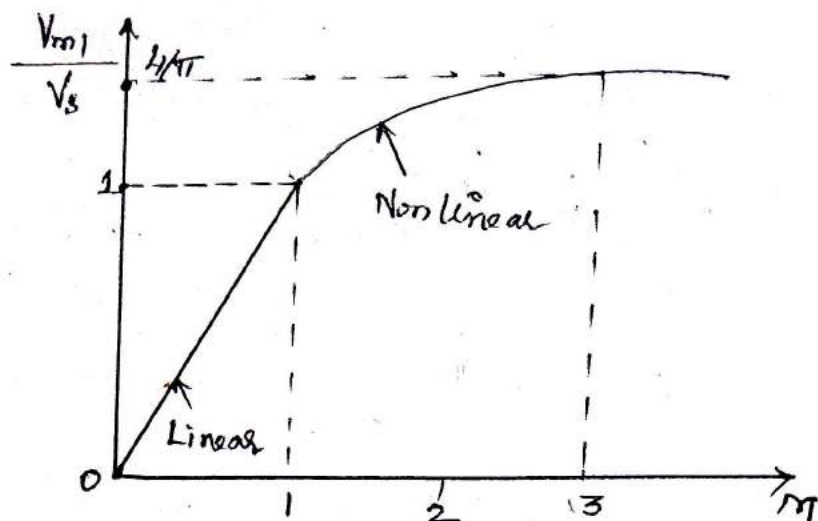
The frequencies at which the voltage harmonics occur can be related by

$$f_n = (j m_f \pm k) f_c$$

where the n th harmonic equals the k th sideband of j th times the frequency to modulation ratio m_f .

$$n = j m_f \pm k.$$

$$= 2j p \pm k \text{ for } j = 1, 2, 3, \dots \text{ and } k = 1, 3, 5, \dots$$



Peak fundamental Output voltage versus modulation index:

(ii) Modified Sinusoidal Pulse-Width Modulation.

The characteristics of a sine wave and the SPWM technique can be modified so that the carrier wave is applied during the first and last 60° intervals per half-cycle (e.g. 0° to 120° to 180°).

The fundamental component is increased and its harmonic characteristics are improved. It reduces the number of switching of power devices and also reduces switching losses.

The m th time t_m and angle α_m of intersection can be determined from

$$t_m = \frac{\alpha_m}{\omega} = k_\alpha + \frac{m\pi}{2} \quad \text{for } m=1,2,3,\dots,p$$

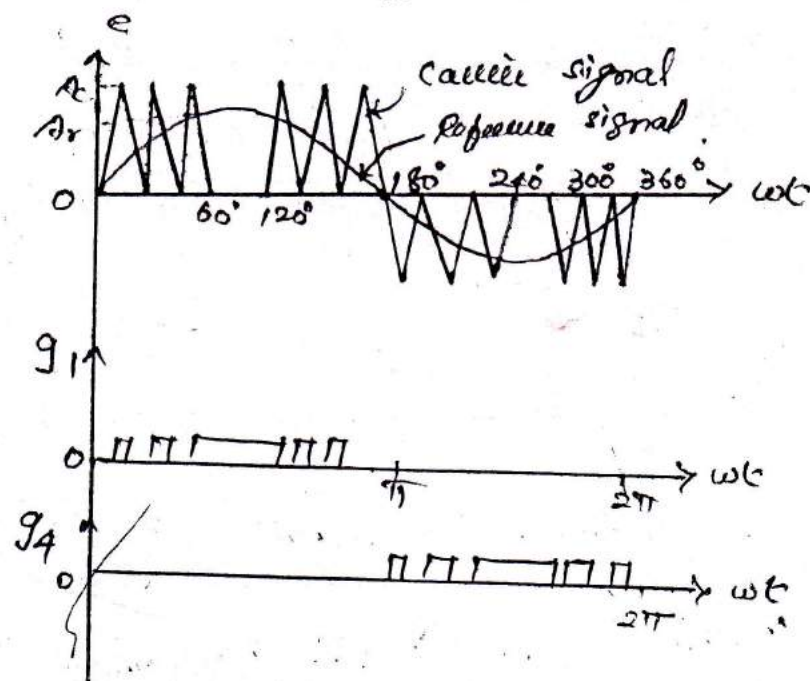
where t_x can be solved from

$$1 - \frac{2k}{T_s} = M \sin \left[\omega \left(t_x + \frac{mT_s}{2} \right) \right] \quad \text{for } m = 1, 3, \dots, p$$

$$\frac{2k}{T_s} = M \sin \left[\omega \left(t_x + \frac{mT_s}{2} \right) \right] \quad \text{for } m = 2, 4, \dots, p$$

The time interval during the last 60° intervals can be found from

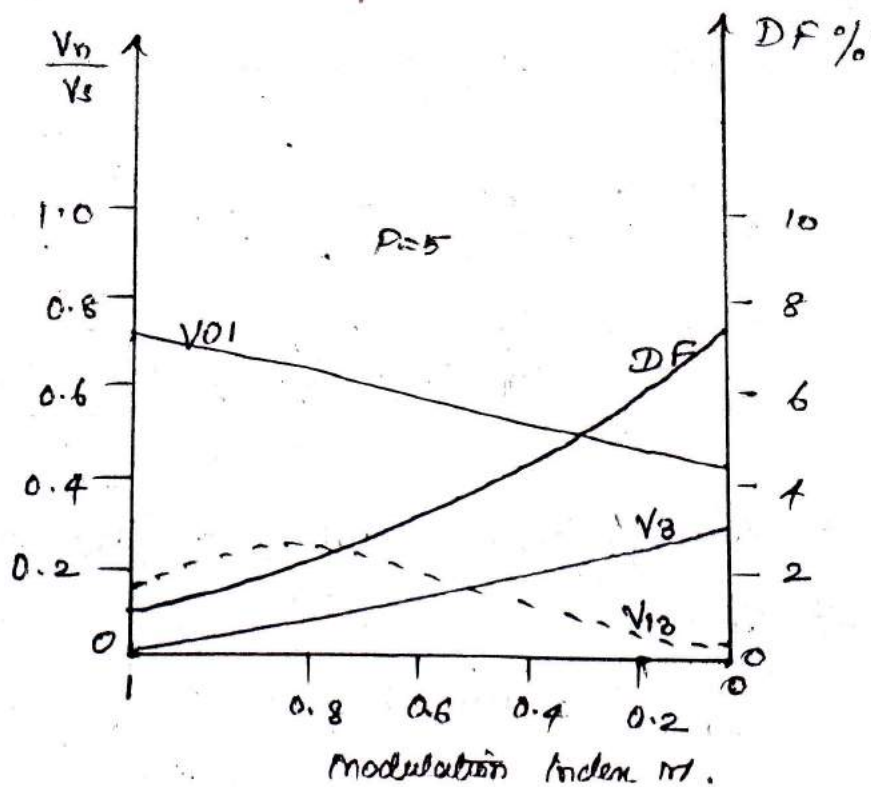
$$t_{m+1} = \frac{d_{m+1}}{\omega} = \frac{T}{2} - t_{2p-m} \quad \text{for } m = p, p+1, \dots, 2p-1$$



Modified Sinusoidal pulse-width modulation

where $T_s = T/b(p+1)$. The width of the m th pulse d_m (or pulse angle δ_m) can be found from

$$d_m = \frac{\delta_m}{\omega} = t_{m+1} - t_m$$



Harmonic Profile of Modified Sinusoidal pulse-width Modulation

The number of pulses q in the 60° period is normally related to the frequency ratio, particularly in three-phase inverters, by

$$\frac{f_c}{f_o} = 6q + 3.$$

The instantaneous output voltage is $v_o = V_s (g_1 - g_2)$.

(iii) Multiple - Pulse - Width Modulation.

The harmonic content can be reduced by using several pulses in each half-cycle of output voltage.

The generation of gating signals for turning on and off of transistors is done by comparing a reference

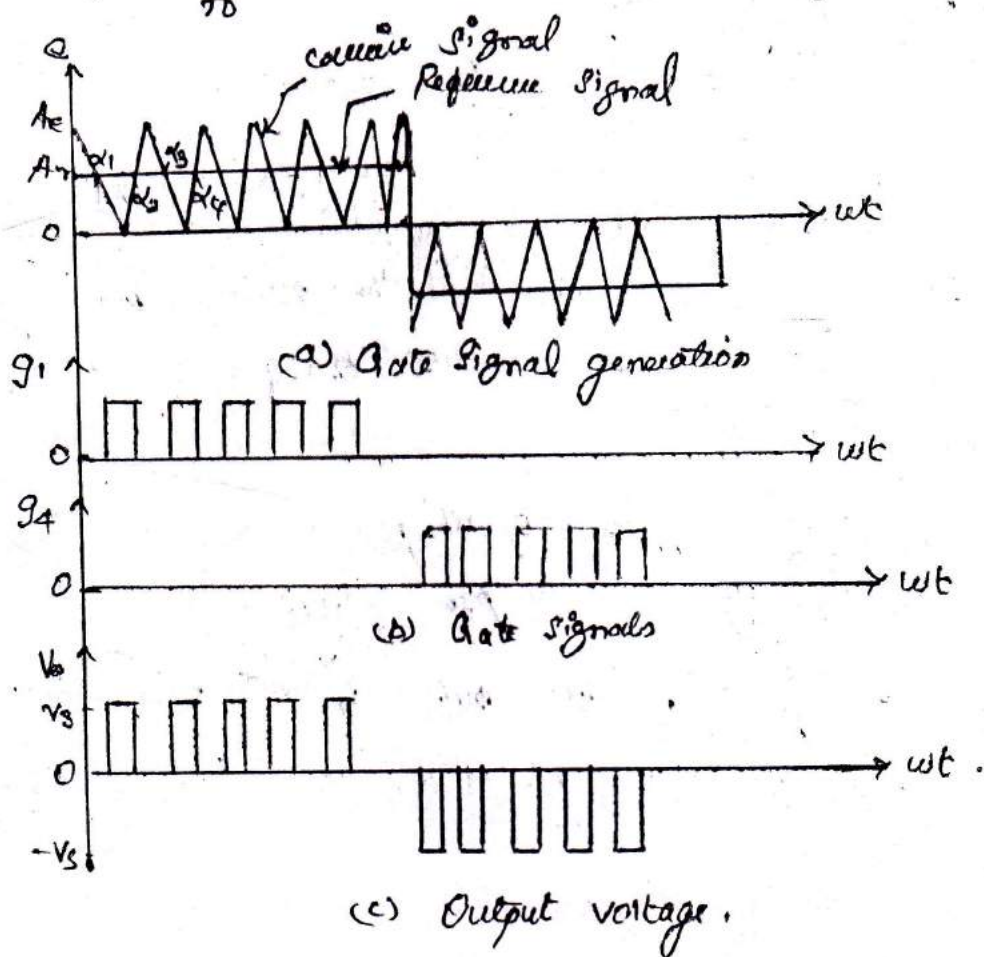
signal with a triangular carrier wave. The frequency of reference signals sets the output frequency f_0 and the carrier frequency f_c determines the number of pulses per half-cycle p .

The modulation index controls the output voltage. This type of modulation is also known as uniform pulse-width modulation (UPWM).

The number of pulses per half cycle is found from

$$p = \frac{f_c}{2f_0} = \frac{m_f}{2}$$

where $m_f = \frac{f_c}{f_0}$ is defined as the frequency modulation ratio.



Multiple pulse-width modulation.

The instantaneous output voltage is $V_o = V_s (g_1 - g_2)$.

If δ is the width of each pulse, the rms output voltage can be found from

$$V_o = \left[\frac{2P}{2\pi} \int_{(\pi/p - \delta)/2}^{(\pi/p + \delta)/2} V_s^2 d(\omega t) \right]^{1/2} = V_s \sqrt{\frac{p\delta}{\pi}}$$

The variation of the modulation index M from 0 to 1, varies the pulse width δ from 0 to $T/2p$ (0 to π/p) and the rms output voltage V_o from 0 to V_s .

The general form of a Fourier series for the instantaneous output voltage is

$$V_o(t) = \sum_{n=1,3,5}^{\infty} B_n \sin n\omega t.$$

The coefficient B_n can be determined by considering a pair of pulses such that the positive pulse of duration δ starts at $\omega t = \alpha$ and the negative one of the same width starts at $\omega t = \pi + \alpha$.

The effects of all pulses can be combined together to obtain the effective output voltage.

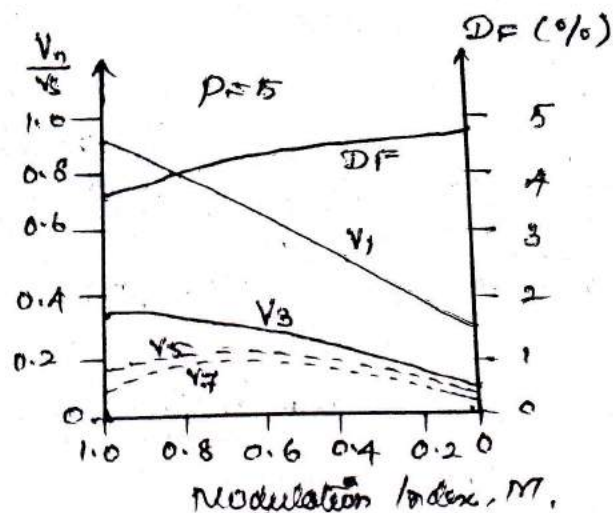
If the positive pulse of m th pair starts at $\omega t = d_m$ and ends at $\omega t = d_m + \delta$, the Fourier coefficient for a pair of pulses is

$$b_n = \frac{2}{\pi} \left[\int_{\alpha_m + \delta/2}^{\alpha_m + \delta} \sin n\omega t d(\omega t) - \int_{\pi + \alpha_m}^{\pi + \alpha_m + \delta/2} \sin n\omega t d(\omega t) \right]$$

$$= \frac{4V_s}{\pi n} \frac{\sin n\delta}{4} \left[\sin n \left(\alpha_m + \frac{3\delta}{4} \right) - \sin n \left(\pi + \alpha_m + \frac{\delta}{4} \right) \right]$$

The coefficient B_n can be found by adding the effects of all pulses.

$$B_n = \sum_{m=1}^{2p} \frac{4V_s}{\pi n} \frac{\sin n\delta}{4} \left[\sin n \left(\alpha_m + \frac{3\delta}{4} \right) - \sin n \left(\pi + \alpha_m + \frac{\delta}{4} \right) \right]$$



Harmonic profile of multiple-pulse width modulation

The m th time t_m and angle α_m of intersection can be determined from

$$t_m = \frac{\alpha_m}{\omega} = (m-1) \frac{T_s}{2} \quad \text{for } m = 1, 3, \dots, 2p.$$

$$t_m = \frac{\alpha_m}{\omega} = (m-1 + M) \frac{T_s}{2} \quad \text{for } m = 2, 4, \dots, 2p.$$

Since all widths are same, pulse width d (or pulse angle δ) becomes,

$$d = \frac{\delta}{\omega} = t_{m+1} - t_m = \pi T_s$$

where $T_s = T/2\pi$.

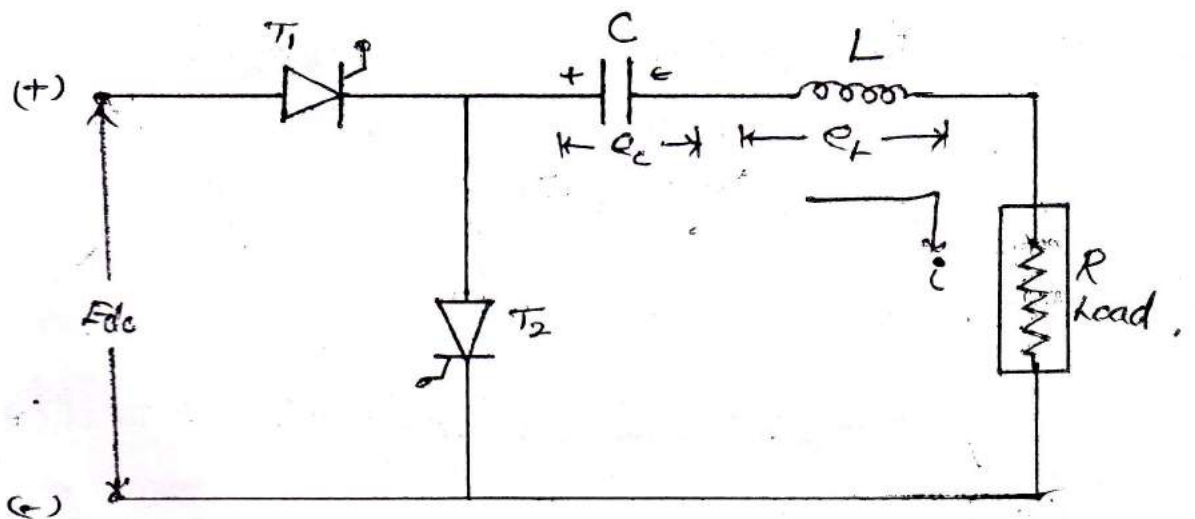
SERIES INVERTERS (SERIES RESONANT INVERTERS)

The commutating elements, L and C are connected in series with load is called series inverter. This constitute a series RLC resonant circuit.

If the load is purely resistive, it only has resistance in the circuit.

In case of load being inductive or capacitive in nature, its inductance or capacitance part is added to the commutating elements.

This type of thyristor inverter produces an approximately sinusoidal waveform at a high output frequency ranging from 200 Hz to 100 KHz and is commonly used in relatively fixed output applications such as ultrasonic generators, induction heating, sonar transmitter, fluorescent lighting, etc. Due to the high-switching frequency, the size of commutating components is small.



Circuit Diagram.

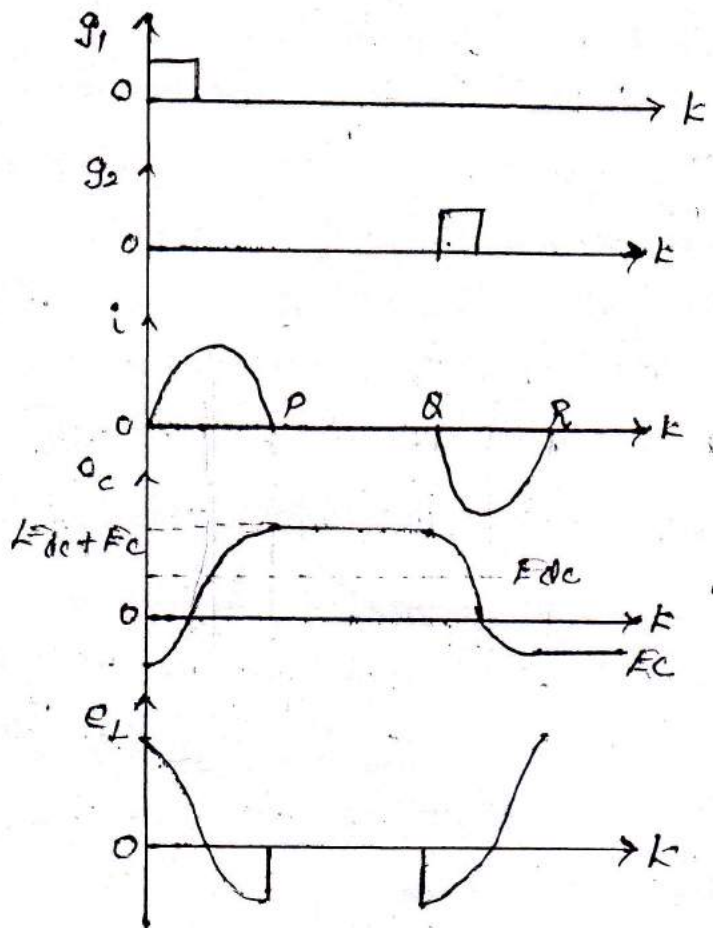
Two thyristors T_1 and T_2 are used to produce two pulses in the output.

The values of L and C are chosen such that, they form an underdamped circuit. This is necessary to produce the required oscillations.

This condition is fulfilled by selecting L and C such that

$$R^2 < \frac{4L}{C}$$

The operation of a basic series inverter circuit can be divided into following three operating modes.



Voltage and current waveforms

Mode: 1.

This mode begins when a d.c. voltage E_{dc} is applied to the circuit and thyristor T_1 is triggered by giving external pulse to its gate.

As soon as T_1 is triggered, it starts conducting and resulting in some current to flow through the RLC series circuit.

Capacitor C gets charged up to voltage, say E_c with positive polarity on its left plate and negative polarity on its right plate. The load current is of alternating nature.

This is due to the underdamped circuit formed by the commutating elements.

It starts building up in the positive half, goes gradually to its peak-value, then starts returning and again becomes zero.

When the current reaches its peak-value, the voltage across the capacitor is approximately the supply voltage E_{dc} . After this, the current starts decreasing but the capacitor voltage still increases and finally the current becomes zero but the capacitor retains the highest voltage ($E_{dc} + E_c$), where E_c is the initial voltage across the capacitor at the instant SCR T_1 was turned-on. At P, SCR is automatically turned-off because the current flowing through it becomes zero.

Mode: 2

During this mode, the load current remains at zero for a sufficient time (T_{off}). Therefore, both the

thyristors T_1 and T_2 are OFF. During this period PQ , capacitance voltage will be held constant.

Mode: 3.

Since the positive polarity of the capacitor C appears on the anode of SCR T_2 , it is in conducting mode and hence triggers immediately.

At Q , SCR T_2 is triggered. When SCR T_2 starts conducting, capacitor C gets discharged through it.

Thus, the current through the load flows in the opposite direction forming the negative alternation. This current builds up to the negative minimum and then decreases to zero at point R .

SCR T_2 will then be turned-off. Now the capacitor voltage reverses to some value depending upon the values of R , L and C .

Again, after some time delay (T_{off}), SCR T_1 is triggered and in the same fashion other cycles are produced. This is a chain process giving rise to alternating output almost sinusoidal in nature.

The output frequency is given by

$$F = \left[\frac{1}{T/2 + T_{off}} \right] \text{ Hz.}$$

where T is the time period for oscillation and is given by

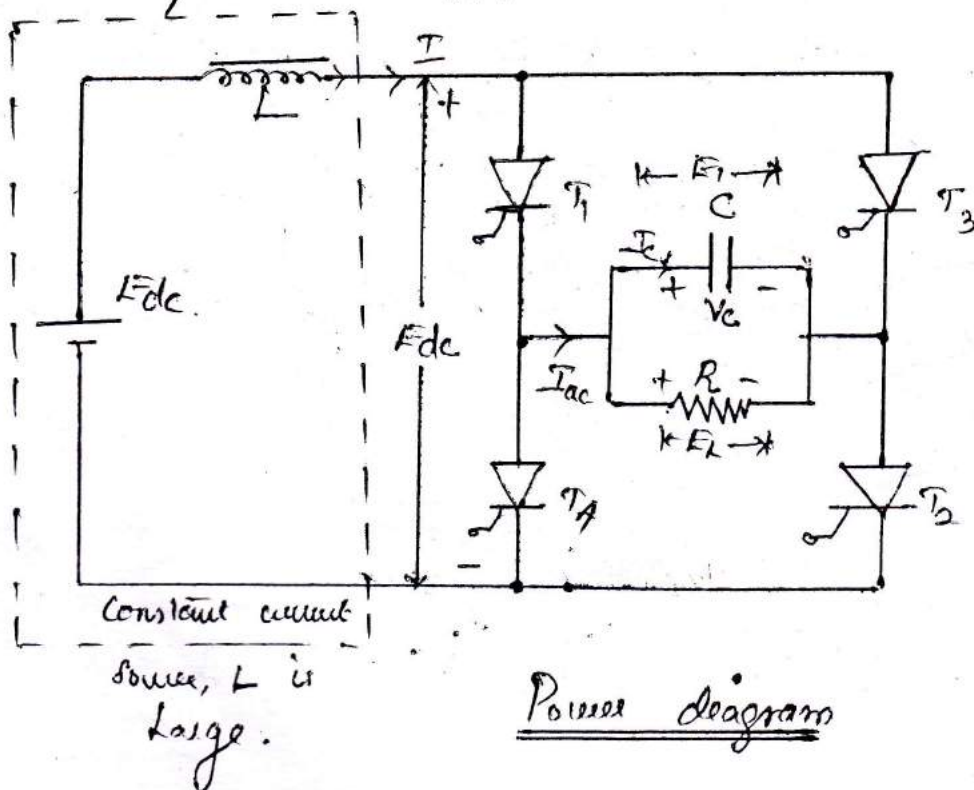
$$\frac{T}{2} = \frac{\pi}{\sqrt{1/LC - R^2/4L}}$$

CURRENT SOURCE INVERTERS.

In a current-source inverter (CSI), the current from the d.c. source is maintained at an effectively constant level, irrespective of load or inverter conditions. This is achieved by inserting a large inductance in series with the d.c. supply to enable changes of inverter voltage to be accommodated at low values of di/dt .

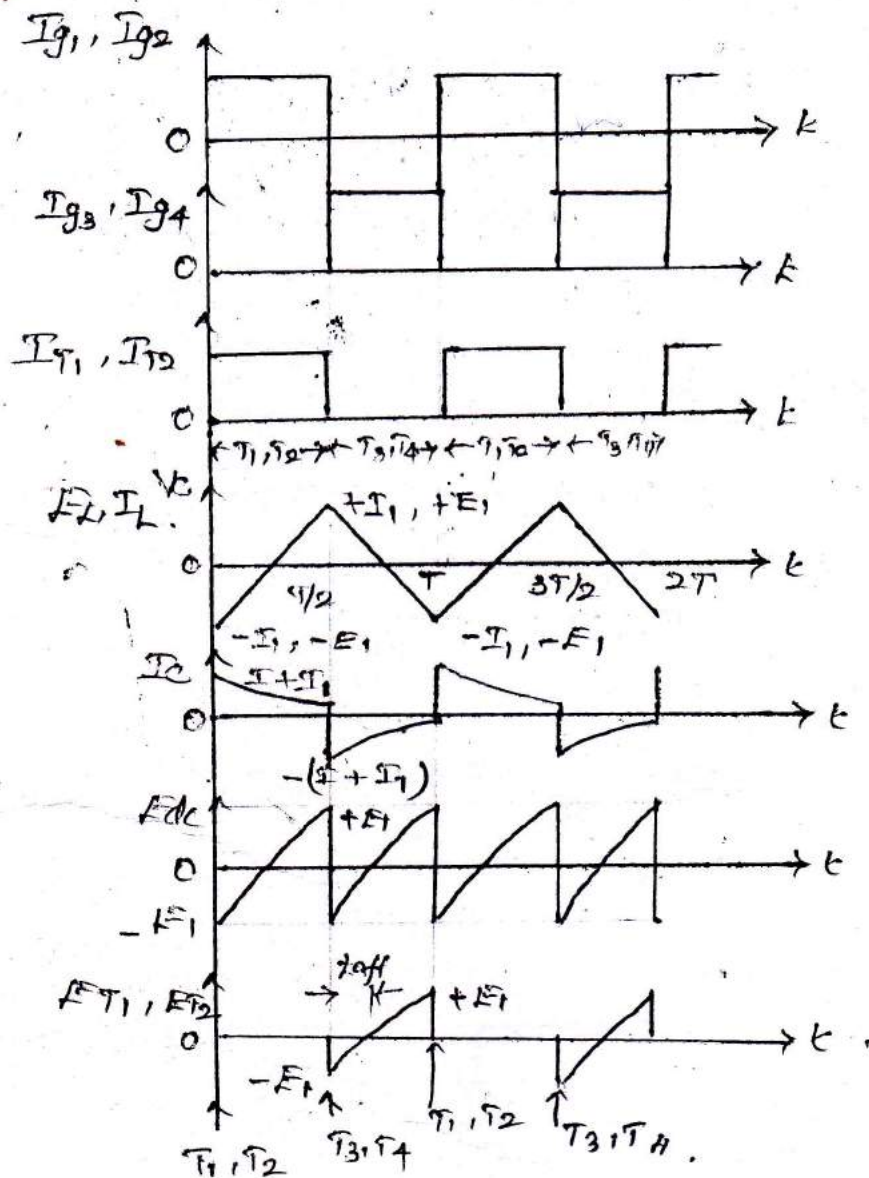
The d.c. input to current-source inverter is obtained from a fixed voltage a.c. source through a controlled rectifier-bridge, or through a diode bridge and a chopper.

In order that current input to CSI is almost ripple free, L-filter is used before CSI. As it is a constant-current system, the current sourced inverter is used typically to supply high-power factor loads whose impedance either remains constant or decreases



that of V_c . When T_3, T_4 are gated at $t = T/2$, $V_c = E_1$ reverse-biases T_1, T_2 . Hence turned-off immediately. The source current now flows through T_3 , parallel combination of R, C and T_4 .

From instant $\frac{T}{2}$ to T , $I_{T_3} = I_{T_4} = I$, but $I_{sc} = -I$. The variation of a.c. current I_{ac} is a square wave of amplitude I .



Voltage and current waveforms

Appl: Speed ctrl of high power ac drive,
 Induction heating,
 Static VAR compensation,
 used as power supplies in aircraft (or)
 shipboard

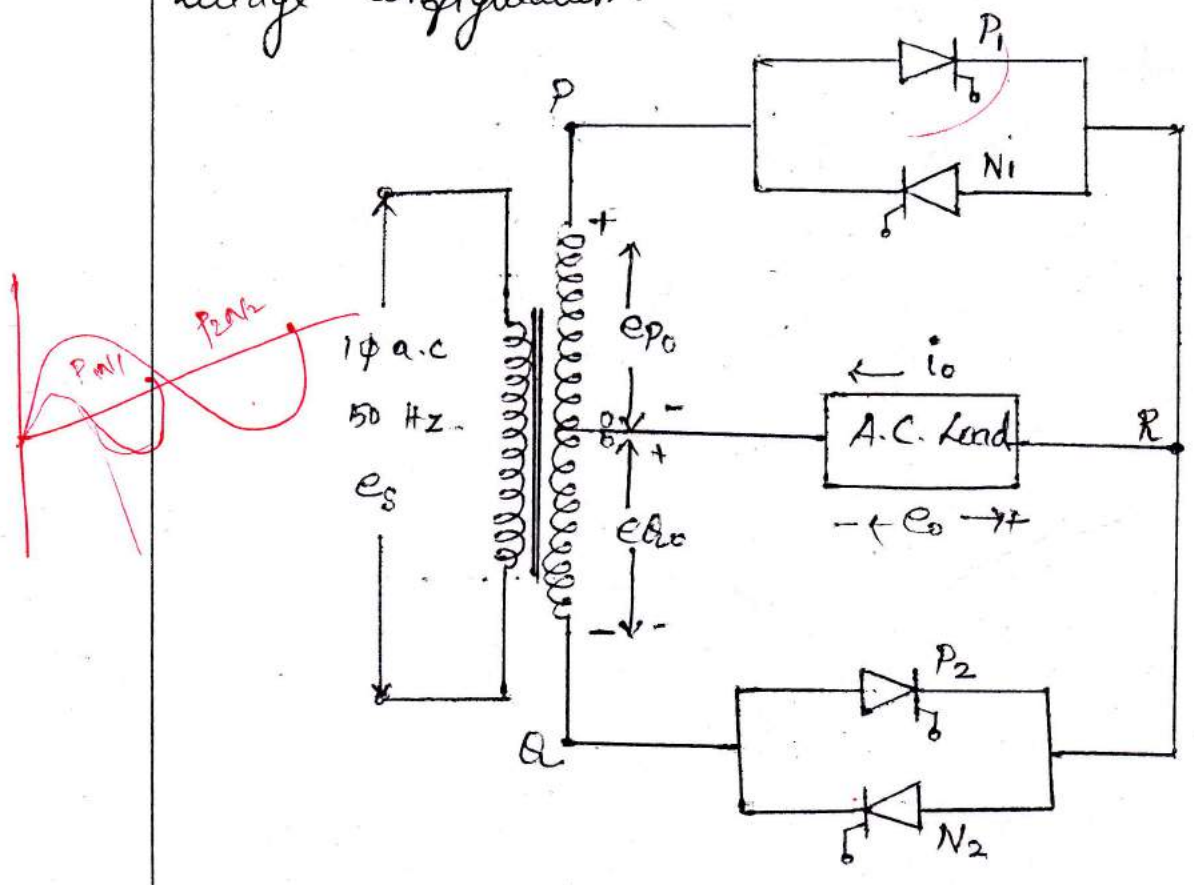
CYCLOCONVERTERS. (One stage freq changer)

A cycloconverter is a type of power controller in which an alternating voltage at supply frequency is converted directly to an alternating voltage at load frequency without any intermediate d.c. stage. types - step down $f_o < f_s \rightarrow$ supply freq \rightarrow load freq
 - step-up $f_o > f_s$

1 ϕ to 1 ϕ CYCLOCONVERTER.

In a single-phase cycloconverter whose input and output are single phase a.c. the input a.c. voltage of supply frequency 50 Hz is converted into lower frequency a.c. output.

There are mainly two configurations for this type of cycloconverter, viz. centre-tapped transformer configuration and bridge configuration.



Single phase to Single phase Cycloconverter circuit.

skoup

harmonic frequencies in order to prevent problems either on switching or with harmonic overvoltages.

Applications of current source inverters are

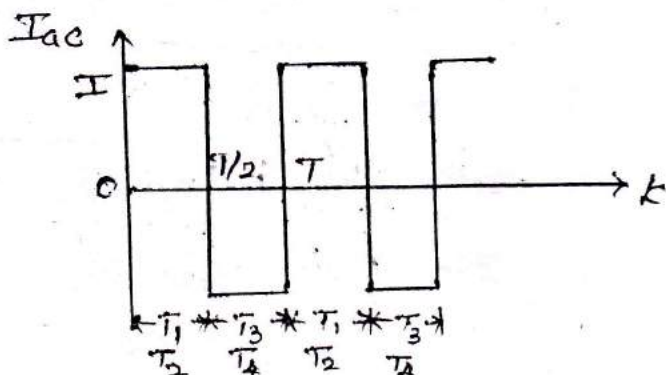
- i) Speed control of a.c. motors
- ii) Inducting heating
- iii) Lagging VAR compensation
- iv) Synchronous motor starting, etc.

Current source inverters may be either load commutated or force commutated. Load commutation is possible when load PF is leading.

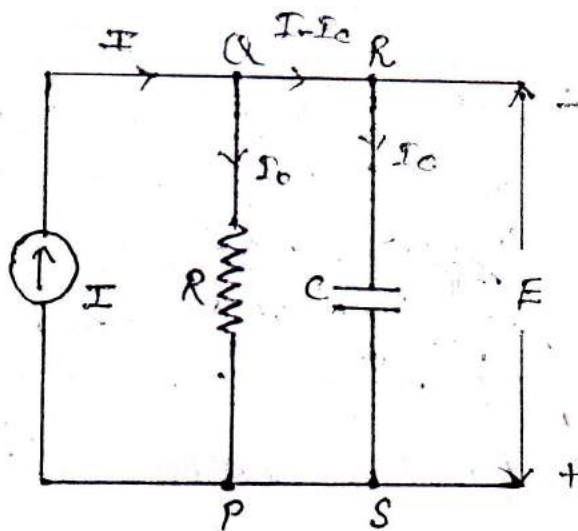
For lagging PF loads, forced commutation is essential. Use of commutating capacitor is an important feature of force-commutated current-source inverters.

Single-Phase Capacitor-Commutated Current Source Inverter with Resistive Load.

Capacitor C in parallel with load is used for storing the charge for force commutating the SCRs. Thyristors T_1, T_2, T_3 and T_4 form the power bridge. These SCRs are triggered in pairs. T_1, T_2 together by gating signal I_{g1}, I_{g2} and T_3, T_4 by I_{g3}, I_{g4} .



a.c. output current waveform.



Equivalent Circuit

Before $t = 0$, let the capacitor voltage be $V_c = -E_1$, i.e. capacitor has right positive and left plate negative. At $t = 0$, thyristors T_1 and T_2 are triggered, and when T_1 and T_2 become turned on, capacitor applies reverse voltage across the previously conducting thyristors T_3, T_4 and hence turn them off.

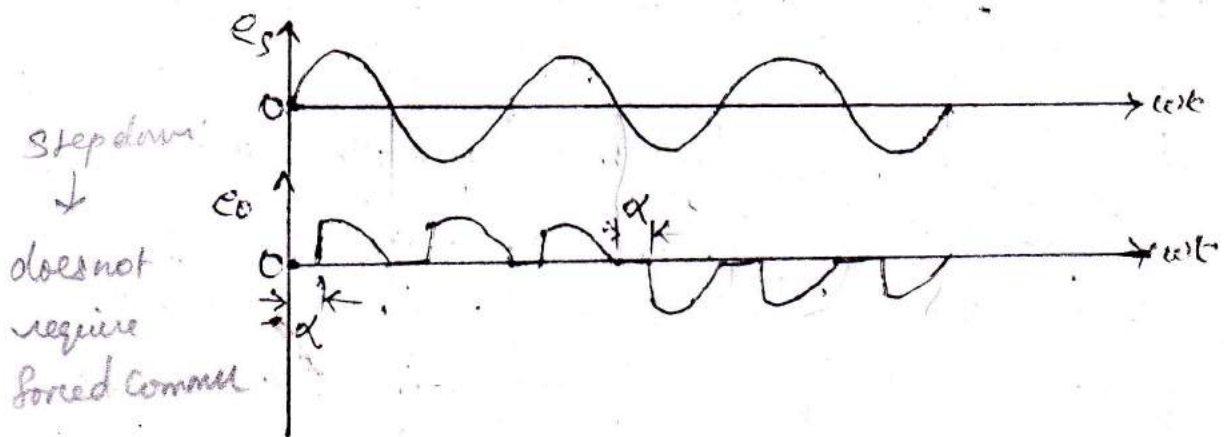
The source current I now flows through T_1 , parallel combination of R and C and through T_2 . From 0 to $T/2$, $I_{T_1} = I_{T_2} = I$, output current $I_{ac} = I$, capacitor voltage V_c changes from $-E_1$ to $+E_1$ through the charging of C by current I_c .

Note that, load voltage $E_L = V_c$. Thus the waveform of $I_L = E_L / R = V_c / R$ has the same nature as

(i) Centre-tapped Transformer Configuration

There are 4 thyristors, namely P_1, N_1, P_2 and N_2 . Out of the four SCRs, SCRs P_1 and P_2 are responsible for generating the positive halves forming the positive group. The other two SCRs N_1 and N_2 , are responsible for producing the negative halves forming the negative group.

This configuration is meant for generating $\frac{1}{3}$ of the input frequency i.e., this circuit generates a frequency of $16\frac{2}{3}$ Hz at its output.

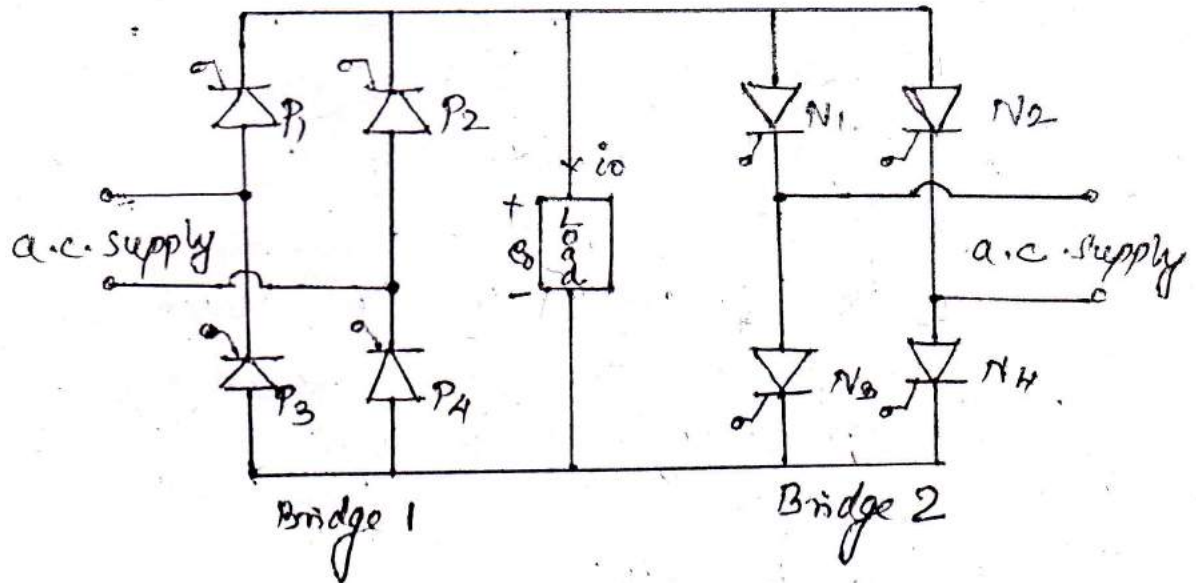


Input and Output waveforms of a $16\frac{2}{3}$ Hz cycloconverter.

(ii) Bridge Configuration.

Two single phase fully-controlled bridges are connected in opposite directions.

Bridge 1 supplies load current in the positive half of the output cycle and bridge 2 supplies load current in the negative half of the output cycle. The two bridges should not conduct together as this will produce a short-circuit at the input.



Bridge Configuration Single phase Cycloconverter.

Instead of one thyristor in the control-tap transformer configuration, two thyristors come in series with each voltage source in the bridge configuration.

For resistive loads, the SCR undergoes natural commutation and produce discontinuous current operation.

For inductive loads, the load current may be continuous or discontinuous, depending upon the firing angle and load power factor.

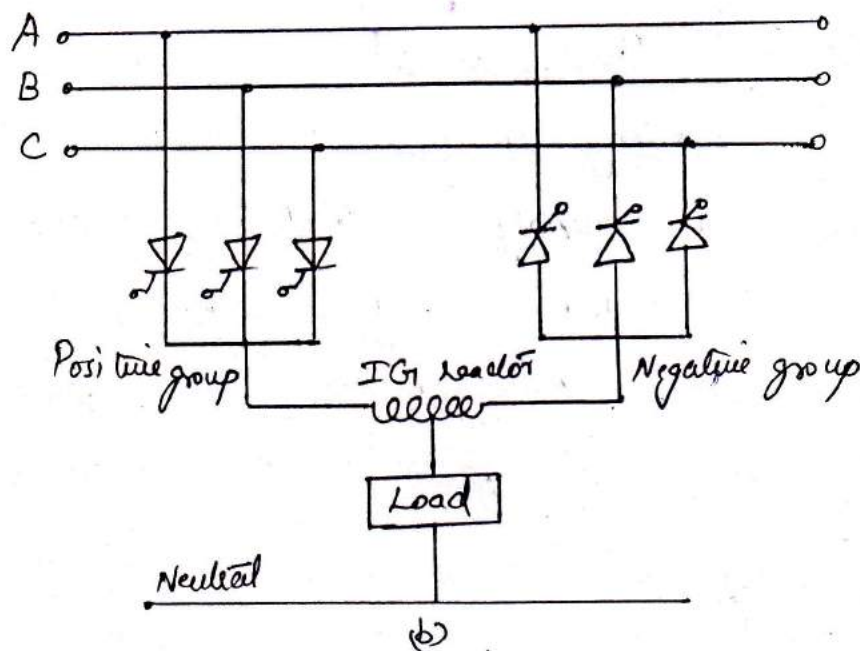
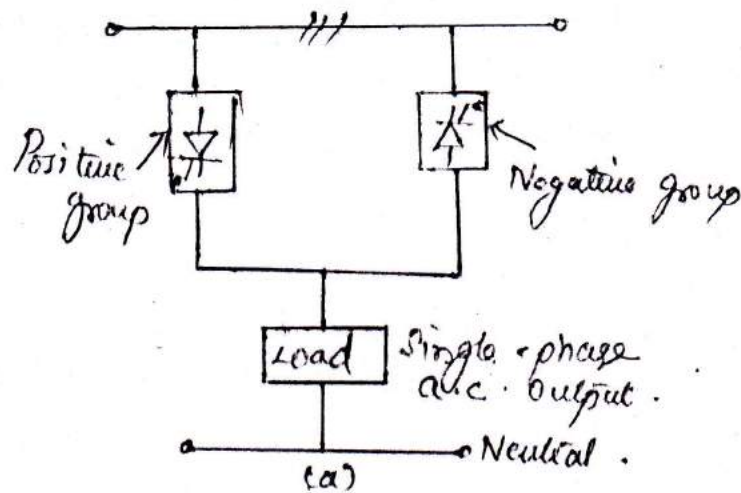
When the load current is positive, the firing pulses to the SCR of bridge 2 will be inhibited and bridge 1 will be gated.

Similarly, when the load current is negative bridge 2 will be gated and the firing pulses will not be applied to the SCR in bridge 1. This is the circulating current free mode of operation.

3 ϕ to 1 ϕ Cycloconverters.

The type of three phase cycloconverter depends on the number of pulses used. The amount of ripple content can be reduced by increasing the number of pulses used.

In a thyristor converter circuit, current can only flow in one direction. For allowing the flow of current in both the direction during one complete cycle of load current, two three-phase half-wave converters must be connected in antiparallel.

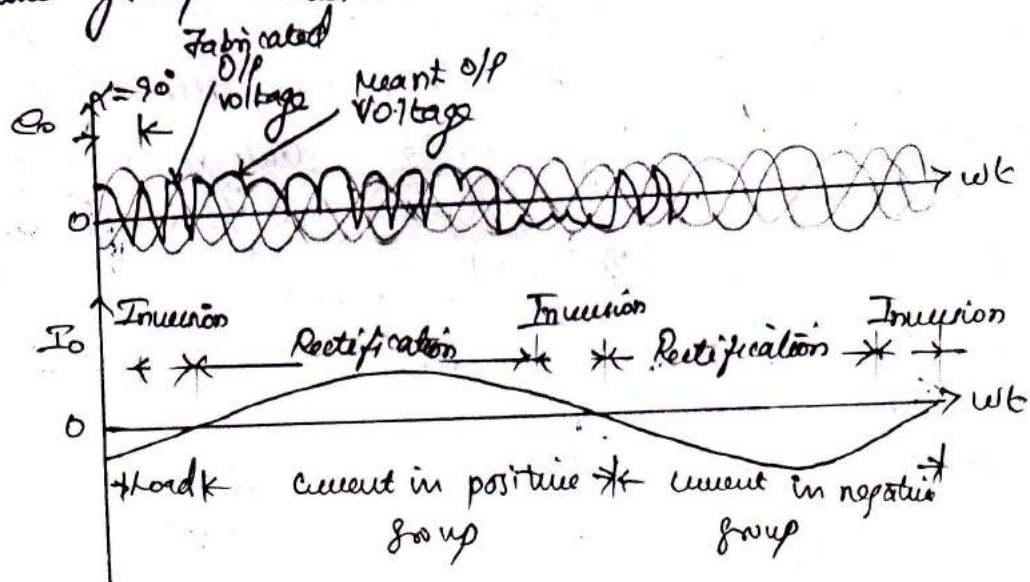


Three-phase to single-phase cycloconverter (a) schematic diagram

(b) basic circuit configuration with IGT reactor.

The converter circuit that permits the flow of current during positive half-cycle of low-frequency output current is called as positive-group converter.

The other group permitting the flow of current during the negative half-cycle of output current is called as negative group converter.



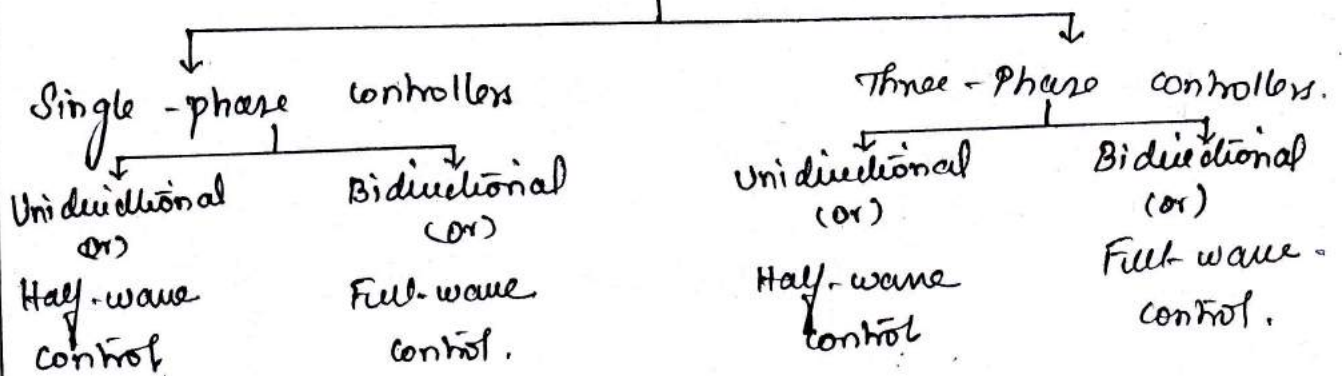
Voltage and current waveforms for a three-phase half-wave thyristor converter

AC VOLTAGE REGULATORS.

By connecting a reverse parallel pair of thyristors or Triac between a.c. supply and load, the voltage applied to the load can be controlled. This type of power controller is known as an a.c. voltage controller or a.c. regulator. Therefore a voltage regulator converts fixed mains voltage directly to variable alternating voltage without a change in the frequency.

- Applications of A.C. Voltage controllers are
1. Speed control of polyphase induction motors,
 2. Domestic and industrial heating,
 3. Light controls
 4. On-load transformer tap changing,
 5. Static reactive power compensators etc.

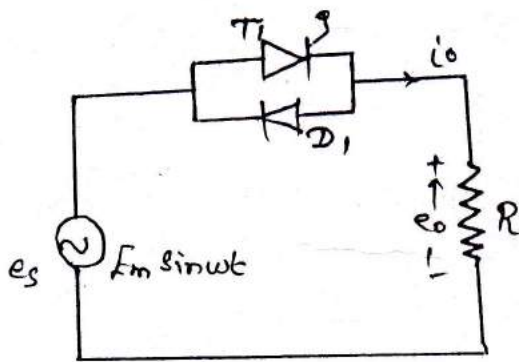
Classification of a.c. voltage controllers



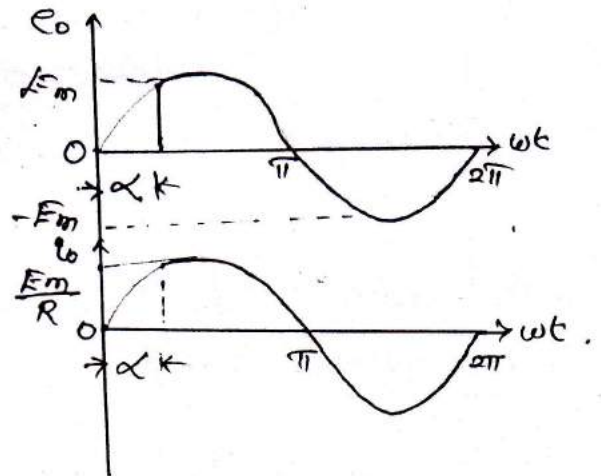
(i) Half-wave a.c. voltage regulators.

One thyristor is connected with one diode in antiparallel condition.

The power flow to the load is controlled by delaying the firing angle of thyristor T_1 .



Power circuit



Voltage and current waveforms.

The RMS output voltage is

$$\begin{aligned}
 E_o &= \left[\frac{1}{2\pi} \left[\int_{\alpha}^{\pi} 2E_s^2 \sin^2 \omega t \, d(\omega t) + \int_{\pi}^{2\pi} 2E_s^2 \sin^2 \omega t \, d(\omega t) \right] \right]^{1/2} \\
 &= \left[\frac{2E_s^2}{4\pi} \left[\int_{\alpha}^{\pi} (1 - \cos 2\omega t) \, d(\omega t) + \int_{\pi}^{2\pi} (1 - \cos 2\omega t) \, d(\omega t) \right] \right]^{1/2} \\
 &= E_s \left[\frac{1}{2\pi} \left(2\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2}
 \end{aligned}$$

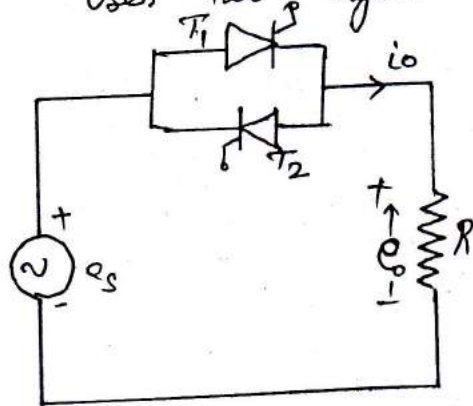
The average value of output voltage is given by

$$\begin{aligned}
 E_{oav} &= \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} \sqrt{2} E_s \sin \omega t \, d(\omega t) + \int_{\pi}^{2\pi} \sqrt{2} E_s \sin \omega t \, d(\omega t) \right] \\
 &= \frac{\sqrt{2} E_s}{2\pi} (\cos \alpha - 1)
 \end{aligned}$$

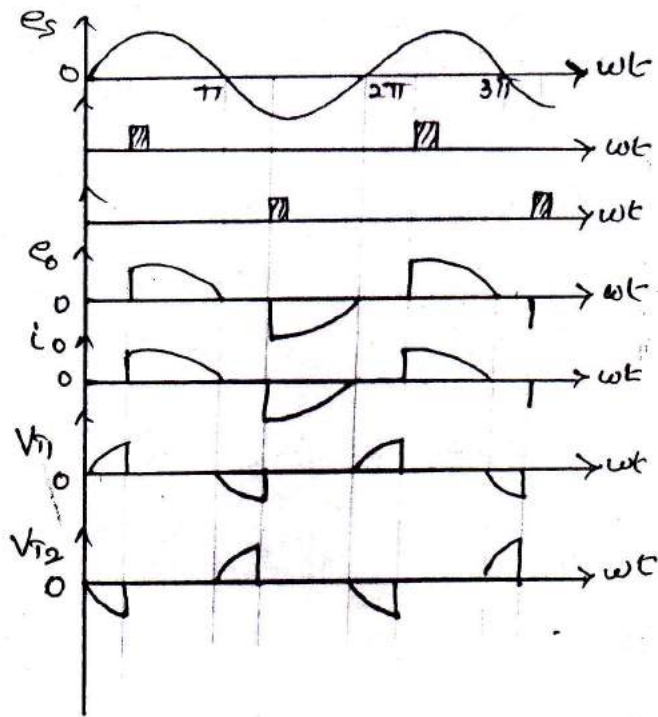
∴ α is varied from 0 to π , E_o varies from E_s to $E_s/\sqrt{2}$ and E_{oav} varies from 0 to $-\frac{\sqrt{2} E_s}{\pi}$.

(ii) Fully-wave (Bidirectional) A.C. voltage controllers.

Uses two thyristors connected in antiparallel.



Single-phase a.c. voltage controller with R Load.



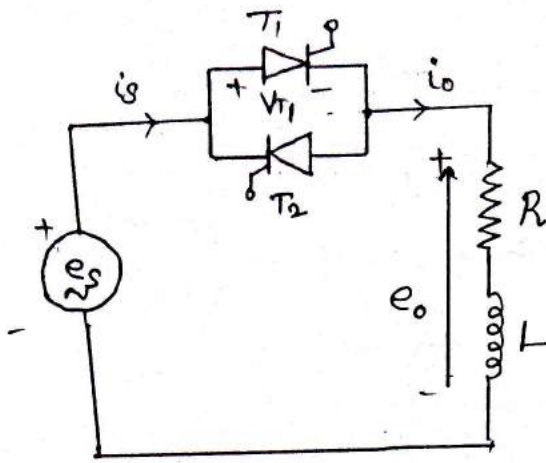
Voltage and Current Waveforms

The RMS output voltage can be obtained from

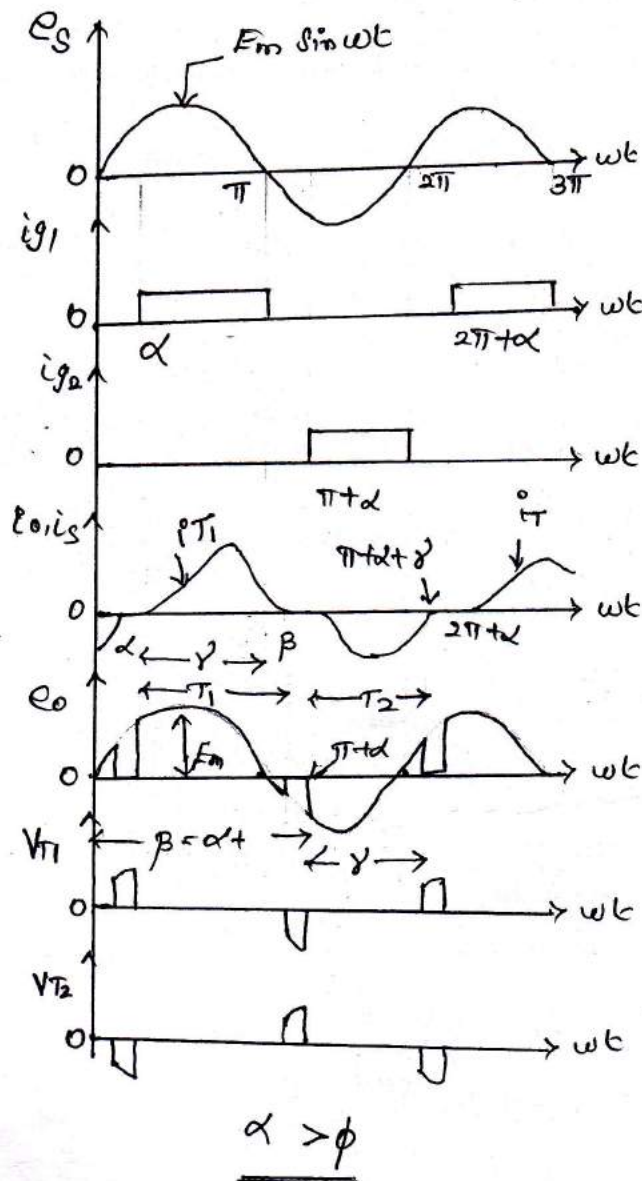
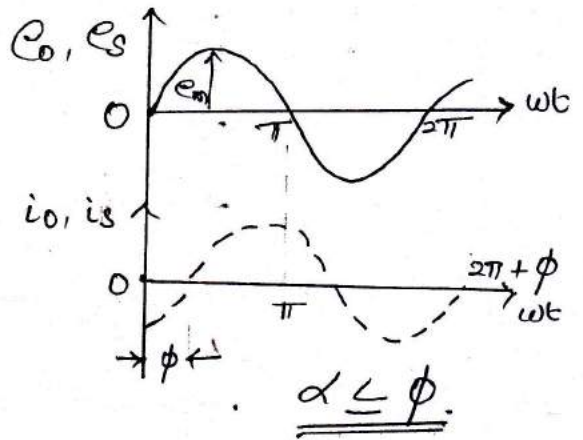
$$\begin{aligned}
 E_o &= \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} 2E_s^2 \sin^2 \omega t \, d(\omega t) \right]^{1/2} \\
 &= \left[\frac{4E_s^2}{4\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) \, d(\omega t) \right]^{1/2} \\
 &= E_s \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2}
 \end{aligned}$$

Thus, by varying α from 0 to π , the RMS output voltage can be controlled from RMS input voltage E_s to zero.

Single-phase a.c. voltage controller with inductive (RL)



circuit diagram



The expression for load current is and β can be obtained as,

For $\alpha \leq \omega t \leq \beta$, the KVL is,

$$e_s = E_m \sin \omega t$$

$$= R \cdot i_o + L \cdot \frac{di_o}{dt}$$

The solution of this equation is of the form,

$$i_o = \frac{E_m}{Z} \sin(\omega t - \phi) + A \cdot e^{-(R/L)t}$$

where

$$Z = (R^2 + \omega^2 L^2)^{1/2}$$

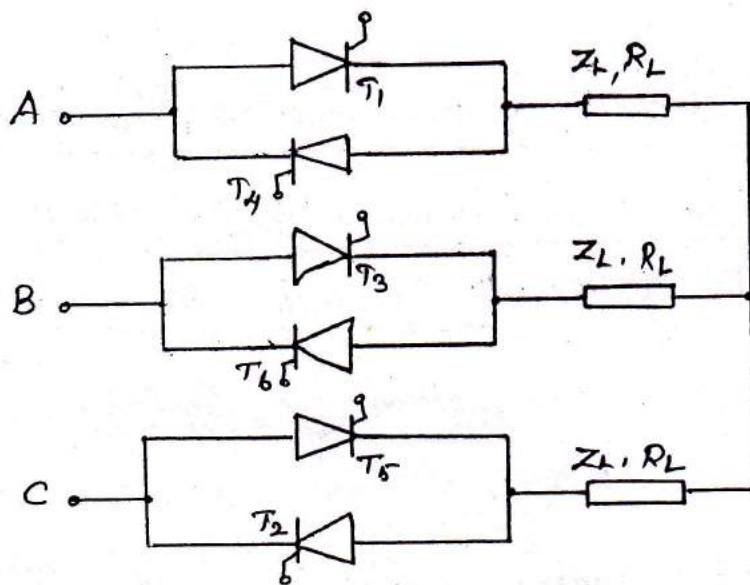
and

$$\phi = \tan^{-1} \frac{\omega L}{R}$$

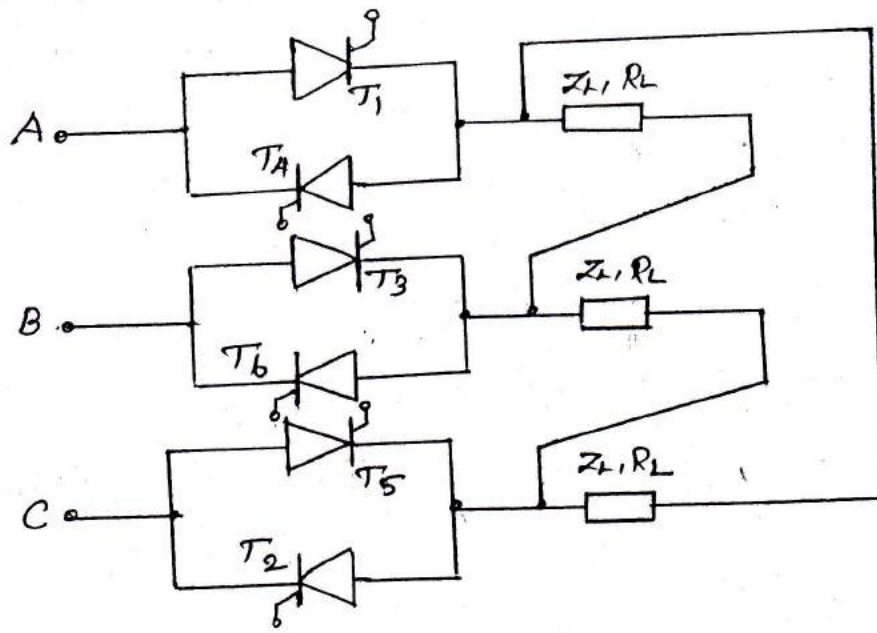
and

$$\gamma = \beta - \alpha$$

Three - Phase A.C. Regulators.



3-wire star load.



3-wire delta load.

Three-phase bidirectional Delta-Connected Regulators:

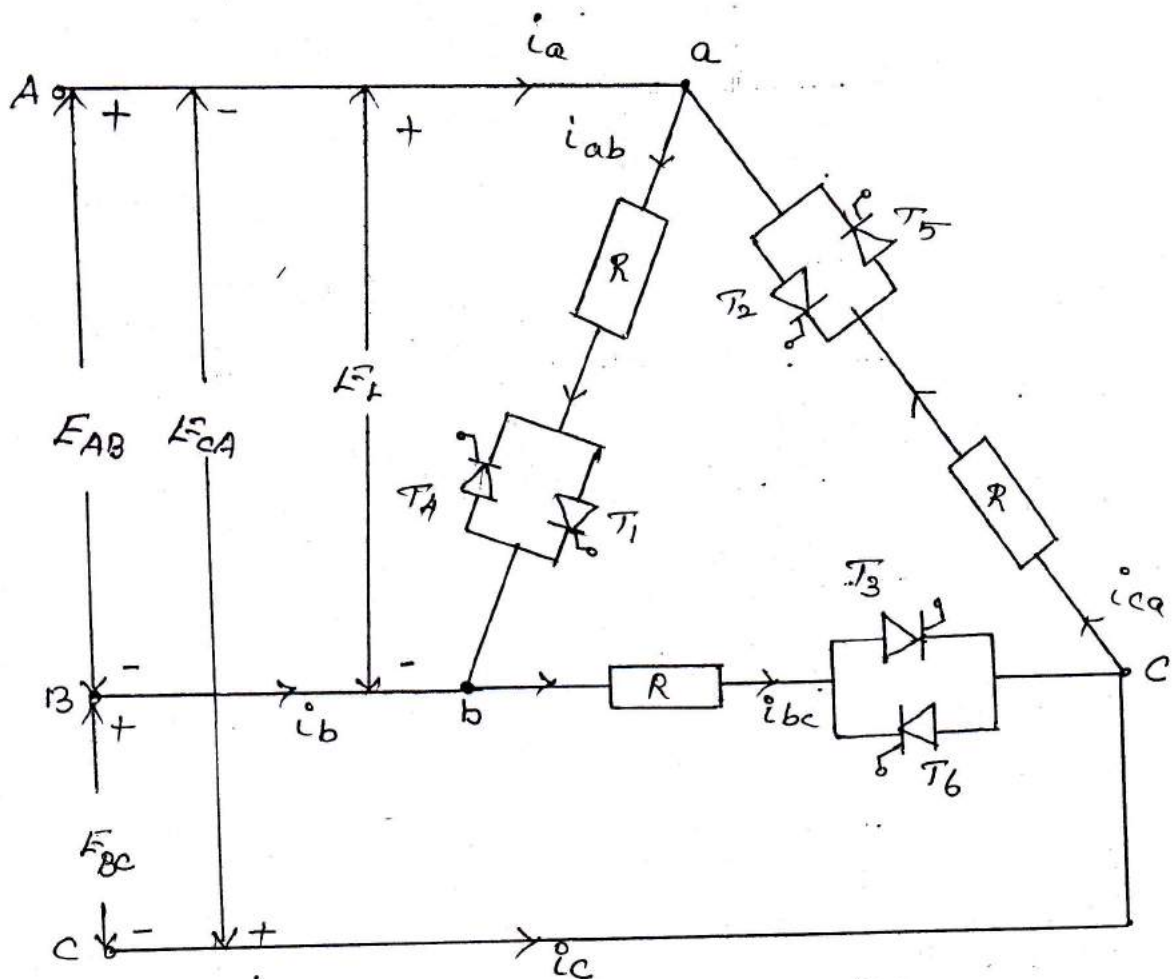
Since the phase current in a normal three-phase system is only $\frac{1}{\sqrt{3}}$ of the line current, the current ratings of thyristors would be less than that if thyristors were placed in the line.

The instantaneous line-to-line voltages are

$$E_{AB} = E_{ab} = \sqrt{2} E_s \sin \omega t.$$

$$E_{BC} = e_{bc} = \sqrt{2} E_s \cdot \sin(\omega t - 2\pi/3)$$

$$E_{CA} = e_{ca} = \sqrt{2} E_s \cdot \sin(\omega t - 4\pi/3)$$



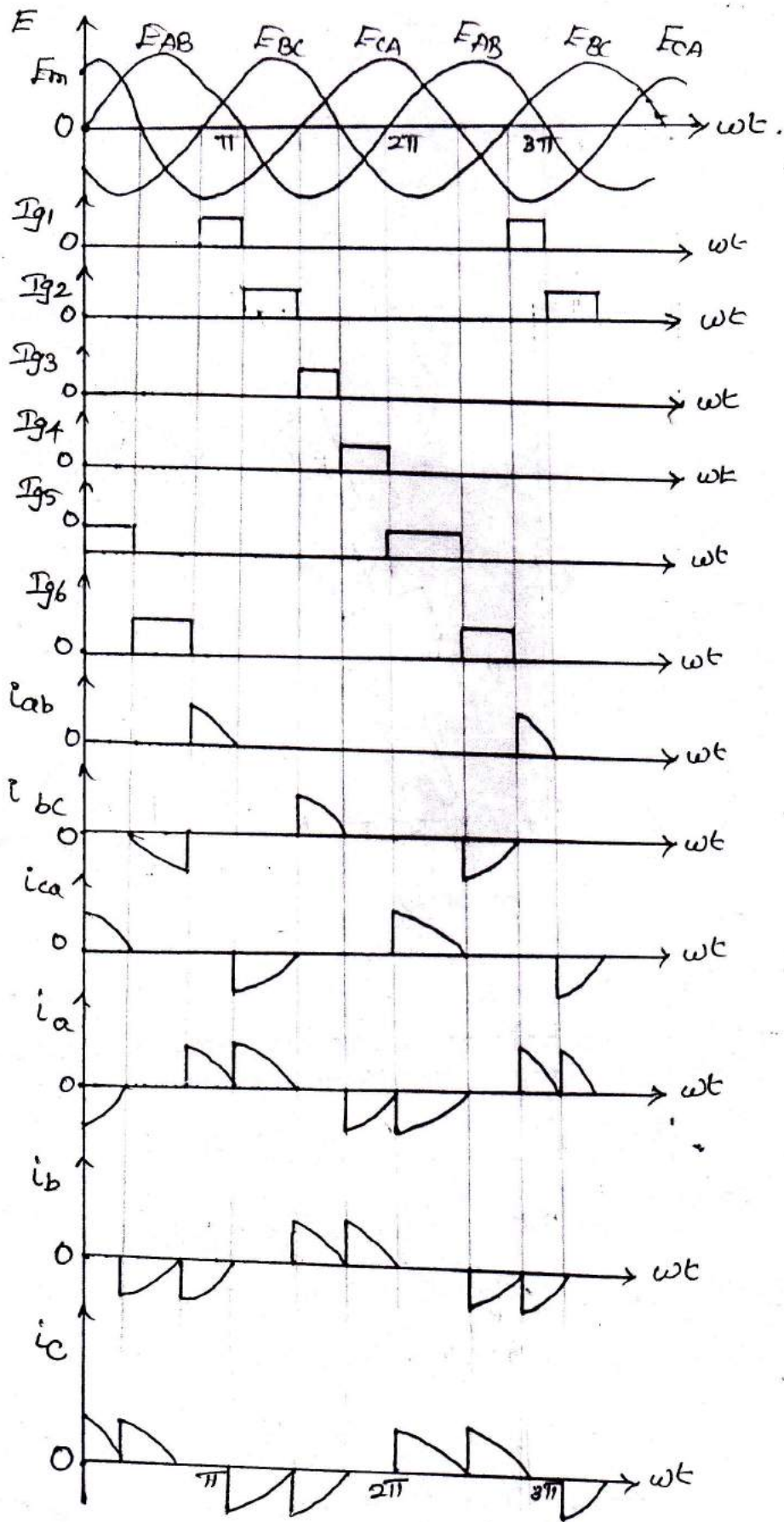
Delta Connected three-phase a.c. regulator.

For $\alpha = 120^\circ$, the input line voltages, phase and line currents, and thyristor gating signals are given below.

For resistive loads, the RMS output phase voltage can be obtained from

$$E_o = \left[\frac{1}{2\pi} \int_{\alpha}^{2\pi} e_{ab}^2 d(\omega t) \right]^{1/2} = \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} 2 E_s^2 \sin^2 \omega t d(\omega t) \right]^{1/2}$$

$$= E_s \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2}$$



For $\alpha = 120^\circ$.

Waveforms for three-phase delta-connected regulator.

When $\alpha = 0$, the maximum output voltage would be obtained, and the control range of delay angle is

$$0 \leq \alpha \leq \pi.$$

The line current, which can be determined from the phase currents, are

$$i_a = i_{ab} - i_{ca}$$

$$i_b = i_{bc} - i_{ab}$$

$$i_c = i_{ca} - i_{bc}.$$

The RMS value of line and phase currents for the load circuits can be determined by numerical solution of (or) Fourier analysis.

If I_n is the rms value of the n^{th} harmonic component of a phase current, the RMS value of phase current can be obtained from

$$I_{ab} = \left[I_1^2 + I_3^2 + I_5^2 + I_7^2 + I_9^2 + I_{11}^2 + \dots + I_n^2 \right]^{\frac{1}{2}}$$

Due to the delta connection, the triplen harmonic components (i.e. those of order $n = 3m$, where m is an odd integer) of the phase currents would flow around the delta and would not appear in the line.

This is due to the fact that the zero sequence harmonics are in phase in all three phases of load.

The RMS line current becomes,

$$I_a = \left[\sqrt{3} \left(I_1^2 + I_5^2 + I_7^2 + I_{11}^2 + \dots + I_n^2 \right) \right]^{1/2}$$

As a result, the RMS value of line current would not follow the normal relationship of a three phase system such that

$$I_a < \sqrt{3} I_{ab}$$