

## CHAPTER 2

### STEPPER MOTORS

#### 2.1.General

Stepper motors are electromagnetic incremental devices that convert electric pulses to shaft motion (rotation). These motors rotate a specific number of degrees as a respond to each input electric pulse. Typical types of stepper motors can rotate  $2^\circ$ ,  $2.5^\circ$ ,  $5^\circ$ ,  $7.5^\circ$ , and  $15^\circ$  per input electrical pulse. Rotor position sensors or sensor less feedback based techniques can be used to regulate the output response according to the input reference command. Stepper motors offers many attractive features such as:

- *Available resolutions ranging from several steps up to 400 steps (or higher) per revolution.*
- *Several horsepower ratings.*
- *Ability to track signals as fast as 1200 pulses per second.*

Stepper motors have many industrial applications such as:

- *Printers.*
- *Disk Drives.*
- *Machine Tools.*
- *Robotics.*
- *Tape Drives*

#### 2.2. Types of Stepper Motors

Stepper motors are usually classified into three main categories, namely,

- 1) *Variable reluctance (single stack and multi stack),*
- 2) *Permanent Magnet, and*
- 3) *Hybrid motors.*

##### 2.2.1. Single Stack Variable Reluctance Stepper Motors

Fig. 1 presents the basic circuit configuration of a typical 4-phase, 2-pole, single-stack, variable reluctance stepper motor. The stator is made of a single stack of steel laminations with the phase windings wound around the stator poles. The rotor is made of stack of steel laminations without any windings. The main principle of operation depends on aligning

one set only of stator and rotor poles by energizing the stator windings. Therefore, the number of poles in the stator and rotor windings has to be different. The stator windings are energized by a DC source in such a sequence to generate a resultant rotating air-gap field around the rotor in steps. The rotor is made of ferromagnetic material that provides a tendency to align the rotor axis along the direction of the resultant air-gap field. Therefore, the rotor tracks the motion of this stepped field.

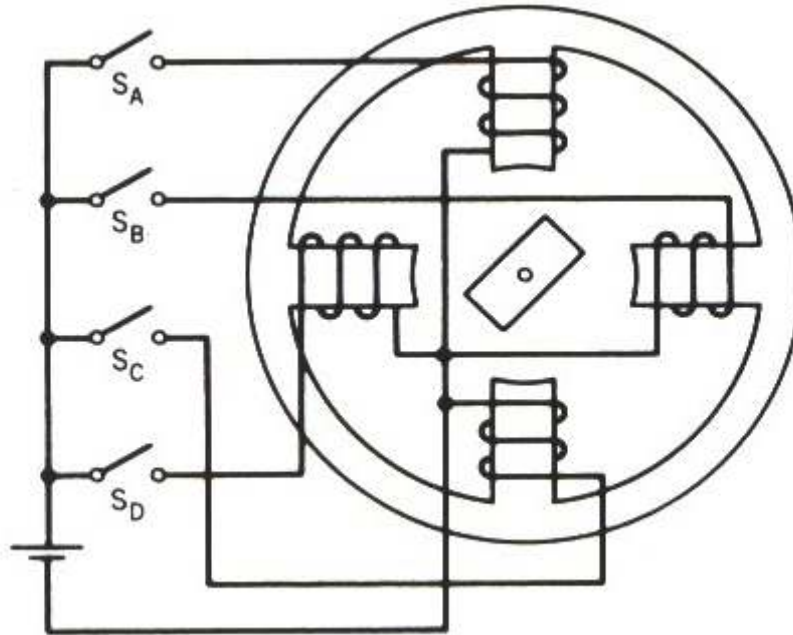


Fig. 1 Basic circuit configuration of a typical 4-phase, 2-pole, single-stack, variable reluctance stepper motor

Fig. 2 illustrates the different modes of operation of the 4-phase, 2-pole, single-stack, variable reluctance stepper motor for 45° step in the following energizing sequence A, A+B, B, B+C, C, C+D, D, and then D+A. Then this switching sequence is repeated.

- **Energizing winding A:** The resultant air-gap flux will be aligned along the axis of pole A windings. Consequently, the rotor aligns itself along the phase A axis as shown in the upper part of Fig. 2.
- **Energizing windings A and B:** The resultant air-gap flux will be oriented in the midway between pole A and pole B i.e., the resultant mmf rotated 45° in the clockwise direction. Consequently, the rotor aligns itself with the resultant mmf (45°) as shown in the middle part of Fig. 2.

- **Energizing winding B:** The resultant air-gap flux will be aligned along the axis of pole  $B$  windings. Consequently, the rotor aligns itself along the phase  $B$  axis as shown in the lower part of Fig. 2.

The direction of rotation can be reversed by reversing the switching sequence to be  $A$ ,  $A+D$ ,  $D$ ,  $D+C$ ,  $C$ ,  $C+B$ ,  $B$ , and then  $B+A$ . Then this switching sequence is repeated.

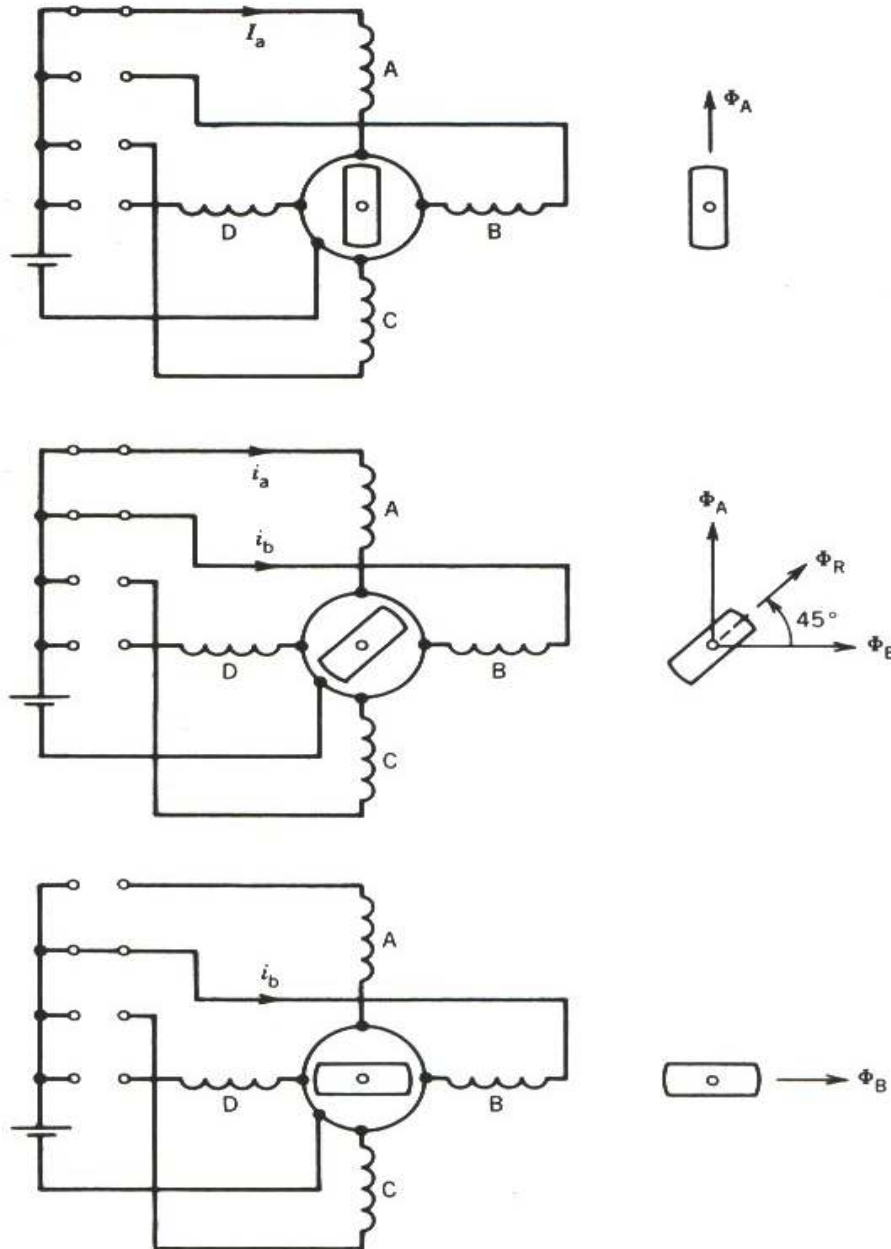


Fig. 2 Operation modes of single-stack, 2-poles, and variable reluctance stepper motor with 45° step

Smaller steps can be obtained by using multi-pole rotor configuration such as the one shown in Fig. 3 that rotate in an anticlockwise direction with a 15° step in the following

energizing sequence A, A+B, B, B+C, C, C+D, D, and then D+A. Then this switching sequence is repeated.

- **Energizing winding A:** The resultant air-gap flux will be aligned along the axis of pole A windings. Consequently, the rotor pole  $P_1$  aligns itself along the phase A axis as shown in the upper part of Fig. 3.
- **Energizing windings A and B:** The resultant air-gap flux will be oriented in the midway between pole A and pole B i.e., the resultant mmf rotated  $45^\circ$  in the clockwise direction. In this case, the nearest rotor pole to this direction is pole  $P_2$ . Consequently, the rotor rotates in an anticlockwise direction to align pole  $P_2$  with the resultant mmf ( $45^\circ$ ). Therefore, the net rotational step is  $15^\circ$  in an anticlockwise direction.
- **Energizing winding B:** The resultant air-gap flux will be aligned along the axis of pole B windings. In this case, the nearest rotor pole to this direction is pole  $P_3$ . Consequently, the rotor rotates in an anticlockwise direction to align pole  $P_3$  with the resultant mmf ( $90^\circ$ ). Therefore, the net rotational step in this stage is also  $15^\circ$  in an anticlockwise direction.
- **and so on.**

The direction of rotation can be reversed by reversing the switching sequence to be A, A+D, D, D+C, C, C+B, B, and then B+A. Then this switching sequence is repeated.

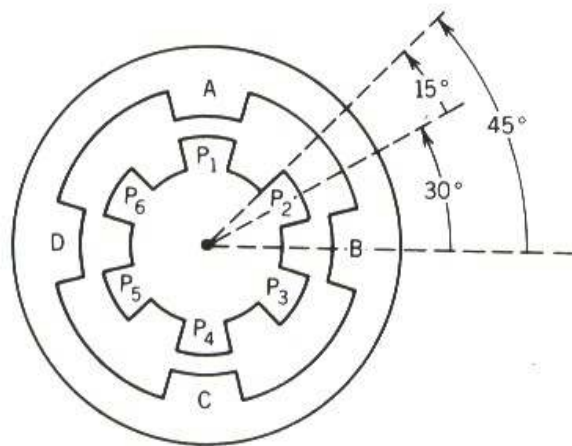


Fig. 3 Construction and operation of 4-phase, 6-pole, single-stack, variable reluctance stepper motor

Fig. 4 presents the circuit configuration and different operation modes for a 3-phase, 4-pole, single stack, variable reluctance stepper motor that rotate in a clockwise direction with a  $30^\circ$  step. Table 1 and Fig. 5 present each phase switching sequence for one revolution of the rotor.

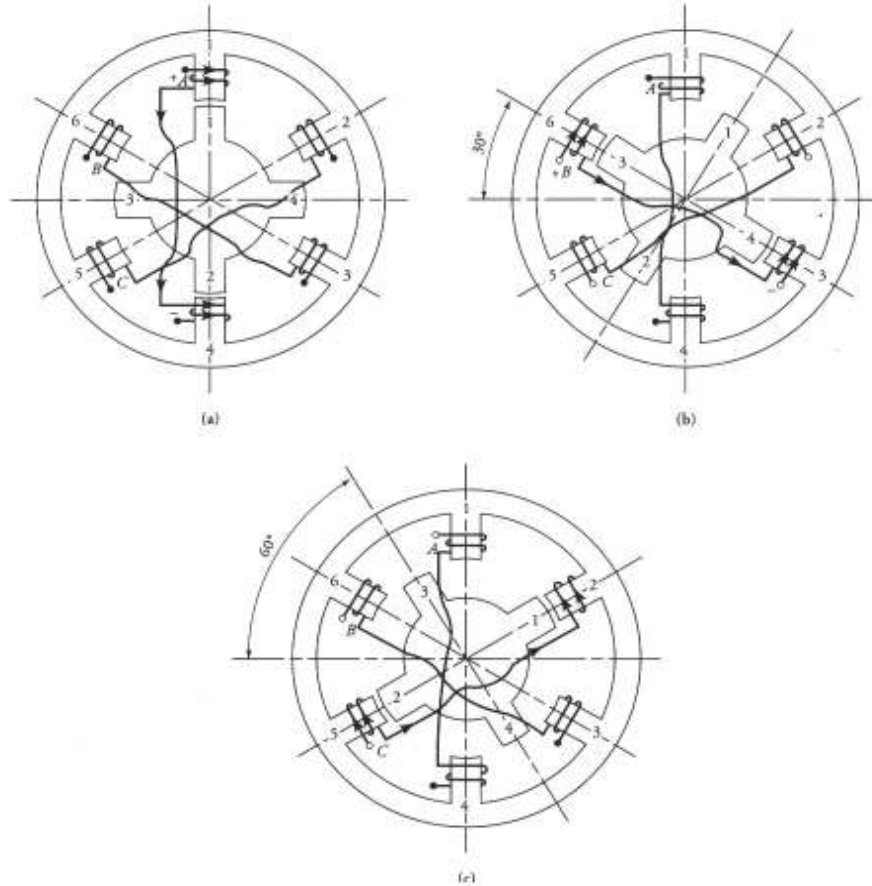


Fig. 4 Construction and operation of 3-phase, 4-pole, single-stack, variable reluctance stepper motor

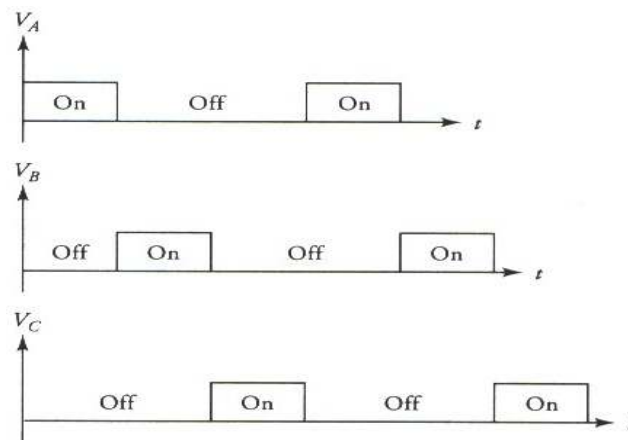


Fig. 5 Phase switching sequence

Table 1 Phase switching sequence: “1” and “0” corresponds to positive and zero phase voltage (currents), respectively

Cycle	Phase			Position $\delta^\circ$
	A	B	C	
1	1	0	0	0
	0	1	0	30
	0	0	1	60
2	1	0	0	90
	0	1	0	120
	0	0	1	150
3	1	0	0	180
	0	1	0	210
	0	0	1	240
4	1	0	0	270
	0	1	0	300
	0	0	1	330
5	1	0	0	360

#### 2.2.4. Multi-Stack Variable Reluctance Stepper Motors

In this type, the motor is divided along its axis into a number of stacks. Each stack is energized by a separate winding (phase) as shown in Fig. 6. These stacks are magnetically isolated from each other. The most common type is the three-stack, three-phase motors; however, number of stacks and phases up to seven are also available.

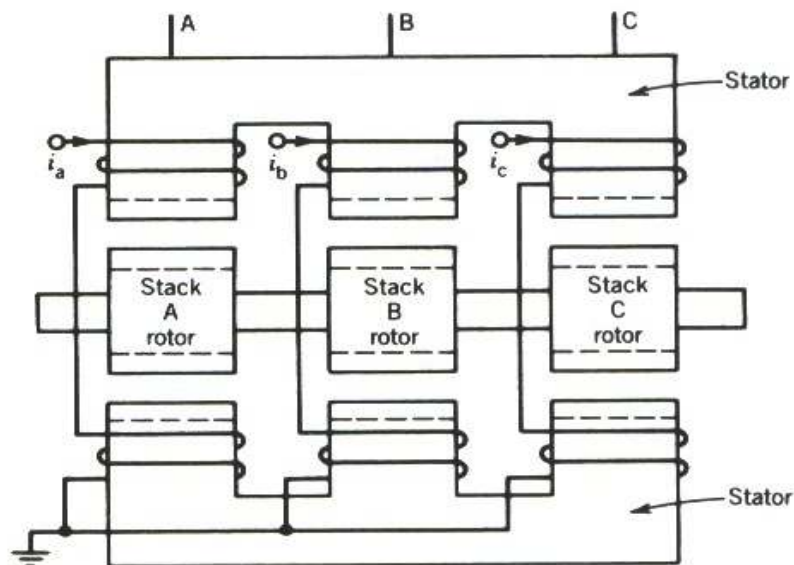


Fig. 6 Cross section view of a typical three-stack variable reluctance stepper motor

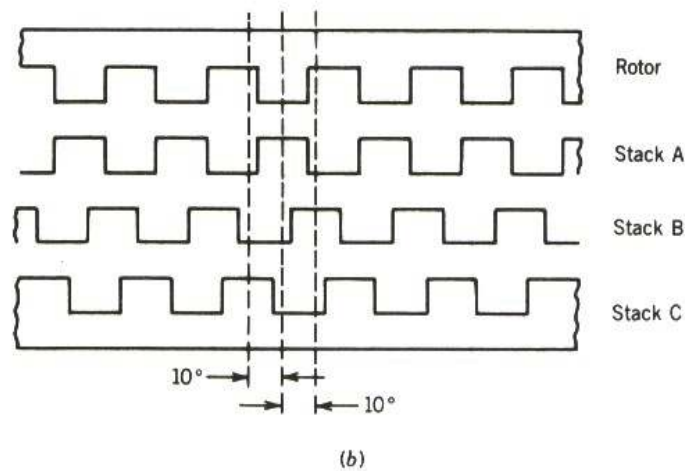
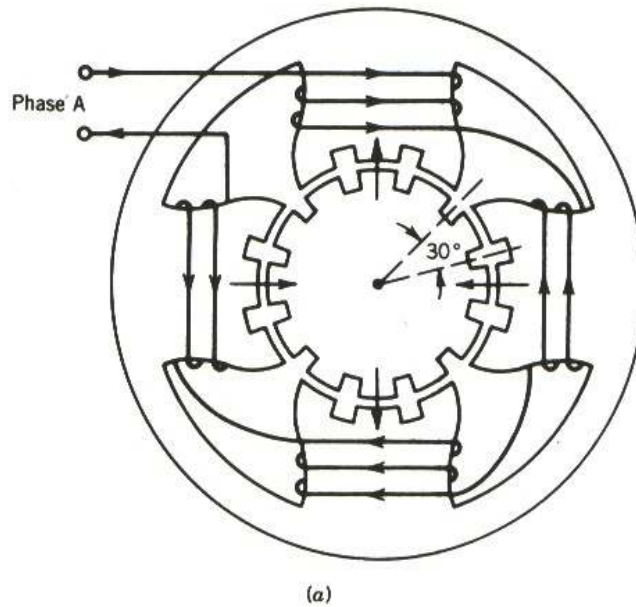


Fig. 7 Teeth position for a 3-phase, 4-pole, 12-teeth, three-stack, variable reluctance stepper motor when phase a is energized

Fig. 7 also illustrates the rotational sequence of a 3-phase, 4-pole, 12-teeth, three-stack, variable reluctance stepper motor for  $10^\circ$  step in a clockwise direction according to the following energizing sequence A, B, and C. Then this switching sequence is repeated.

- **Energizing phase (stack) A:** when *stack A* winding is energized, the rotor teeth will move to align themselves with the stator teeth is *stack A* as shown in Fig. 7.
- **Energizing phase (stack) B:** when *stack B* winding is then energized while *stack A* winding is de-energized, the rotor teeth will move to align themselves with the

stator teeth is *stack B*. This will result in a clockwise rotation of the rotor by  $10^\circ$  as shown in Fig. 7.

- **Energizing phase (stack) C:** when *stack C* winding is then energized while *stack B* winding is de-energized, the rotor teeth will move to align themselves with the stator teeth is *stack C*. This will result in another clockwise rotation of the rotor by  $10^\circ$ . After this stage the rotor has moved one rotor tooth pitch as illustrated by Fig. 7.
- **And So on**

The direction of rotation can be reversed by reversing the switching sequence. Assume that the total number of stacks (phases) is  $N$  while the total number of teeth in each stack is  $x$ . The tooth pitch ( $\tau_p$ ) can be expressed by,

$$\tau_p = \frac{360^\circ}{x}$$

Moreover, the step size ( $\Delta\theta$ ) can be expressed by,

$$\Delta\theta = \frac{360^\circ}{xN}$$

Consequently, the number of steps per revolution ( $n$ ) is given by

$$n = \frac{360^\circ}{\Delta\theta} = xN$$

As an example, for the motor in Fig. 7,  $x = 12$  and  $N = 3$ . Therefore,

$$\begin{aligned}\tau_p &= \frac{360^\circ}{x} = \frac{360^\circ}{12} = 30^\circ \\ \Delta\theta &= \frac{360^\circ}{xN} = \frac{360^\circ}{12 * 3} = 10^\circ \\ n &= \frac{360^\circ}{10} = 36\end{aligned}$$

#### **2.2.4. Permanent Magnet Stepper Motors**

Permanent magnet (PM) stepper motors are similar in construction to that of single-stack; variable reluctance stepper motors except that the rotor is made of permanent magnet. Fig. 8 presents the circuit configuration and different operation modes for a 2-phase, permanent magnet stepper motor that rotate in an anticlockwise direction with a  $90^\circ$  step. Table 2 and



Fig. 9 present each phase switching sequence for one revolution of the rotor. Reversing the switching sequence will result in reversing the direction of rotation.

PM stepper motors offer many features compared to variable reluctance type such as

- Higher inertia and consequently lower acceleration (deceleration) rates.
- Maximum step pulse rate is 300 pulses per second compared to 1200 pulses per second for variable reluctance stepper motors.
- Larger step sizes, ranging from  $30^\circ$  to  $90^\circ$  compared to step sizes as low as  $1.8^\circ$  for variable reluctance stepper motors.
- Generate higher torque per ampere of stator currents than variable reluctance stepper motors.

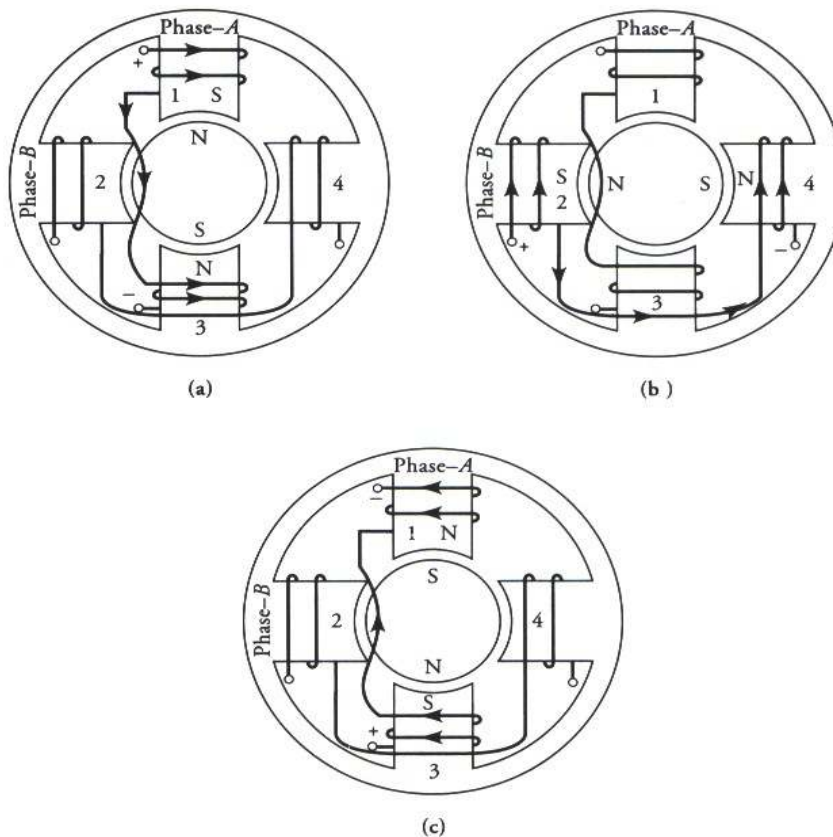


Fig. 8 Construction and operation of 2-phase, permanent magnet stepper motor

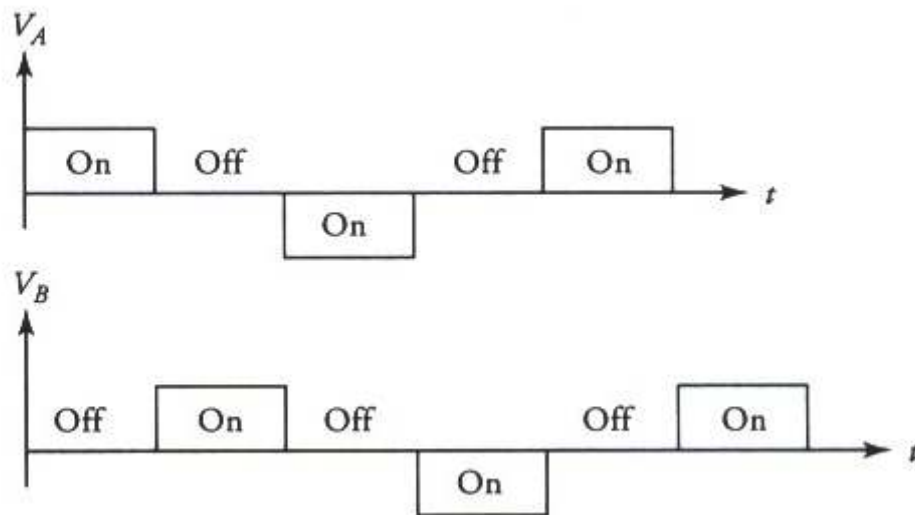


Fig. 9 Phase switching sequence

Table 2 Phase switching sequence: “1”, “-1” and “0” corresponds to positive, negative, and zero phase voltage (currents), respectively

Cycle	Phase		Position $\delta^\circ$
	A	B	
+	1	0	0
	0	1	90
-	-1	0	180
	0	-1	270
+	1	0	360

#### 2.2.4. Hybrid Stepper Motors

Hybrid stepper motors have similar stators' construction to that of variable reluctance stepper motors. However, their rotors constructions combine both variable reluctance and permanent magnet constructions. The rotors are made of an axial permanent magnet at the middle and two identical stacks of soft iron poles at the outer ends attached to the north and south poles of the permanent magnet. The rotor poles connected to the north pole of the permanent magnet forms North Pole, while the other forms the south poles as shown in Fig. 10. This figure also presents two different views of these motors types. Fig. 11 presents a complete cross section view of 4-pole stator and 5-pole rotor hybrid stepper motor while Fig. 12 presents the different components of standard hybrid stepper motor. These types of motors have similar operation modes as the permanent magnet types.

Moreover, they are characterized by smaller step sizes but they are very expensive compared to variable reluctance stepper motors.

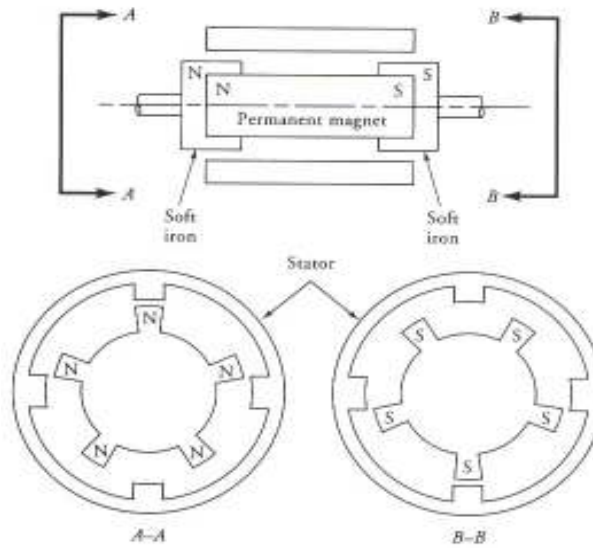


Fig. 10 Construction of 4-pole stator and 5-pole rotor hybrid stepper motor

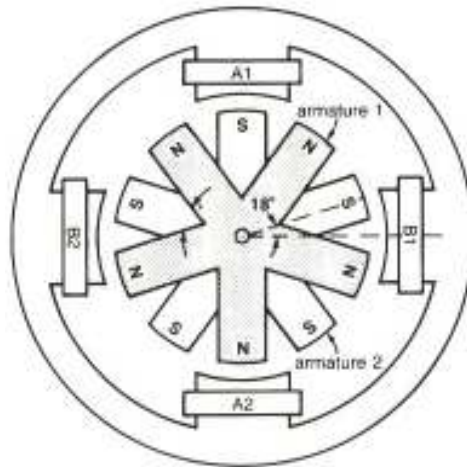


Fig. 11 Cross section view of 4-pole stator and 5-pole rotor hybrid stepper motor

### **Working Principle of Hybrid Motors**

Consider a 2-phase, 4-poles stepper motor as shown in Fig. 13. When energizing *phase B*, the rotor starts to rotate in an anticlockwise direction to align itself with *pole 2*. It is supposed theoretically, that the rotor will come to rest once its axis is aligned with *pole 2* axis. However, practically, due to the inertia of the rotor, the rotor will overshoot and pass the central line of *pole 2*. After that, the magnetic field generated from *pole 2* pulls the rotor in the opposite direction. The rotor will swing around the central line of *pole 2* until finally

it comes to rest after being aligned. Fig. 14 a presents the variation of the rotor position and the rotor speed as a result of energizing *phase B*.



Fig. 12 Different components of standard hybrid stepper motor

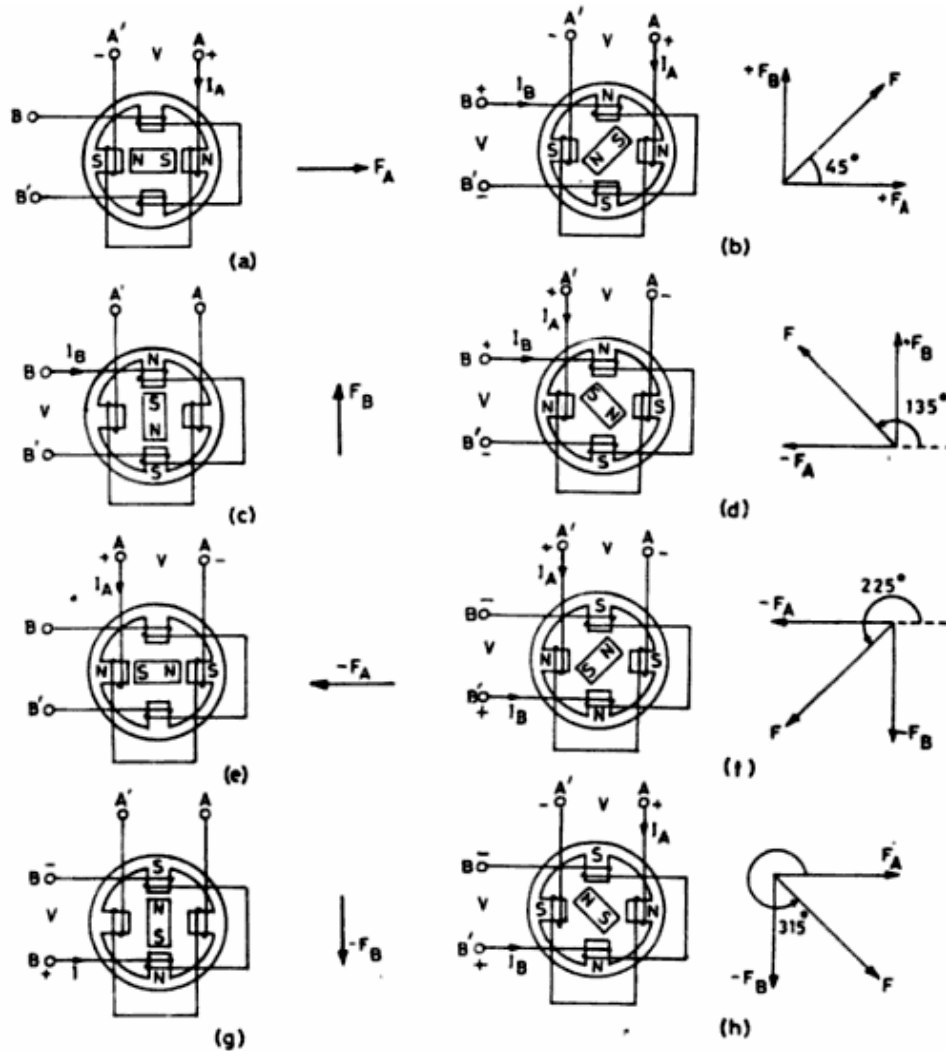


Fig. 13 2-phase, 4-poles stepper motor

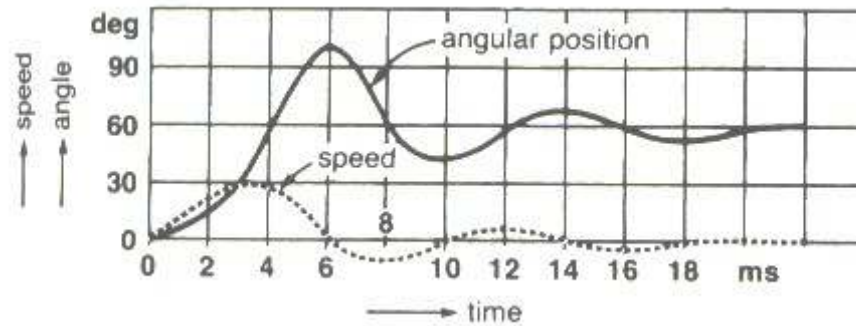


Fig. 14 a Effect of inertia on rotor angular speed and angular position

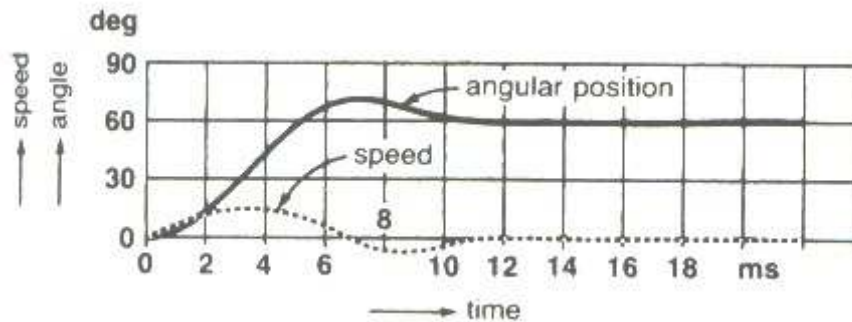


Fig. 14 b Effect of viscous damping on rotor angular speed and angular position

### 2.3. Modes of Operation

There are two main modes of operation of stepper motors that can be summarized as follows:

**Start-Stop Mode:** In this mode the motor is controlled to settle down (rest) after each step before advancing to the next step. The rotational speed will be in the form of pulses that drops to zero at the end each step while the rotor position will be in the form of pulses also but with an increasing steady state value with time as shown in Fig. 15. This mode is sometimes referred to by the *start without error mode*. A maximum permissible stepping rate is required for this mode of operation; otherwise, the motor will not be able to track the control current pulses and the step will be lost. This minimum rate depends on the motor inertia and the loading condition. Fig. 16 presents the torque speed (steps per second, where each step equivalent to  $1.8^\circ$ ) characteristic for this mode of operation represented by:

- **Curve 1:** Low inertia. If the motor drives a load of 1.4 N.m then the maximum permissible pulse rate is 500 steps per second.
- **Curve 2:** Higher inertia. If the motor drives a load of 1.4 N.m then the maximum permissible pulse rate is 400 steps per second.

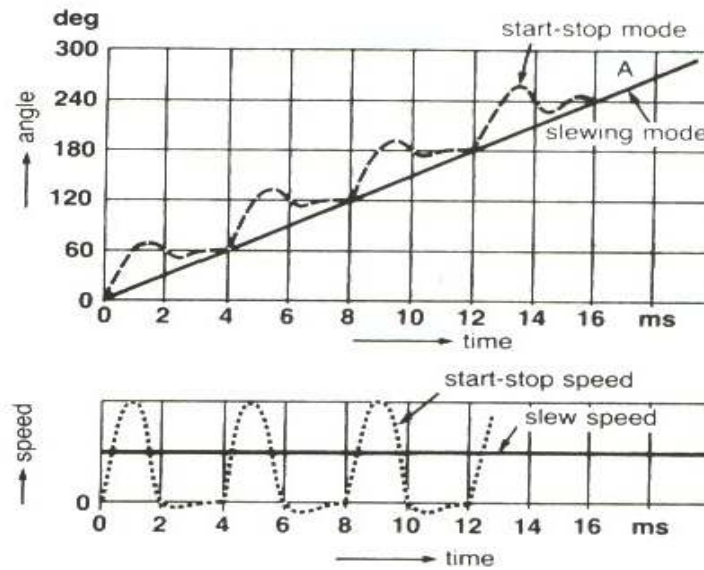


Fig. 15 Rotor angular speed and angular position for different operating modes

**Slewing Mode:** In this mode the motor is controlled to rotate at a constant uniform speed without stopping at the end of each step and the rotor position varies linearly with time as shown in Fig. 15. The torque speed characteristic of this mode will not be affected by the system inertia because of the constant speed. Moreover, for a specific pulse rate (500 steps per second) this mode allows the motor to drive higher torque load as in the start-stop mode as shown in Fig. 16.

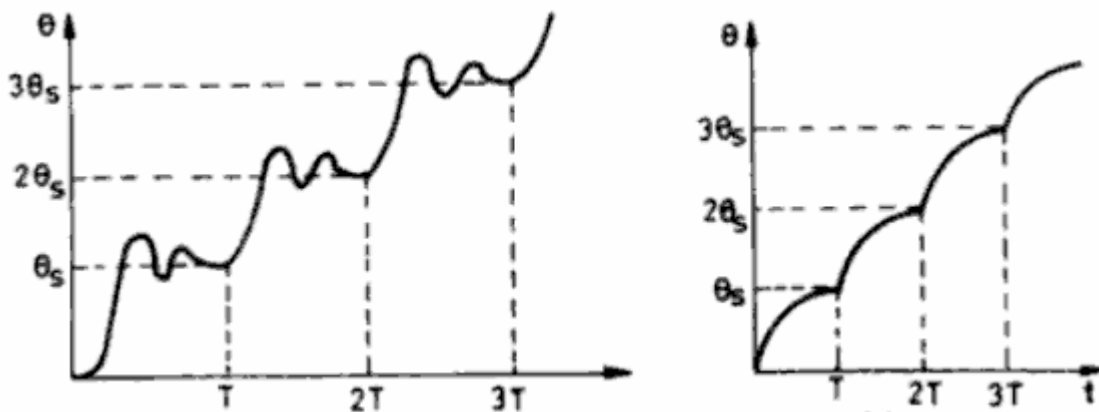


Fig. 16 (a) Start Stop Mode (b) Slewing Mode

## 2.7. Drive Circuit

There are two main drive circuits for stepper motors, namely; *Uni-polar* and *Bi-polar* drive circuits.

### Uni-polar Drive Circuit

Fig. 17 presents a schematic diagram for a uni-polar drive circuit. This circuit is suitable for three phase variable reluctance stepper motors. Each phase winding of the motor is controlled by a separate drive circuit with a transistor as its controllable power switch. All drive circuits are energized by the same DC source. The transistor (power switch) of each winding has two modes of operation as follows:

- ✓ **On Mode:** When sufficiently high base current flow through the transistor base it turn ON and acts ideally like a short circuit. Consequently, the supply voltage will be applied across the phase winding and the external resistor ( $R_{ext}$ ) connected in series with the phase winding. The DC source magnitude is adjusted to produces the rated phase current when the switch is turned ON. Therefore,

$$V_s = I(R_{ph} + R_{ext})$$

Where  $V_s$  is the DC source voltage in  $V$ ,  $I$  is the phase winding rated current in  $A$ ,  $R_{ph}$  is the phase winding resistance in  $\Omega$ , and  $R_{ext}$  is the external resistance connected in series to the phase winding in  $\Omega$ .

The phase winding inductance is very large and consequently results in slow rate of building the phase winding current that might result in unsatisfactory operation of the stepper motor at high stepping rates. Therefore, the external resistance is connected in series with the phase winding to reduce the time constant. The net *ON Mode* circuit time constant will be very large and can be expressed by,

$$\tau_{ON} = \frac{L_{ph}}{(R_{ph} + R_{ext})}$$

Where  $L_{ph}$  is the phase winding average inductance in  $H$ .

- ✓ **OFF Mode:** In this mode, the base drive current of the transistor is removed and the switch is turned *OFF* and acts as an open circuit. The phase winding current will continue to flow through the freewheeling path formed by the freewheeling diode ( $D_f$ ) and the freewheeling resistance ( $R_f$ ). The maximum *OFF state* voltage appears across the transistor (switch) ( $V_{CE(max)}$ ) can be expressed by,

$$V_{CE(max)} = V_s + I R_f$$



During this mode of operation, phase current decays in the *OFF mode* circuit with a net *OFF Mode* circuit time constant that can be expressed by,

$$\tau_{OFF} = \frac{L_{ph}}{(R_{ph} + R_{ext} + R_f)}$$

The energy stored in the phase inductance during the *ON mode* is dissipated in the *OFF* mode circuit resistances during the switch turn OFF period.

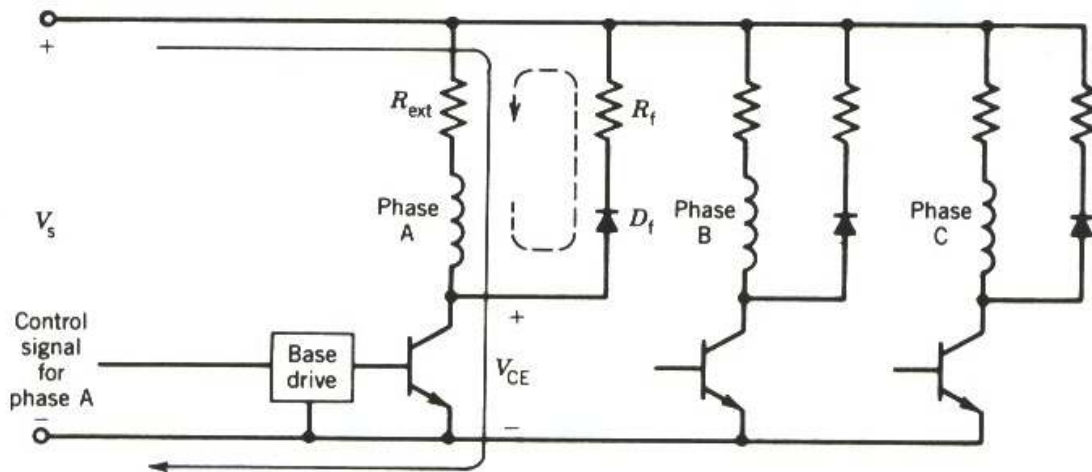


Fig. 17 Uni-polar drive circuit for three-phase variable reluctance stepper motor

### Bi-polar Drive Circuit

Fig. 18 presents a schematic diagram for one phase of a bi-polar drive circuit. This circuit is suitable for permanent magnet or hybrid stepper motors. Each phase winding of the motor is controlled by a separate drive circuit with a transistor as its controllable power switch. All drive circuits are energized by the same DC source. Each two transistors (power switches) of each phase winding are turned ON simultaneously. Two modes of operation occur as follows:

- ✓ ***T<sub>1</sub> and T<sub>2</sub> are in the On Mode:*** This is done by injecting sufficiently high base current through their bases simultaneously. Each transistor acts ideally like a short circuit. Consequently, the current will flow as indicated by the solid line in Fig. 18. The inductor is then energized.
- ✓ ***D<sub>3</sub> and D<sub>4</sub> are in the On Mode:*** This mode follows the switching *OFF* of *T<sub>1</sub>* and *T<sub>2</sub>*. In this mode, the phase winding current cannot change its direction or decay to zero instantaneously after turning *OFF* of *T<sub>1</sub>* and *T<sub>2</sub>* because of the phase winding



inductances. Thus the current continue to flow through of  $D_3$  and  $D_4$  as indicated by the dotted line in Fig. 18. The inductor discharges and the energy is returned back to the DC source.

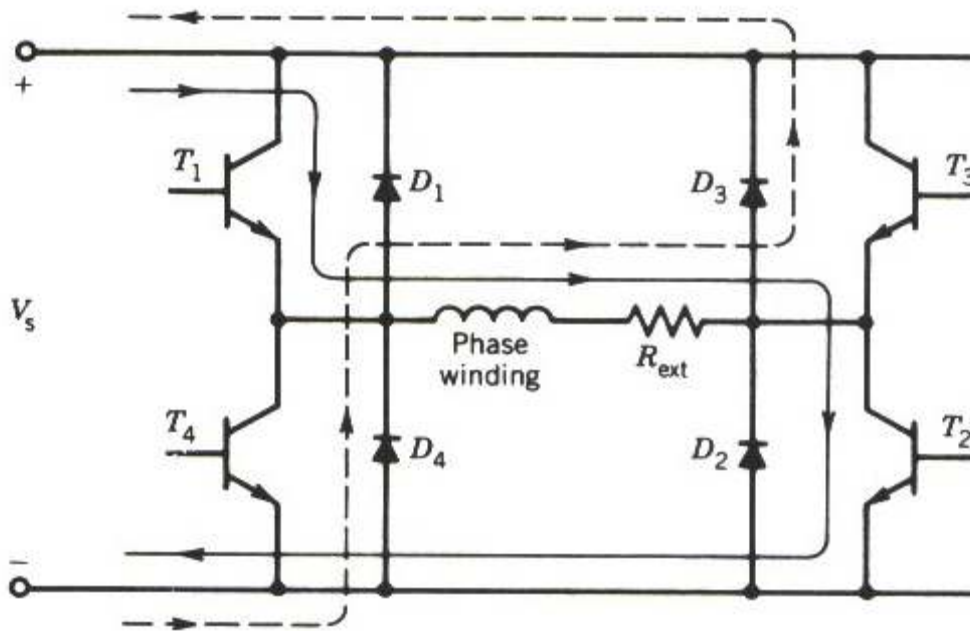


Fig. 18 One phase of a Bi-polar drive circuit for permanent magnet or hybrid stepper motors

A reverse flow of current in the phase windings and hence a reverse direction of rotation of the motor can be achieved by activating  $T_3$  and  $T_4$ . When  $T_3$  and  $T_4$  are turned OFF the freewheeling path will provided through  $D_1$  and  $D_2$ . The bi-polar circuit is characterized by,

- ✓ Higher efficiency than the uni-polar drive circuit as part of the stored energy in the phase winding returns back to the DC source during the power switches turn OFF mode.
- ✓ Fast decaying of the freewheeling current as the inductor discharge through the phase winding resistance, phase external resistance and the DC source.
- ✓ No freewheeling resistance is required.
- ✓ More power switches (devices) than the uni-polar drive circuit.
- ✓ More expensive than the uni-polar drive circuit.
- ✓ Most of the large stepper motors types ( $> 1$  kW) are driven by the bi-polar drive circuit including variable reluctance types.

### Switching Sequence

Consider the 4-pole hybrid stepper motor shown in Fig. 11. The motor is driven by a Bi-polar drive circuit where the power switches are represented by contacts as illustrated by

Fig. 19. Four contacts are used with each coil set of the motor (*A1* and *A2*) and (*B1* and *B2*). The two coil sets are energized by the same DC source. There are three main switching techniques for controlling these contacts namely; *Wave Switching*, *Normal Switching*, and *Half-Step Switching*.

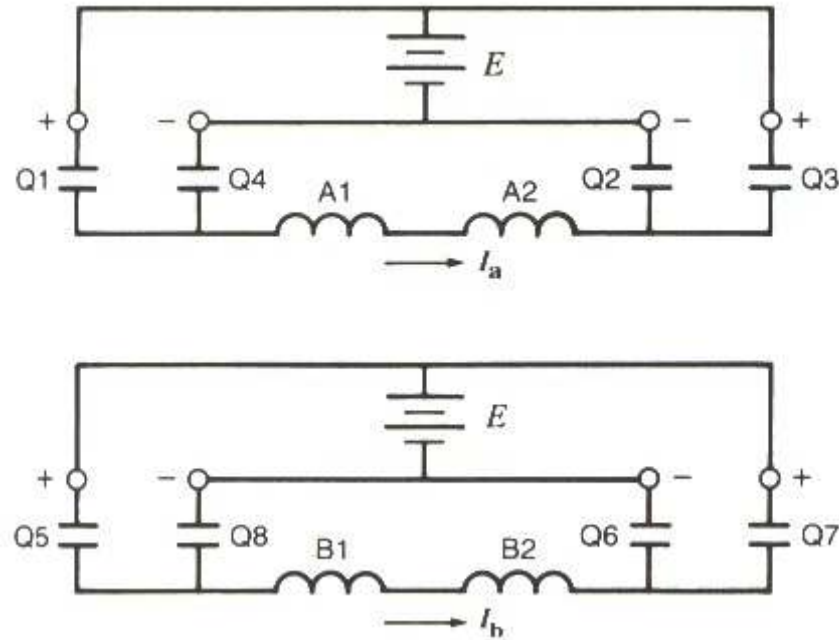


Fig. 19 Drive circuit for the hybrid motor under consideration

✓ **Wave Switching Sequence:** In this technique, only one set of coils is switches each step and the generated flux rotates by  $90^\circ$  per step. Table 3 presents the switching sequence for clockwise rotation. The corresponding coils' current pulses and the generated fluxes are shown in Fig. 20.

Table 3 Wave switching sequence for clockwise rotation

WAVE SWITCHING SEQUENCE FOR CW ROTATION						
Step		1	2	3	4	1
Q1	Q2	on	—	—	—	on
Q3	Q4	—	—	on	—	—
Q5	Q6	—	on	—	—	—
Q7	Q8	—	—	—	on	—

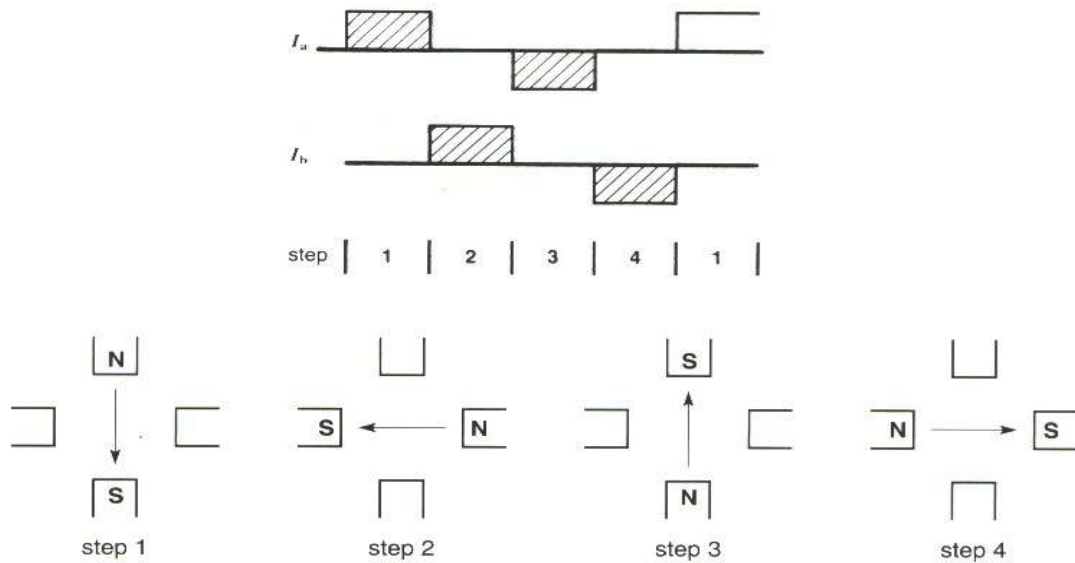


Fig. 20 Current pulses and generated fluxes for wave switching sequence

✓ **Normal Switching Sequence:** In this technique, the two sets of coils are switches each step. The generated flux also rotates by  $90^\circ$  per step; however, it is oriented in the midway between the stator's poles. Table 4 presents the switching sequence for clockwise rotation. The corresponding coils' current pulses and the generated fluxes are shown in Fig. 21. This technique is characterized by slightly greater torque than the wave switching sequence.

Table 4 Normal switching sequence for clockwise rotation

TABLE 19B NORMAL SWITCHING SEQUENCE FOR CW ROTATION						
Step		1	2	3	4	1
Q1	Q2	on	—	—	on	on
Q3	Q4	—	on	on	—	—
Q5	Q6	on	on	—	—	on
Q7	Q8	—	—	on	on	—

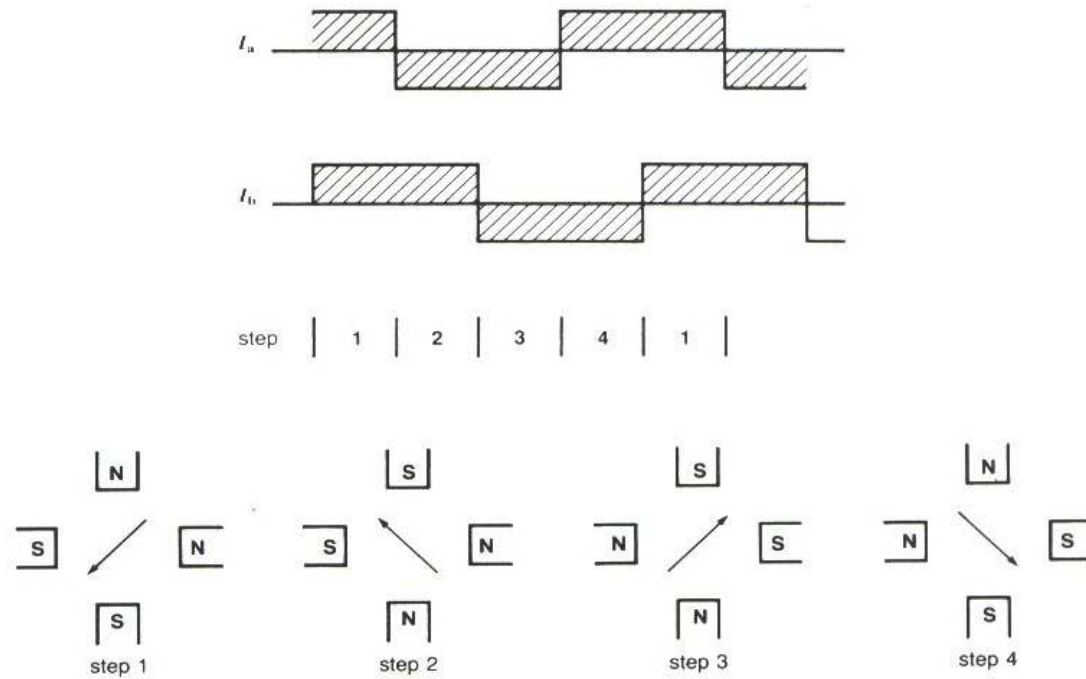


Fig. 21 Current pulses and generated fluxes for normal switching sequence

✓ **Half-Step Switching Sequence:** In this technique, both the wave and the normal switching sequence are combined. The generated flux also rotates by  $45^\circ$  per step. Table 5 presents the switching sequence for clockwise rotation. The corresponding coils' current pulses and the generated fluxes are shown in Fig. 22. This technique is characterized by better resolution of position and reduction in the resonance problem.

Table 5 Half-step switching sequence for clockwise rotation

TABLE 19C HALF-STEP SWITCHING SEQUENCE FOR CW ROTATION										
Step		1	2	3	4	5	6	7	8	1
Q1	Q2	on	on	—	—	—	—	—	on	on
Q3	Q4	—	—	—	on	on	on	—	—	—
Q5	Q6	—	on	on	on	—	—	—	—	—
Q7	Q8	—	—	—	—	—	on	on	on	—

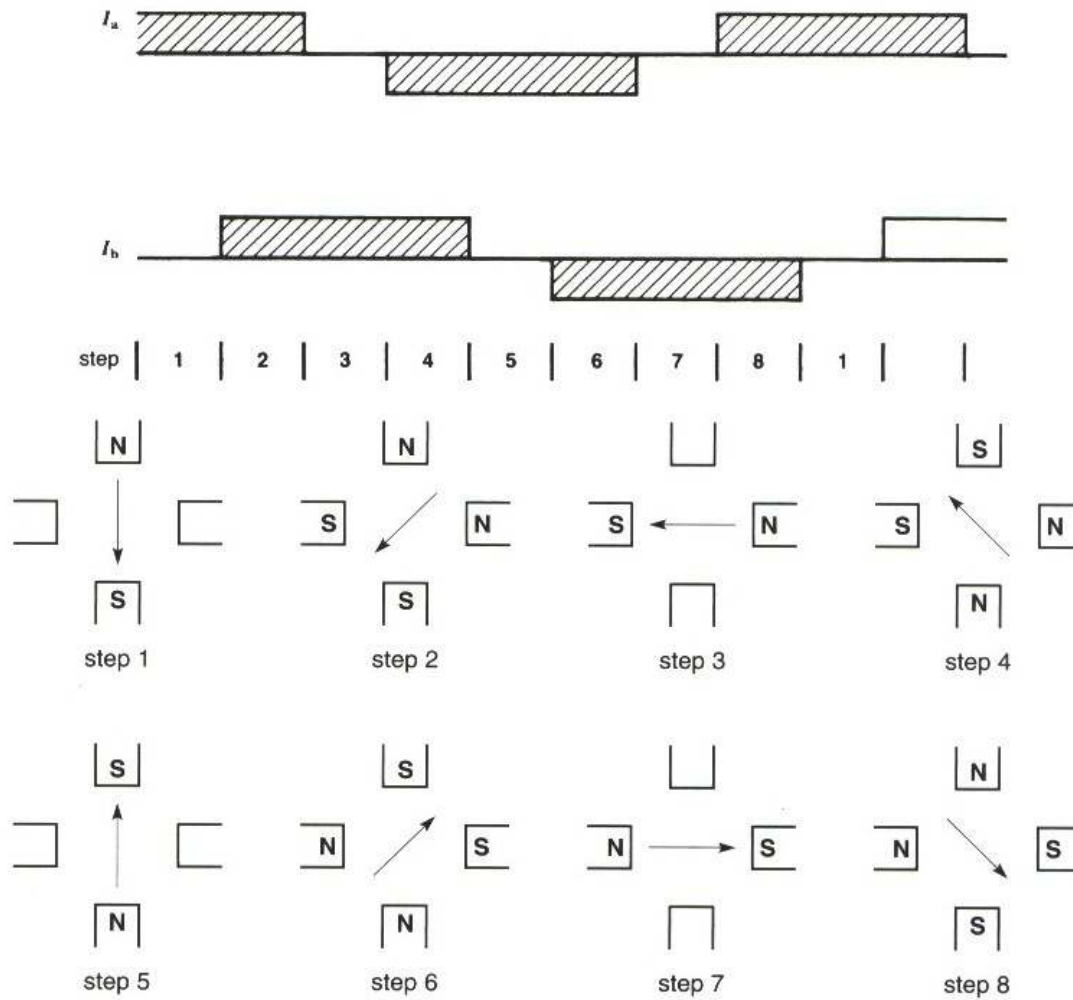


Fig. 22 Current pulses and generated fluxes for half-step switching sequence

### High Speed Operation

In the previous analysis, the current waveforms is considered to be in the form of rectangular pulses (ideal case) that reach its peak value and drops from peak to zero in no time as shown in Fig. 23-a. However, in practice, because of the system inductance, the current can not change its value instantaneously. This can be explained by considering the circuit shown in Figs. 23-b and 23-c. In this configuration, an inductive load is connected to a DC source via transistor. A freewheeling diode is also used to provide a continuous path for the load current when the transistor is switched *OFF*. Fig.

23-d presents the shape of the current pulse in this case. There are two operation modes for this circuit that can be explained as follows:

- ✓ **Mode 1 (Transistor is ON):** The transistor behaves as short circuit. The load (motor's phase) current ( $i = i_1$ ) starts to flow with an increasing magnitude until it reaches its rated value  $I$  (where  $I = \frac{E}{R}$ ) in approximately 3 times the circuit time constant ( $\tau_o = \frac{L}{R}$ ) i.e. time to reach rated current is  $3\tau_o = \frac{3L}{R}$ . The load current continues to flow with its rated value ( $i = I$ ) until the transistor is switched OFF at  $t = T_p$ .
- ✓ **Mode 2 (Transistor is OFF):** The transistor behaves as open circuit. The load current ( $i = i_2$ ) will continue to circulate in the freewheeling circuit path as shown in Fig. 23-c. This current starts to decay until it reaches zero in approximately 3 times the circuit time constant i.e. time to decay to zero is also  $3\tau_o = \frac{3L}{R}$ .

This practical pulse shape is characterized by:

- ✓ The total current period is  $T_p + 3\tau_o$ . This results in delaying the switching ON process of the next phase in stepper motor.
- ✓ The initial torque developed by stepper motors is less than its ideal value because the current doesn't reach its rated value instantly.
- ✓ The short current pulse period required to allow the current to reach its rated value is approximately 6 times the circuit time constant ( $6\tau_o = \frac{6L}{R}$ ) as shown in Fig. 23-e. Normally, stepper motors have time constants ranging from 1 to 8 ms. Therefore, the minimum permissible pulse period (minimum duration for one step) is 6 ms which corresponds to a maximum stepping rate of about 166 steps per second.

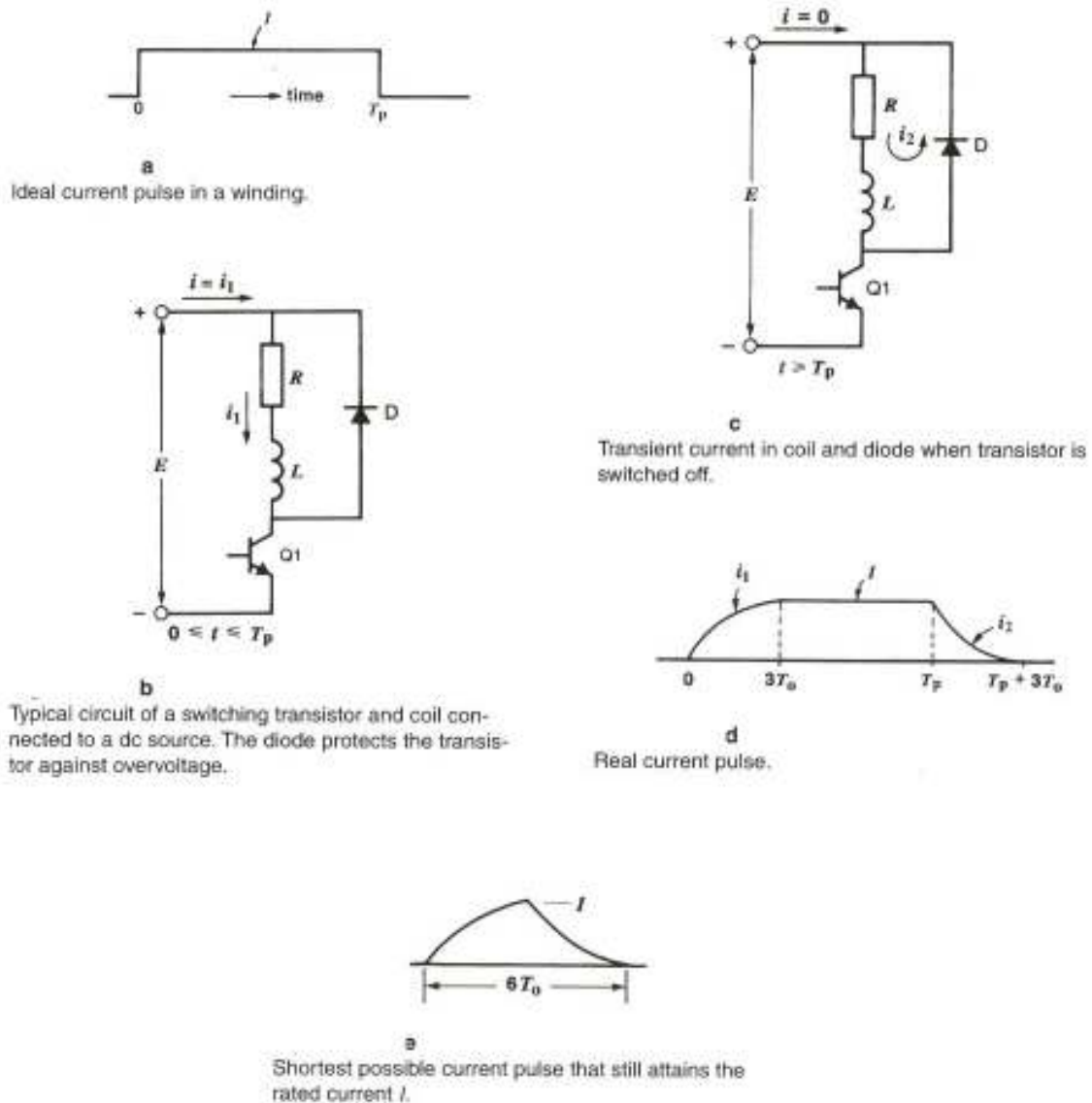


Fig. 23 High speed operation equivalent circuits and current waveforms

There two methods that can be applied to increase the switching rate. These methods can be explained briefly as follows:

- ✓ **External Resistance:** In this method, an external resistance is inserted in series with the motor's phase winding, as shown in Fig. 24, to reduce the circuit time constant. This figure shows that an external resistance with a resistive value four times the phase winding resistance is added. In this case, the new time constant is  $\tau_o = \frac{L}{5R}$  consequently, the new minimum permissible pulse period is 1.2 ms which corresponds to a maximum stepping rate of about 833 steps per second. However,

this method requires increasing the DC source rating to five times its initial value  $5E$  as it has to deliver five times the original power. Moreover, the power losses in the resistances are very high.

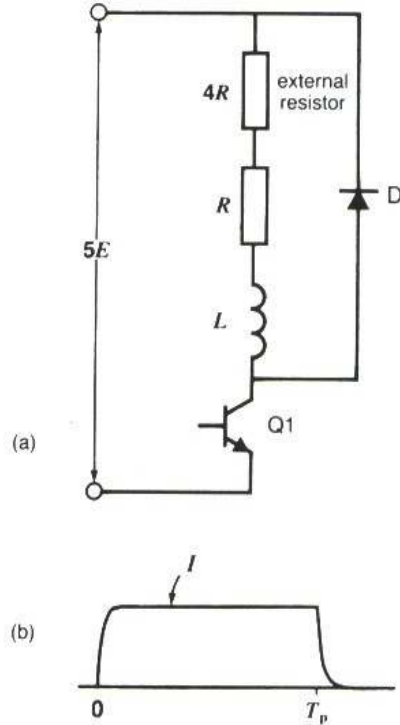


Fig. 24 Circuits to modify the rise and fall time of the current pulse and the associated current

✓ **Bi-level Drive:** In this method, two controllable switches (transistors) and two diodes are used in each phase as shown in Fig. 25-a. Moreover, an additional DC source ( $E_{ext}$ ) with higher magnitude than the original source ( $E$ ) is also used. This circuit has three modes of operation that can be explained briefly, with numerical values, as follows:

✓  **$Q1$  and  $Q2$  are switched ON:** This is represented by the equivalent circuit shown in Fig. 25-b. The transistors behave as short circuits.

The load (motor's phase) current starts to flow with an increasing magnitude until it reaches its rated value of,

$$I = \frac{E}{R} = \frac{3}{0.3} = 10A$$

The circuit time constant is given by,



$$\tau_o = \frac{L}{R} = \frac{2.4 \times 10^{-3}}{0.3} = 8ms$$

Therefore the current approximately increases linearly with an increasing rate ( $Rate_1$ ) as shown in Fig. 25-c that can be expressed by,

$$Rate_1 = \frac{\frac{E + E_{est}}{R}}{\tau_o} = \frac{\frac{3 + 57}{0.3}}{0.008} = 25,000 A/sec$$

The time required to reach the phase winding's rated current ( $t_1$ ) is therefore approximated to,

$$t_1 = \frac{I}{Rate_1} = \frac{10}{25,000} = 0.4ms$$

✓ **Q1 is switched OFF while Q2 is still ON:** This mode is activated once the phase current reaches its rated value of 10 A. The switch  $Q1$  is switched *OFF* while the switch  $Q2$  remains conducting. This is represented by the equivalent circuit shown in Fig. 25-d. The switch  $Q1$  behaves as an open circuit. In this case the diode  $D1$  will conduct and the current flows as shown in Fig. 25-d with a constant magnitude of 10 A ( $I = \frac{E}{R} = \frac{3}{0.3} = 10A$ )

✓ **Q1 and Q2 are switched OFF:** The current will remain flowing in the phase winding circuit until switch  $Q2$  is turned *OFF*. Both switches are now in their *OFF* state and behave as open circuits. In this case the two diodes  $D1$  and  $D2$  will conduct and the current flows as shown in Fig. 25-e.

Assume that the switch  $Q2$  is turned *OFF* after 5 ms from the instant at which the current reached its rated value. The circuit time constant is given by,

$$\tau_o = \frac{L}{R} = \frac{2.4 \times 10^{-3}}{0.3} = 8ms$$

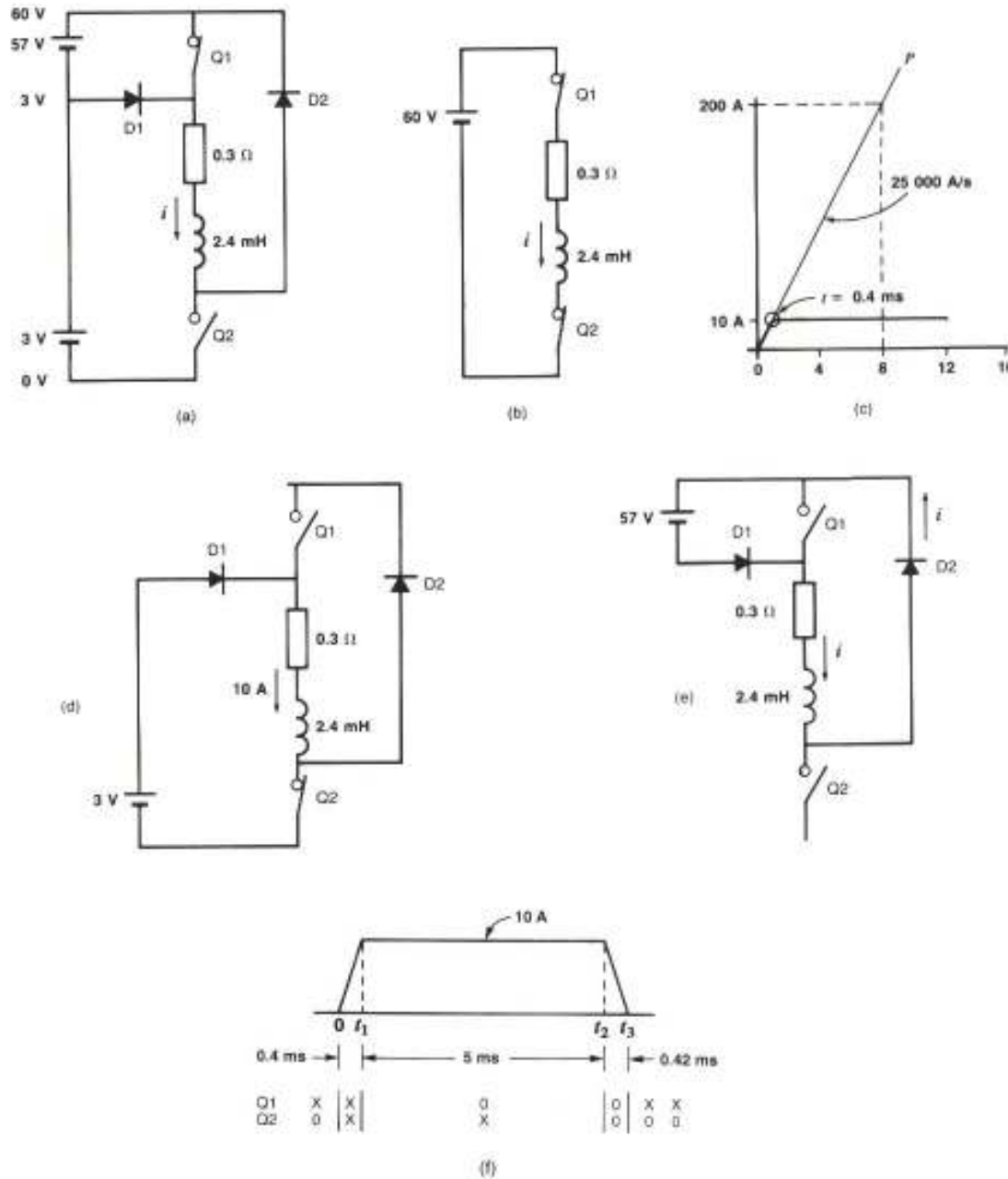


Fig. 25 Bi-level drive circuits to modify the rise and fall time of the current pulse and the associated current

Therefore the current approximately decreases linearly with an decreasing rate ( $Rate_2$ ) as shown in Fig. 25-f that can be expressed by,

$$Rate_2 = \frac{\frac{E_{est}}{R}}{\tau_o} = \frac{\frac{57}{0.3}}{0.008} = 23,750 A/sec$$

The time required for the current to decay to zero ( $t_2$ ) is therefore approximated to,

$$t_2 = \frac{I}{\text{Rate}_2} = \frac{10}{23,750} = 0.42\text{ms}$$

Once the current reaches zero, switch  $Q1$  is switched *ON* to force the phase current to remain zero until the next pulse.

### Chopper Drive Circuit

This drive circuit - illustrated in its unipolar form in Fig.26 - has a high supply voltage which is applied to the phase winding whenever the current falls below its rated value. If the phase excitation signal is present, the base drive for transistor T2 is controlled by the voltage  $V_c$  dropped across the small resistance  $R_c$  by the winding current. At the beginning of the excitation interval the transistor T1 is switched on and the base drive to T2 is enabled. As the phase current is initially zero there is no voltage across  $V_c$  and the transistor T2 is switched on. The full supply voltage is therefore applied to the phase winding, as shown in the timing diagram, Fig.27.

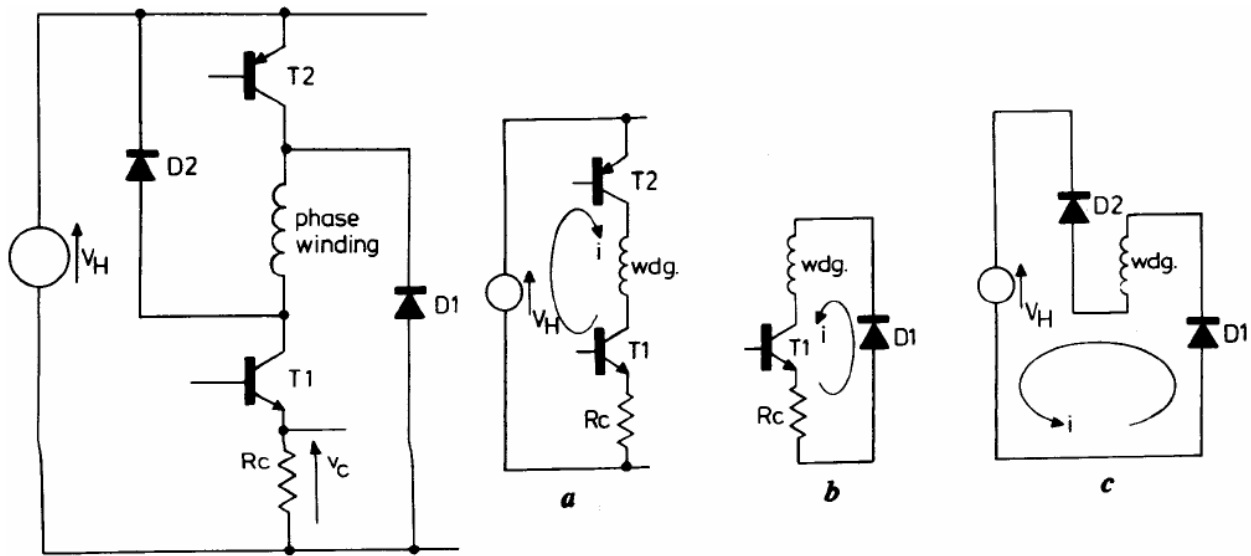


Fig.26 Chopper drive current waveform and transistor switching times

The phase current rises rapidly until it slightly exceeds its rated value ( $I$ ). Consequently the control voltage is  $R_c I + e$  and this is sufficient to switch off transistor T2. There is now no voltage applied to the phase winding and the current decays around a path which includes T1,  $R_c$  and diode D1. This current path has a small resistance and no opposing voltage, so the decay of current is relatively slow. As the resistance  $R_c$  is still included in the circuit the winding current can be monitored and when the control voltage has fallen to  $R_c I - e$  the transistor T2 is switched on again. The full supply voltage is applied to the winding and the

current is rapidly boosted to slightly above rated. This cycle is repeated throughout the excitation time, with the winding current maintained near its rated value by an 'on-off' closed-loop control.

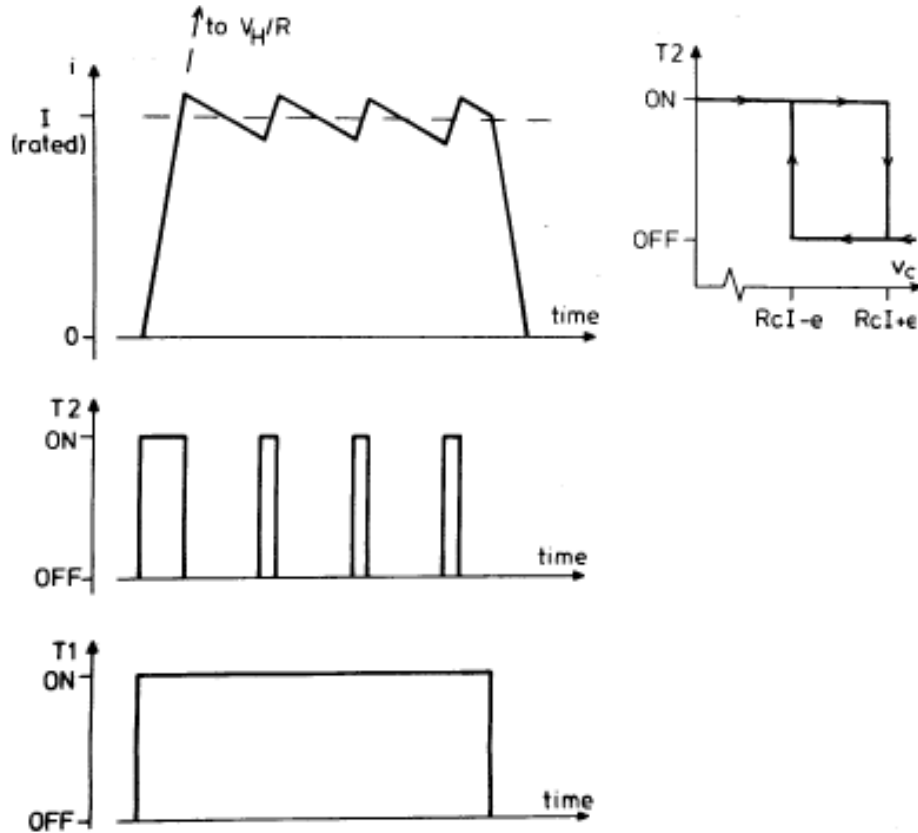


Fig.27 Copper drive current waveform and transistor switching times

At the end of the excitation interval both transistors are switched off and the winding current freewheels via diodes D1 and D2. The current is now opposed by the supply voltage and is rapidly forced to zero. A high proportion of the energy stored in the winding inductance at turn-off is returned to the supply and therefore the system has a high efficiency.

The chopper drive incorporates more sophisticated control circuitry, e.g. the T2 base drive requires a Schmitt triggering of the control voltage  $V_c$  to produce the transition levels. If these levels are not well-separated the transistor T2 switches on and off at a very high frequency, causing interference with adjacent equipment and additional iron losses in the motor. However the chopper drive does have the advantage that the available supply voltage is fully utilized, enabling operation over the widest possible speed range, and the power losses in forcing resistors are eliminated, giving a good system efficiency.

## 2.7. Characteristics of Stepper motor

### Static Characteristics

The characteristics relating to stationary motors are called static characteristics.

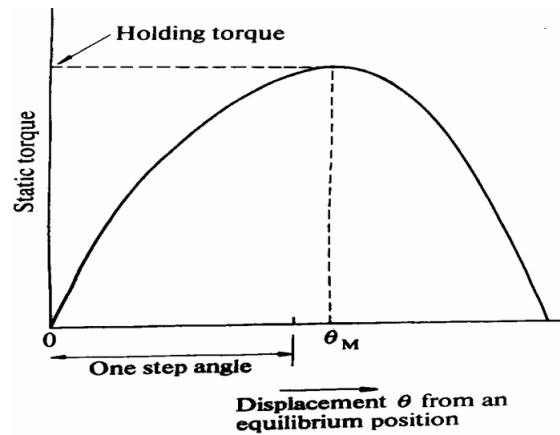


Fig.28  $T/\theta$  characteristics

#### ✓ $T/\theta$ characteristics

The stepping motor is first kept stationary at a rest (equilibrium) position by supplying a current in a specified mode of excitation, say, single-phase or two phase excitation. If an external torque is applied to the shaft, an angular displacement will occur. The relation between the external torque and the displacement may be plotted as in Fig.28. This curve is conventionally called the  $T/\theta$  characteristic curve, and the maximum of static torque is termed the 'holding torque', which occurs at  $\theta = \theta_M$  in Fig.28. At displacements larger than  $\theta_M$ , the static torque does not act in a direction towards the original equilibrium position, but in the opposing direction towards the next equilibrium position. The holding torque is rigorously defined as 'the maximum static torque that can be applied to the shaft of an excited motor without causing continuous motion'. The angle at which the holding torque is produced is not always separated from the equilibrium point by one step angle.

#### ✓ $T/I$ characteristics

The holding torque increases with current, and this relation is conventionally referred to as  $T/I$  characteristics. Fig.29 compares the  $T/I$  characteristics of a typical hybrid motor with those of a variable-reluctance motor, the step angle of both being  $1.8^\circ$ . The maximum static torque appearing in the hybrid motor with no current is the detent torque, which is defined as the maximum static torque that can be applied to the shaft of an unexcited motor without causing continuous rotation.

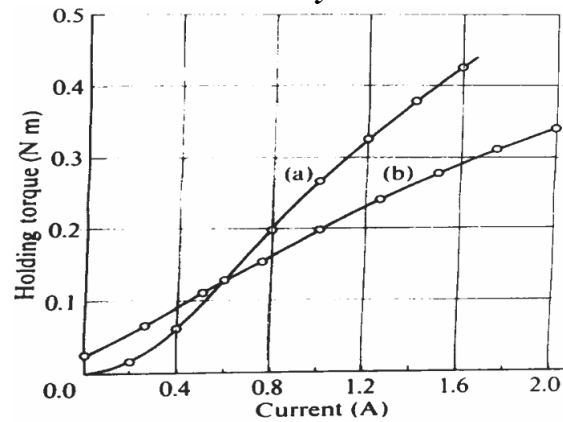


Fig.29 Examples of T/I characteristics: (a) a 1.8° four phase variable reluctance motor; and (b) a 1.8° four phase hybrid motor.

### Dynamic Characteristics

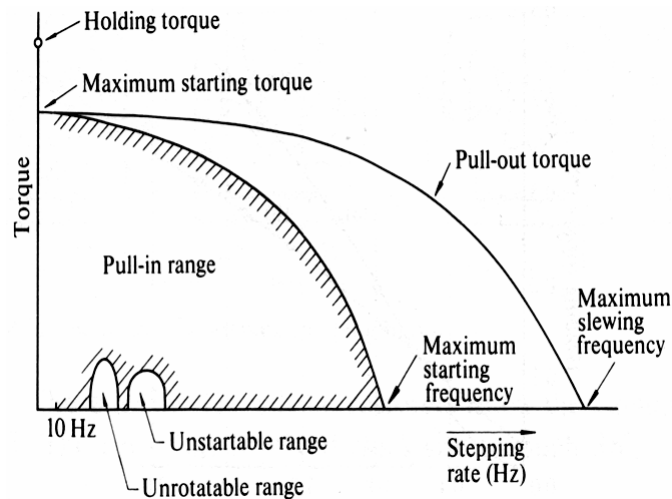


Fig.30 Dynamic characteristics

The characteristics relating to motors which are in motion or about to start are called dynamic characteristics.

#### (1) Pull-in torque characteristics

These are alternatively called the starting characteristics and refer to the range of frictional load torque at which the motor can start and stop without losing steps for various frequencies in a pulse train. The number of pulses in the pulse train used for the test is 100 or so. The reason why the word 'range' is used here, instead of 'maximum', is that the motor is not capable of starting or maintaining a normal rotation at small frictional loads in certain frequency ranges as indicated in Figure. When the pull-in torque is measured or discussed, it is also necessary to specify clearly the driving circuit, the measuring method, the coupling method, and the inertia to be coupled to the shaft. In general, the self-starting range decreases with increases in inertia.

**(2) Pull-out torque characteristics**

This is alternatively called the slewing characteristic. After the test motor is started by a specified driver in the specified excitation mode in self-starting range, the pulse frequency is gradually increased; the motor will eventually run out of synchronism. The relation between the frictional load torque and the maximum pulse frequency with which the motor can synchronize is called the pull-out characteristic (see Figure). The pull-out curve is greatly affected by the driver circuit, coupling, measuring instruments, and other conditions.

**(3) The maximum starting frequency**

This is defined as the maximum control frequency at which the unloaded motor can start and stop without losing steps.

**(4) Maximum pull-out rate**

This is defined as the maximum frequency (stepping rate) at which the unloaded motor can run without losing steps, and is alternatively called the 'maximum slewing frequency'

**(5) Maximum starting torque**










This is alternatively called 'maximum pull-in torque' and is defined as the maximum frictional load torque with which the motor can start and synchronize with the pulse train of a frequency as low as 10Hz.

**2.7. Modes of Excitation**

**Excitation Sequence in the single-phase-on operation**










In this mode of operation only one phase is excited at any time so it is called “single-phase-on operation”.The below table shows the sequences of a single phase excitation mode for three and four-phase motors. In the table the shaded parts represent the excited state and white the blanks show the un-excited state.

**1) Three-phase motor**

<b><i>Clock State</i></b>	<b><i>R</i></b>	<b><i>1</i></b>	<b><i>2</i></b>	<b><i>3</i></b>	<b><i>4</i></b>	<b><i>5</i></b>	<b><i>6</i></b>	<b><i>7</i></b>	<b><i>8</i></b>
<b><i>Phase 1</i></b>									
<b><i>Phase 2</i></b>									
<b><i>Phase 3</i></b>									

To rotate a motor into clockwise direction, the excitation sequence is  $Ph1 \rightarrow Ph2 \rightarrow Ph3 \dots$ , for counter clockwise rotation the excitation sequence is  $Ph3 \rightarrow Ph2 \rightarrow Ph1 \dots$ . It is also known as “one-phase-on drive”

## 2) Four-phase motor





















<i>Clock State</i>	<i>R</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>Phase 1</i>									
<i>Phase 2</i>									
<i>Phase 3</i>									
<i>Phase 4</i>									

### Excitation Sequence in the two-phase-on operation



















In this mode of operation two phases are always in excited condition so it is called “two-phase-on operation”. The below table shows the sequences of a two phase excitation mode for three and four-phase motors. It is seen in these tables that when an excitation current is switched from one phase to another (e.g. as shown by the arrow in table Ph2 is turned off and Ph1 is turned on) the third phase (Ph3 in the above example) remains excited. It is also known as “Two-phase-on drive”

- ✓ In two-phase-on drive the oscillation damps more quickly than the case of the one-phase-on drive mode

## 1) Three-phase motor

<i>Clock State</i>	<i>R</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>Phase 1</i>									
<i>Phase 2</i>									
<i>Phase 3</i>									

## 2) Four-phase motor

<i>Clock State</i>	<i>R</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>Phase 1</i>									
<i>Phase 2</i>									
<i>Phase 3</i>									
<i>Phase 4</i>									



**Half-Step mode****Excitation Sequence in the half-step operation (three-phase motor)**

<i>Clock State (A)</i>	<i>R</i>	<i>1</i>		<i>2</i>		<i>3</i>		<i>4</i>		<i>5</i>	
<i>Clock State (B)</i>	<i>R</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>
<i>Phase 1</i>											
<i>Phase 2</i>											
<i>Phase 3</i>											

**Excitation Sequence in the two-phase-on operation for a bifilar-wound three-phase VR motor**

<i>Clock State</i>	<i>R</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>Phase 1 (S1)</i>									
<i>Phase 2 (S2)</i>									
<i>Phase 3 (S3)</i>									
<i>Phase 4 (S4)</i>									
<i>Phase 5 (S5)</i>									
<i>Phase 6 (S6)</i>									

**Excitation Sequence in the bridge operation for a two-phase motor**

<i>Clock State</i>	<i>R</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>S1</i>									
<i>S2</i>									
<i>S3</i>									
<i>S4</i>									
<i>S5</i>									
<i>S6</i>									
<i>S7</i>									
<i>S8</i>									

**2.7. Windings Used in Stepper Motors**

The transistor bridge bipolar drive circuit requires four transistor/diode pairs per phase, where as the simple unipolar drive requires only one pair per phase, so drive costs for a hybrid stepping motor are potentially higher than for the variable reluctance type; a

two phase hybrid motor drive has eight transistors and diodes, but a three-phase variable reluctance motor drive has only three transistors and diodes. The bridge configuration has the additional complication of base drive isolation for the pair of switching transistors connected to the positive supply rail. From the view point of drive costs the conventional hybrid motor has a severe disadvantage and therefore many manufacturers have introduced 'bifilar-wound' hybrid motors, which can be operated with a unipolar drive.

A bidirectional current flowing in the hybrid motor windings produces a bidirectional field in the stator poles. With a bifilar winding the same results is achieved by two pole windings in opposite senses, as illustrated for one pole in Fig.31. Depending on the field direction, one of the windings is excited by a unidirectional current; in Fig.31 the field produced by a positive current in the conventional arrangement is available by exciting the bifilar +winding with positive current. The effect of negative current in the conventional winding is then achieved by positive excitation of the bifilar -winding.

Each of the bifilar pole windings must have as many turns as the original winding and the same rated current, so a bifilar winding has twice the volume of a conventional winding. This additional volume does, of course, increase the manufacturing costs but for small size of hybrid motor this is outweighed by resultant reduction in drive costs.

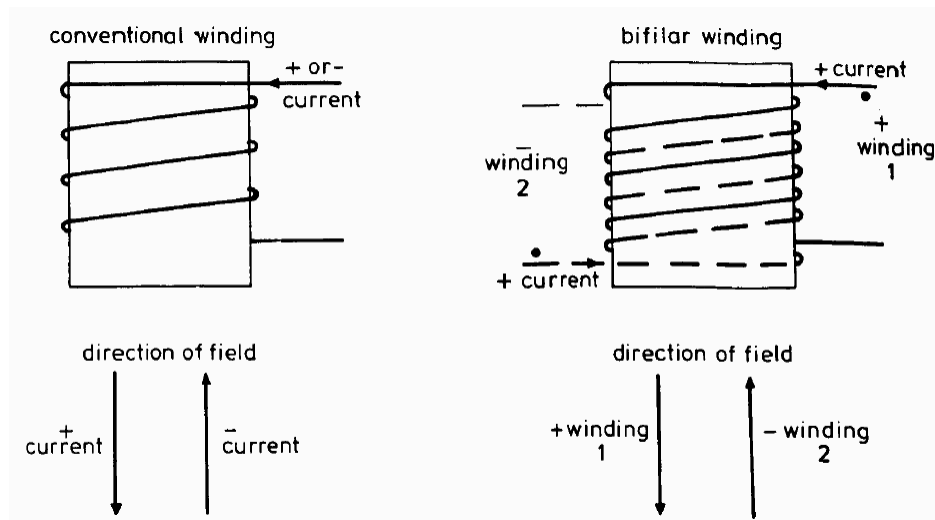


Fig.31 Comparison of conventional (monofilar) and bifilar windings

## **2.8 Linear and Non-Linear Analysis**

The linear and non-linear analysis of the motor performance with respect to the torque produced by the motor is explained below. Let

$T_m$  be the motor torque produced by the rotor in NM

- $J$  be the inertia of the rotor and load combination in  $\text{Kgm}^2$
- $\omega$  be the angular velocity of the rotor
- $D$  be the damping co-efficient or viscous frictional co-efficient
- $T_f$  be the frictional load torque independent of the speed
- $\theta_s$  be the step angle in radians
- $f$  be the stepping rate in steps/sec or pps

Frictional load torque  $T_f = K\theta$

According to rotor dynamics

$$T_m = J \frac{d\omega}{dt} + D\omega + T_f \quad \text{----- (1)}$$

Also  $\theta_s = \theta = \omega t = \text{step angle}$

$$\omega = \frac{\theta_s}{t} = f\theta_s \quad \text{----- (2)}$$

Where

$$f = \frac{1}{T} \quad \text{----- (3)}$$

Substitute  $\omega = f\theta_s$  in (1)

$$T_m = J \frac{d(f\theta_s)}{dt} + D(f\theta_s) + T_f \quad \text{----- (4)}$$

$\theta_s = \frac{360}{mN_r}$  is fixed for a particular type of motor.

So  $\theta_s$  can be considered as constant therefore

$$T_m = J\theta_s \frac{d}{dt}(f) + D\theta_s(f) + T_f \quad \text{----- (5)}$$

In the above equation if viscous friction constant is neglected the equation will be linear equation, the corresponding analysis is a linear analysis. If the damping co-efficient is also considered the corresponding acceleration will be non-linear and the equation will be a nonlinear which given rise to non-linear analysis.

### **Linear acceleration on Linear Analysis:**

If the damping co-efficient is neglected  $D=0$ . The expression for motor torque becomes

$$T_m = J \frac{d\omega}{dt} + T_f \quad \text{----- (6)}$$

$$T_m - T_f = J \frac{d\omega}{dt}$$

$$\frac{T_m - T_f}{J} = \frac{d\omega}{dt}$$

$$d\omega = \left( \frac{T_m - T_f}{J} \right) dt \quad \text{----- (7)}$$

Integrating on both sides

$$\omega = \left( \frac{T_m - T_f}{J} \right) t + \omega_1 \quad \text{----- (8)}$$

Where  $\omega_1$  = Integrating Constant

Mathematically  $\omega_1$  is the constant of integration but it indicates the initial angular velocity of the motor before the occurrence of acceleration. Therefore

$$\omega = \theta_s f_0$$

$$\omega_1 = \theta_s f_1$$

Sub  $\omega$  and  $\omega_1$  in (8)

$$\left( \frac{T_m - T_f}{J} \right) t + \theta_s f_1 = \theta_s f$$

Divide by  $\theta_s$  we get

$$\left( \frac{T_m - T_f}{J\theta_s} \right) t + f_1 = f$$

Therefore stepping rate  $f = \left( \frac{T_m - T_f}{J\theta_s} \right) t + f_1 \quad \text{----- (9)}$

And  $T_f = K \theta$

Figure shows the linear acceleration from  $\omega_1$  and  $\omega_2$

### **Non Linear (Exponential) acceleration on non linear Analysis:**

Considering the torque produced by the motor

$$T_m = J\theta_s \frac{d}{dt}(f) + D\theta_s(f) + T_f \quad \text{----- (10)}$$

$$(T_m - T_f) = J\theta_s \frac{d}{dt}(f) + D\theta_s(f)$$

Divide by  $J\theta_s$  we get

$$\frac{d}{dt}(f) + \frac{D}{J}(f) - \frac{T_m - T_f}{J\theta_s} = 0$$

Or

$$\frac{df}{dt} + \left(\frac{D}{J}\right)f = \frac{T_m - T_f}{J\theta_s} \text{ ----- (11)}$$

The above equation is of the form

$$\frac{dy}{dx} + PY = Q$$

Which have the solution

$$ye^{\int P dx} = \int Qe^{\int P dx} + C \text{ ----- (12)}$$

Here

$$y=f, x=t, P=D/J$$

$$\theta = \frac{T_m - T_f}{J\theta_s}$$

$$f e^{\int \frac{D}{J} dt} = \int \frac{T_m - T_f}{J\theta_s} e^{\int \frac{D}{J} dt} + c \text{ ----- (13)}$$

$$f e^{\int \frac{D}{J} t} = \int \frac{T_m - T_f}{J\theta_s} e^{\int \frac{D}{J} t} + c \text{ ----- (14)}$$

$$f e^{\int \frac{D}{J} t} = \frac{T_m - T_f}{J\theta_s} \left[ \frac{e^{\int \frac{D}{J} t}}{\frac{D}{J}} \right] + c \text{ ----- (15)}$$

Where 'c' is integration constant.

To find 'c' substitute initial conditions,

At  $t=0$  and  $f(0)=f_1$

$$f_1 e^0 = \left( \frac{T_m - T_f}{J\theta_s} \right) \frac{1}{\frac{D}{J}} + c$$

$$f_1 = \left( \frac{T_m - T_f}{\theta_s} \right) \frac{1}{D} + c$$

$$f_1 = \left( \frac{T_m - T_f}{D\theta_s} \right) + c \quad c = f_1 - \left( \frac{T_m - T_f}{D\theta_s} \right) \text{ ----- (16)}$$

Sub (16) in (15) we get

$$f e^{\int \frac{D}{J} t} = \left( \frac{T_m - T_f}{J\theta_s} \right) \frac{1}{\frac{D}{J}} e^{\int \frac{D}{J} t} + f_1 - \left( \frac{T_m - T_f}{D\theta_s} \right) \text{ ----- (17)}$$

$$f e^{\frac{D}{J}t} = \left( \frac{T_m - T_f}{D\theta_s} \right) e^{\frac{D}{J}t} + \left[ f_1 - \left( \frac{T_m - T_f}{D\theta_s} \right) \right]$$

Divide by  $e^{\frac{D}{J}t}$

$$f = \left( \frac{T_m - T_f}{D\theta_s} \right) + \left[ f_1 - \left( \frac{T_m - T_f}{D\theta_s} \right) \right] e^{-\frac{D}{J}t}$$

There fore Stepping frequency

$$f = \left( \frac{T_m - T_f}{D\theta_s} \right) + \left[ f_1 - \left( \frac{T_m - T_f}{D\theta_s} \right) \right] e^{-\frac{D}{J}t}$$

The above equation is a nonlinear exponential equation which gives rise to non-linear acceleration of the rotor of the motor.

## **2.9 THEORY OF TORQUE PREDICTION**

### **Mechanism of static torque production in a VR Motor**

Basic field theory approach of torque production depends on

- Magnetic energy and Co-energy
- Ideal case of torque production, in which stator and rotor cores have infinite permeability
- Cores when subjected to magnetic saturation

### **The case of infinitely permeable cores**

To analysis the situation of an iron piece being drawn into a magnetic field created by electromagnet as shown in figure below.

A current  $I$  is flowing in the coil of 'n' turns to yield magnetic flux and force 'f' is acting on the iron piece in the x-direction. The iron piece may be regarded as a tooth of the rotor of a stepping motor, and the electromagnet corresponds to a pair of teeth of the stator in a VR motor. Ampere's circuit law along the closed loop is expressed as

$$\oint H \cdot dl = nI \quad \text{----- (1)}$$

When 'H' is the magnetic field intensity

The left hand side of the above equation can be written as

$$\oint H \cdot dl = H_g \left( \frac{g}{2} \right) + H_g \left( \frac{g}{2} \right) + H_i(l) \\ = H_g(g) + H_i(l) \quad \text{----- (2)}$$

Where

$H_g$  = Magnetic field intensity in airgaps

$H_i$  = Magnetic field intensity in cores

$l$  = Total magnetic path in cores

When, the permeability of cores is extremely large,  $H_i$  is so low that it is allowable to put  $H_i = 0$ . If  $H_i = 0$  and the core permeability  $\mu$  is  $\infty$  namely that  $B = \mu H_i = \infty$  in the cores. So, equ (2) becomes,

$$\oint H \cdot dl = H_g(g)$$

Therefore substitute the above equation in (1), we get

$$H_g(g) = nl$$

$$H_g = \frac{nl}{g} \quad \text{----- (3)}$$

The airgap flux density is given by

$$B = \mu_0 \frac{nl}{g} \quad (\because B = \mu H) \quad \text{----- (4)}$$

Where  $\mu_0$  = Permeability of airgap =  $4\pi \times 10^{-7} \text{ NA}^{-2}$

Let

$w$  = Transverse length of iron piece

$x$  = Distance by which rotor tooth and iron piece overlap

The overlapped area is  $xw$

The  $B_g$  in (4) multiplied by the overlapped area is the magnetic flux

$$\varphi = xw\mu_0 \frac{nl}{g} \quad \text{----- (5)}$$

Hence the flux linkages  $\psi$  is given by

$$\psi = n\varphi = xw\mu_0 \frac{n^2 l}{g} \quad \text{----- (6)}$$

Now let us assume that there is an incremental displacement  $\Delta x$  of the tooth during a time interval  $\Delta t$  then the incremental flux linkage,  $\Delta\psi$  is given by

$$\Delta\psi = \Delta x \cdot w \cdot \mu_0 \cdot \frac{n^2 I}{g} \text{-----} (7)$$

The emf induced in the coils by change in flux linkage is given by,

$$\begin{aligned} e &= -\frac{\Delta\psi}{\Delta t} = -\Delta x \cdot w \cdot \mu_0 \cdot \frac{n^2 I}{g} \cdot \frac{1}{\Delta t} \\ &= -w \cdot \mu_0 \cdot \frac{n^2 I}{g} \frac{\Delta x}{\Delta t} \text{-----} (8) \end{aligned}$$

The minus sign in the equation implies that the direction of the emf is opposing the current.

Since the current  $I$  is supplied by the power source for the time interval  $\Delta t$  overcoming the counter-emf, the work (energy) done  $\Delta P_i$  by the source is

$$\begin{aligned} \Delta P_i &= I |e| \Delta t = I \cdot w \cdot \mu_0 \cdot \frac{n^2 I}{g} \cdot \frac{\Delta x}{\Delta t} \cdot \Delta t \\ &= \frac{w \mu_0 n^2 I^2}{g} \Delta x \text{-----} (9) \end{aligned}$$

The coil resistance is assumed to be zero, to simplify the analysis using (4),  $\Delta P_i$  is expressed in terms of  $B_g$  as follows

$$\begin{aligned} \Delta P_i &= \frac{w \mu_0 n^2 I^2}{g} \Delta x \\ \left( \therefore B_g^2 &= \mu_0^2 \frac{n^2 I^2}{g^2} \right) \\ &= w \frac{\mu_0^2 n^2 I^2}{\mu_0 g^2} g \Delta x \\ &= \frac{w}{\mu_0} g \frac{\mu_0^2 n^2 I^2}{g^2} \Delta x \\ &= \frac{w}{\mu_0} g B_g^2 \Delta x \\ &= \frac{B_g^2}{\mu_0} w g \Delta x \text{-----} (10) \end{aligned}$$

The work done by the source is converted partly to mechanical work, and the rest is spent in increasing the magnetic field energy in the gaps. The increase in the gap field energy is given by

$$\Delta w_m = \frac{1}{2} \frac{B_g^2}{\mu_0} \cdot (\text{The increase in the gap space})$$



$$\Delta w_m = \frac{1}{2} \frac{B_g^2}{\mu_0} g w \Delta x \text{----- (11)}$$

From the observations of the equations (10) & (11) we find that a half of  $\Delta P_i$  is converted into magnetic energy in the airgaps, consequently we are allowed to say that the other half of  $\Delta P_i$  is converted into mechanical work. Since the mechanical work is the force 'f' multiplied by the displacement  $\Delta x$ , we obtain

$$f \Delta x = \frac{1}{2} \left[ \frac{B_g^2}{\mu_0} g w \Delta x \right] \text{----- (12)}$$

Eliminating  $\Delta x$  from both sides,

$$f = \frac{1}{2} \left[ \frac{B_g^2}{\mu_0} g w \right] \text{----- (13)}$$

Which by use of equation (4), may be put in the form,

$$f = \frac{1}{2} \frac{w \mu_0 n^2 I^2}{g} \text{----- (14)}$$

On the other hand, the magnetic energy  $w_m$  in the gap is

$$w_m = \frac{1}{2} \frac{B_g^2}{\mu_0} g x w \text{----- (15)}$$

From equation (13) and (15) we derive

$$f = \left( \frac{dw_m}{dx} \right) \text{----- (16)}$$

Attention must, however, be paid to the assumption that the current  $I$  is kept constant during the displacement. Hence (16) must be described in the rigorous form

$$f = \left( \frac{dw_m}{dx} \right)_{I=\text{constant}}$$

### **The case of constant Permeability's**

In the model with infinitely permeable cases, the magnetic field appears only in the gaps and its magnetic treatment is simple. When cores are of finite permeability, on the other hand, magnetic energy appears not only in the airgaps, but also in the cores and spaces other than the gaps. If the coil inductance is  $L$  in the model of figure the flux linkages  $\psi$  is given by

$$\psi = LI \text{----- (1)}$$

The magnetic energy  $w_m$  in the system is expressed as

$$w_m = \frac{1}{2} L I^2 \text{-----} (2)$$

If the iron piece undergoes a displacement  $\Delta x$  during the time interval  $\Delta t$ , the inductance  $L$  will increase by  $\Delta L$ . The emf induced in the coil is

$$e = -\frac{\Delta \psi}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} \text{-----} (3)$$

If the power supply is a current source and provides a current  $I$  during displacement (3) become

$$e = -I \frac{\Delta L}{\Delta t} \text{-----} (4)$$

Since the voltage at the source is equal but opposite to the counter emf of equation (4), the work  $\Delta P_i$  done by the source on the circuit is,

$$\begin{aligned} \Delta P_i &= I |e| \Delta t = I^2 \Delta L \\ &= I \left| I \cdot \frac{\Delta L}{\Delta t} \right| \Delta t = I^2 \Delta L \text{-----} (5) \end{aligned}$$

On the other hand, the increase in the magnetic energy

$$\Delta \omega_m = \frac{1}{2} I^2 \Delta L \text{-----} (6)$$

From comparison of equations (5) and (6) it is seen that half of the work done on the circuit by the source is converted into mechanical energy. Hence it is supposed that the other half is converted to mechanical work

$$\Delta P_o = f \Delta x = \frac{1}{2} I^2 \Delta L \text{-----} (7)$$

Then the force is

$$f = \frac{1}{2} I^2 \frac{\Delta L}{\Delta x} \text{-----} (8)$$

## **TWO MARKS QUESTIONS AND ANSWERS**

### **1. Define stepper motor.**

Stepper motor is an electrical motor which receives the digital pulses as input and produce the mechanical rotation in terms of steps.

### **2. What is meant by step angle?**

The fixed angle through which the rotor rotates for each pulse is called step angle.

**3. Write the expression for step angle in terms of stator and rotor teeth.**

$$\text{Step angle } \theta_s = \frac{N_s \sim N_r}{N_s N_r} \times 360^\circ$$

Where  $N_s$  = Number of Stator teeth

$N_r$  = Number of Rotor teeth

**4. Define resolution.**

The factor which defines the fine movement of the rotor to complete one revolution smoothly is called resolution. If number of steps increased per revolution resolution will be increased.

**5. Mention the types of stepper motor.****With PM**

- Claw pole
- PM motor
- Disc magnet motor
- Hybrid PM Motor.

**With out PM**

- Variable reluctance stepper motor (VRSM)

**6. What is REA?**

REA means Rare Earth Alloy. It is the special type of material used for the PM which is a combination of Neodymium, Samarium Cobalt and ferrite.

**7. State the advantages and disadvantages of VRSM.****Advantages**

- High torque to inertia ratio
- Rotor inertial is less
- High stepping rate and high speed slewing capability
- Less weight
- Freewheeling ability

**Disadvantages**

- At low voltages low stepping rate and low efficiency
- Presence of mid range resonance frequency under certain drive conditions.
- Normal step angle is 30 degree.
- Absence of detent torque with winding de energized.

**8. State the classification of VRSM.**

- Single stack VRSM
- Multi stack VRSM

**9. List the advantages of multi stack VRSM.**

- Smaller step size
- High torque per unit volume
- High efficiency

**10. State the types of exciting mode.**

- Single phase ON mode
- 2 phase on mode
- Half step mode
- 2 phase on drive of bifilar wound motor
- Excitation by a bridge circuit.
- Micro step mode.

**11. Differentiate between the single phase and two phase on mode operation.**

In single phase on mode one phase is excited at a time. But in two phase on mode two phase windings are excited simultaneously to produce the torque.

**12. Define micro stepping.**

It is defined as the stepping rate at which the rotor has to move number of steps to complete one revolution. It improves the resolution.

**13. What is meant by hybrid stepper motor?**

The motor which is operated under the combined principles of the PM and VRSM is called as hybrid stepper motor.

**14. State the design parameters for stepper motor.**

- Resistance
- Inductance
- Series and parallel connection
- Driver voltage
- Motor stiffness
- Motor heat
- Accuracy

- Resonance.

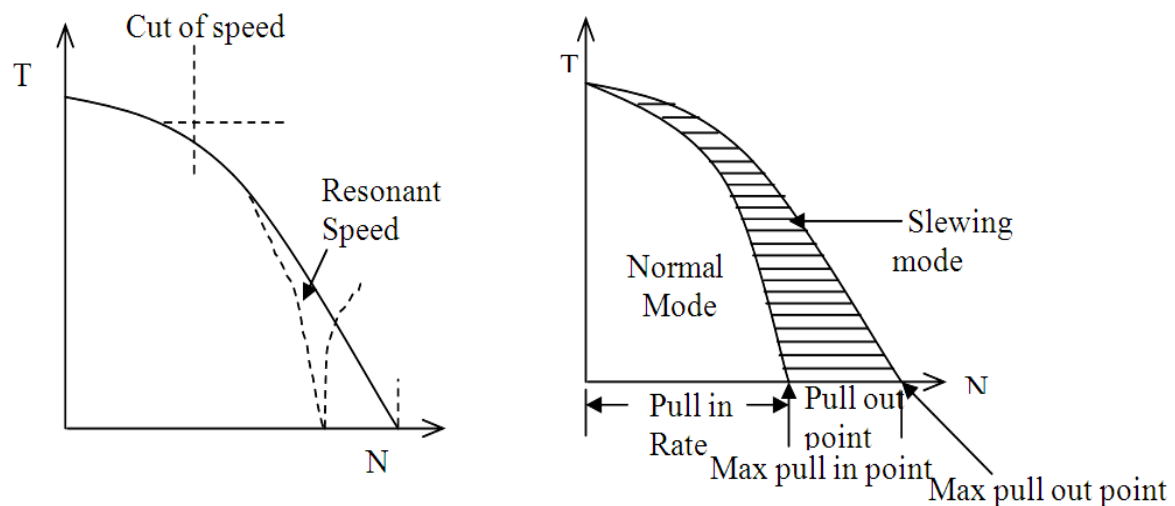
**15. State the advantages of hybrid stepper motor.**

- Combined advantages of VRSM and PM stepper motor.

**16. List the applications of stepper motor.**

- Floppy disk drive
- Hard disk drive
- Printers
- Plotters
- Taps and video recorders.

**17. Draw the speed torque characteristics of stepper motor.**



**18. Define stepping rate.**

It is defined as the speed of rotation of a stepper motor in terms of the number of steps per second where the number of steps equals the number of input pulses.

**19. Define the following terms: Slew range, Pull in/out torque and pull in/out rate.**

**Slew range:**

It is defined as the range of switching rates between pull in and pull out in which a motor will run in synchronism but cannot start or reverse.

**Pull in torque:**

It is defined as the maximum torque against which a motor will start at a given pulse rate and reach synchronism without losing step.

**Pull out torque:**

It is defined as the maximum torque which can be applied to a motor running at a given stepping rate without losing synchronism.

**Pull in rate:**

It is defined as the maximum switching rate at which a loaded motor can start without losing steps.

**Pull out rate:**

It is defined as the maximum switching rate at which a motor will remain in synchronism without missing steps against load torque T.

**20. Define detent torque and holding torque.**

**Detent torque:**

It is defined as the maximum load torque which the un energized stepper motor can with stand without slipping.

**Holding Torque:**

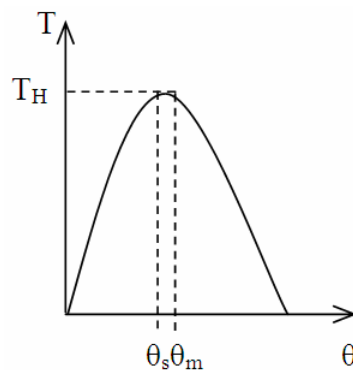
It is the maximum load torque which the energized stepper motor can with stand without slipping from the equilibrium state.

**21. Classify the static characteristics of stepper motor.**

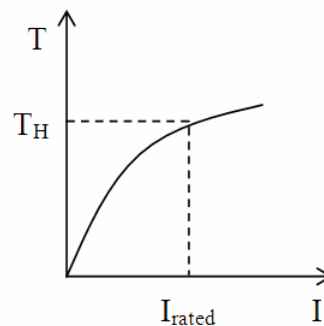
- Torque angle characteristics
- Torque current curve.

**22. Draw the 2 types of static characteristics of stepper motor.**

Torque – Angle characteristics



Torque – Current characteristics



**23. Write the torque equation for stepper motor.**

$$T = \frac{1}{2} i^2(t) \cdot \frac{dl(\theta)}{d(\theta)}$$

**24. Write the merits and demerits of permanent magnet stepper motor.****Merits:**

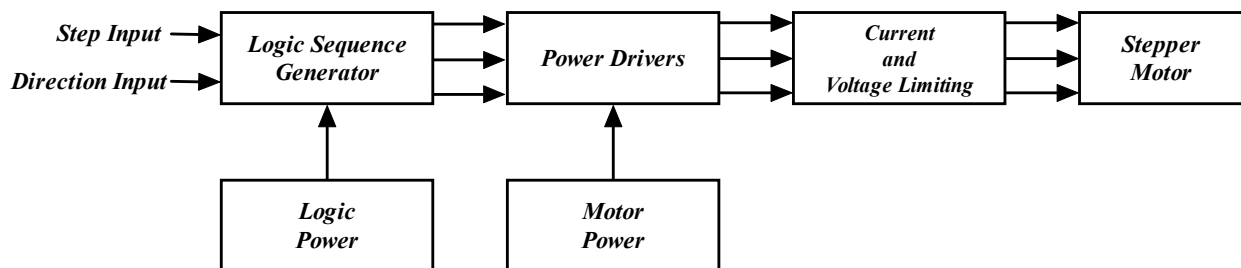
- Provides detent torque when winding is de energized
- Less resonance tendency
- High holding torque capability
- High efficiency at lower speeds and stepping rates
- High stepping rate capability
- Better damping due to the presence of rotor magnet.

**Demerits:**

- Higher inertia and weight due to the rotor magnet
- Performance will be affected by change in magnet strength
- Costlier than VRSM.

**25. Classify the driver circuits for stepper motor.**

- Resistance drive (L/R drive)
- Bilevel drive or dual voltage drive
- Chopper drive
- Linear constant current drive
- Variable voltage control

**26. Draw the controller block diagram of stepper motor.****27. What is the need for logic sequencer?**

It generates programmed logic sequences required for operation of a stepper motor. This is a finite state sequential logic circuit which generates the particular logic sequence selected by the user for operation of his stepper motor. The selected logic sequence is treated as a truth table, and is implemented with the help of flip-flops and logic gates.

**28. State the advantages of each drive circuits.**

**L/R drive:****Advantages:**

1. The motor current rises much more because of the reduction in the electrical time constant.
2. The motor develops greater torque at high stepping rate.

**Disadvantages:**

1. Since the current through the series resistance increases, the copper loss also increased, which results in heating.
2. In order to reach the same steady state current as before the voltage required to be applied is very high as,  $V' = V_R (R_S + R_m) / R_m$ .

**Bilevel drive:****Merits:**

It is good and energy efficient.

**Demerits:**

1. It requires two regulated power supplies one for high voltage and another for rated voltage.
2. Two power transistors and the current sensor are needed.
3. Complex switching logic.

**Chopper drive:****Merits:**

1. It is applied for high torque stepper motor.
2. Energy efficient
3. Very simple circuit
4. So popular.

**29. What do you understand by full step and half step?**

If the stepper motor rotates for the desired step angle then it is called full step mode. If it is rotated for half of the full step angle with the alternate excitation of single phase on mode and two phase on mode is called as half step mode.

**30. Write down the limitations of open loop operation and need for closed loop operation of stepper motor.****Limitation of open loop:**



- The stepper motor may fail to follow a pulse command when the frequency of the pulse train is too high.
- Motor motion tends to be oscillatory

**Need for closed loop:**

- To avoid the step failures
- To have the quicker and smoother operation

**31. How is the step of the permanent magnet stepper motor controlled?**

By changing the polarity of the exciting current sequence for the winding the step of the PM stepper motor can be controlled.