UNIT-I

DRIVE CHARACTERISTICS

Electrical Drives:

Motion control is required in large number of industrial and domestic applications like *transportation systems, rolling mills, paper machines, textile mills, machine tools, fans, pumps, robots, washing machines etc.*

Systems employed for motion control are called DRIVES, and may employ any of prime movers such as *diesel or petrol engines, gas or steam turbines, steam engines, hydraulic motors and electric motors,* for supplying mechanical energy for motion control. Drives employing electric motors are known as **ELECTRICAL DRIVES.**

An ELECTRIC DRIVE can be defined as an electromechanical device for converting electrical energy into mechanical energy to impart motion to different machines and mechanisms for various kinds of process control.

Classification of Electric Drives

According to **Mode of Operation**

- ✓ Continuous duty drives
- ✓ Short time duty drives
- ✓ Intermittent duty drives

According to **Means of Control**

- ✓ Manual
- ✓ Semi automatic
- ✓ Automatic

According to **Number of machines**

- ✓ Individual drive
- ✓ Group drive
- ✓ Multi-motor drive

According to **Dynamics and Transients**

- ✓ Uncontrolled transient period
- ✓ Controlled transient period

According to **Methods of Speed Control**

- ✓ Reversible and non-reversible uncontrolled constant speed.
- ✓ Reversible and non-reversible step speed control.
- ✓ Variable position control.
- ✓ Reversible and non-reversible smooth speed control.

Advantages of Electrical Drive

1. They have *flexible control* characteristics. The steady state and dynamic characteristics of electric drives can be *shaped* to satisfy the load requirements.
2. Drives can be provided with *automatic fault detection systems*. Programmable logic controller and computers can be employed to *automatically control* the drive operations in a *desired sequence*.
3. They are available in *wide range* of torque, speed and power.
4. They are *adaptable to almost any operating conditions* such as explosive and radioactive environments
5. It can operate in all the *four quadrants* of speed-torque plane
6. They can be *started instantly* and can immediately be *fully loaded*
7. Control gear requirement for speed control, starting and braking is usually *simple and easy to operate.*

Choice (or) Selection of Electrical Drives

Choice of an electric drive depends on a number of factors. Some of the important factors are.

1. *Steady State Operating conditions requirements*

   Nature of speed torque characteristics, speed regulation, speed range, efficiency, duty cycle, quadrants of operation, speed fluctuations if any, ratings etc
2. *Transient operation requirements*

   Values of acceleration and deceleration, starting, braking and reversing performance.
3. *Requirements related to the source*

   Types of source and its capacity, magnitude of voltage, voltage fluctuations, power factor, harmonics and their effect on other loads, ability to accept regenerative power
4. *Capital and running cost, maintenance needs life.*
5. *Space and weight restriction if any.*
6. *Environment and location.*
7. *Reliability.*

Group Electric Drive

This drive consists of a single motor, which drives one or more line shafts supported on bearings. The line shaft may be fitted with either pulleys and belts or gears, by means of which a group of machines or mechanisms may be operated. It is also some times called as *SHAFT DRIVES.*

Advantages

- ✓ A single large motor can be used instead of number of small motors

Disadvantages

- ✓ There is no flexibility. If the single motor used develops fault, the whole process will be stopped.

Individual Electric Drive

In this drive each individual machine is driven by a separate motor. This motor also imparts motion to various parts of the machine.

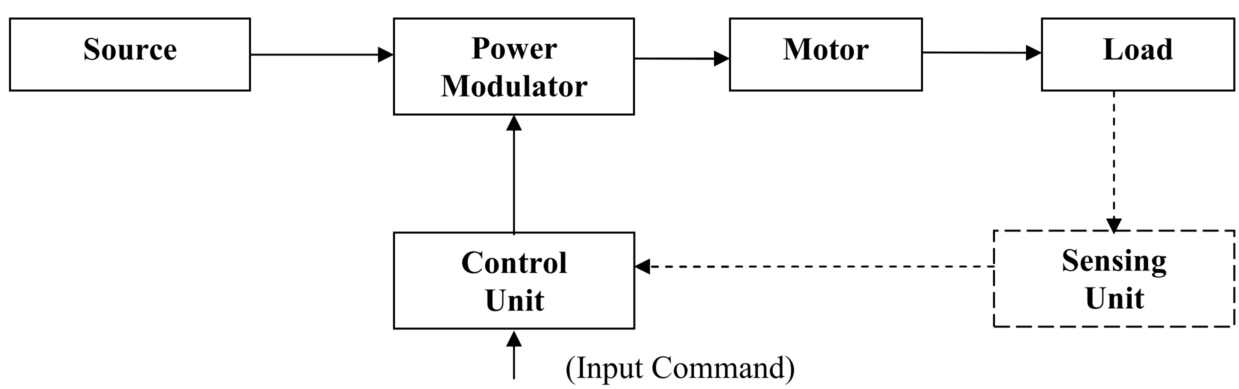
Multi Motor Electric Drive

In this drive system, there are several drives, each of which serves to actuate one of the working parts of the drive mechanisms.

E.g.: Complicated metal cutting machine tools
Paper making industries,
Rolling machines etc.

General Electric Drive System

Block diagram of an electric drive system is shown in the figure below.



A modern variable speed electrical drive system has the following components

- ✓ Electrical machines and loads
- ✓ Power Modulator
- ✓ Sources
- ✓ Control unit
- ✓ Sensing unit

Electrical Machines

Most commonly used electrical machines for speed control applications are the following

DC Machines

Shunt, series, compound, separately excited DC motors and switched reluctance machines.

AC Machines

Induction, wound rotor, synchronous, PM synchronous and synchronous reluctance machines.

Special Machines

Brush less DC motors, stepper motors, switched reluctance motors are used.

Power Modulators

Functions:

- ✓ Modulates flow of power from the source to the motor in such a manner that motor is imparted speed-torque characteristics required by the load
- ✓ During transient operation, such as starting, braking and speed reversal, it restricts source and motor currents with in permissible limits.
- ✓ It converts electrical energy of the source in the form of suitable to the motor
- ✓ Selects the mode of operation of the motor (i.e.) Motoring and Braking.

Types of Power Modulators

In the electric drive system, the power modulators can be any one of the following

- ✓ Controlled rectifiers (ac to dc converters)
- ✓ Inverters (dc to ac converters)
- ✓ AC voltage controllers (AC to AC converters)
- ✓ DC choppers (DC to DC converters)
- ✓ Cyclo converters (Frequency conversion)

Electrical Sources

Very low power drives are generally fed from single phase sources. Rest of the drives is powered from a 3 phase source. Low and medium power motors are fed from a 400v supply. For higher ratings, motors may be rated at 3.3KV, 6.6KV and 11 KV. Some drives are powered from battery.

Sensing Unit

- ✓ Speed Sensing (From Motor)
- ✓ Torque Sensing
- ✓ Position Sensing
- ✓ Current sensing and Voltage Sensing from Lines or from motor terminals

From Load

- ✓ Torque sensing
- ✓ Temperature Sensing

Control Unit

Control unit for a power modulator are provided in the control unit. It matches the motor and power converter to meet the load requirements.

Classification of Electrical Drives

Another main classification of electric drive is

- ✓ DC drive
- ✓ AC drive

Comparison between DC and AC drives

DC DRIVES	AC DRIVES
The power circuit and control circuit is simple and inexpensive	The power circuit and control circuit are complex
It requires frequent maintenance	Less Maintenance
The commutator makes the motor bulky, costly and heavy	These problems are not there in these motors and are inexpensive, particularly squirrel cage induction motors
Fast response and wide speed range	In solid state control the speed range is wide

of control, can be achieved smoothly by conventional and solid state control	and conventional method is stepped and limited
Speed and design ratings are limited due to commutations	Speed and design ratings have upper limits

Applications

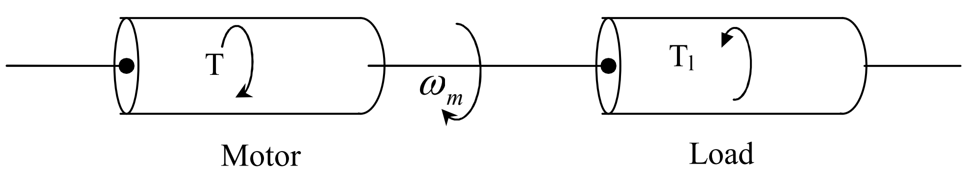
- ✓ Paper mills
- ✓ Cement Mills
- ✓ Textile mills
- ✓ Sugar Mills
- ✓ Steel Mills
- ✓ Electric Traction
- ✓ Petrochemical Industries
- ✓ Electrical Vehicles

Dynamics of Motor Load System

Fundamentals of Torque Equations

A motor generally drives a load (Machines) through *some transmission system*. While motor always rotates, the load may rotate or *undergo a translational motion*.

Load speed may be different from that of motor, and if the load has many parts, their speed may be different and while some parts rotate others may go through a translational motion. Equivalent rotational system of motor and load is shown in the figure.



Notations Used:

J = Moment of inertia of motor load system referred to the motor shaft $kg-m^2$

ω_m = Instantaneous angular velocity of motor shaft, rad/sec.

T = Instantaneous value of developed motor torque, N-m

T_l = Instantaneous value of load torque, referred to the motor shaft N-m

Load torque includes friction and wind age torque of motor. Motor-load system shown in figure can be described by the following fundamental torque equation.

$$T - T_l = \frac{d}{dt}(J\omega_m) = J\frac{d\omega_m}{dt} + \omega_m \frac{dJ}{dt} ------------ (1)$$

Equation (1) is applicable to variable inertia drives such as mine winders, reel drives, Industrial robots.

For drives with constant inertia $\frac{dJ}{dt} = 0$

$$\therefore T = T_l + J\frac{d\omega_m}{dt} ------------ (2)$$

Equation (2) shows that torque developed by motor is counter balanced by load torque T_l and a dynamic torque $\left(J\frac{d\omega_m}{dt}\right)$. Torque component $\left(J\frac{d\omega_m}{dt}\right)$ is called dynamic torque because it is present only during the transient operations.

Note:

Energy associated with dynamic torque $\left(J\frac{d\omega_m}{dt}\right)$ is stored in the form of kinetic energy given by $\frac{J\omega_m^{2}}{2}$.

Classification of Load Torques:

Various load torques can be classified into broad categories.

- ✓ Active load torques
- ✓ Passive load torques

Load torques which has the *potential to drive the motor under equilibrium conditions* are called active load torques. Such load torques usually retain their sign when the drive rotation is changed (reversed)

Eg: Torque due to force of gravity

Torque due tension

Torque due to compression and torsion etc

Load torques which *always oppose the motion and change their sign on the reversal of motion* are called passive load torques

Eg: Torque due to friction, cutting etc.

Components of Load Torques:

The load torque T_l can be further divided in to following components

(i) Friction Torque (T_F)

Friction will be present at the motor shaft and also in various parts of the load. T_F is the equivalent value of various friction torques referred to the motor shaft.

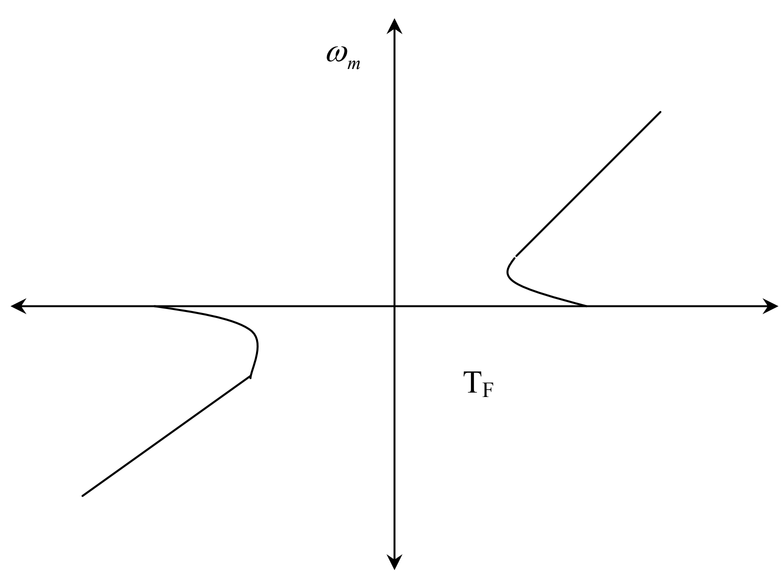
(ii) **Windage Torque (T_W)**

When motor runs, wind generates a torque opposing the motion. This is known as windage torque.

(iii) **Torque required to do useful mechanical work**.

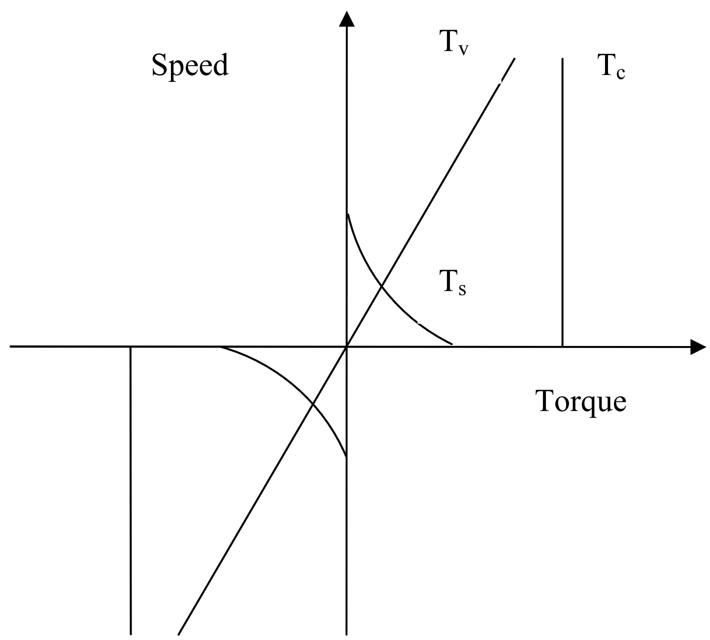
Nature of this torque depends upon particular application. It may be constant and independent of speed. It may be some function of speed, it may be time invariant or time variant, its nature may also change with the load's mode of operation.

Value of friction torque with speed is shown in figure below



Its value at stand still is much higher than its value slightly above zero speed. Friction at zero speed is called stiction or static friction. In order to start the drive the motor should at least exceed stiction.

Friction torque can also be resolved into three components



Component *T_v varies linearly with speed* is called VISCOUS friction and is given by

$$T_v = B\omega_m$$

Where B is viscous friction co-efficient.

Another component *T_C, which is independent of speed*, is known as COULOMB friction. Third component T_s accounts for additional torque present at stand still. Since Ts is present only at stand still it is not taken into account in the dynamic analysis. Windage torque, *T_W which is proportional to speed squared* is given by

$$T_w = C\omega_m^{\ 2} \qquad \text{C is a constant}$$

From the above discussions, for finite speed

$$T_l = T_L + B\omega_m + T_C + C\omega_m^{\ 2}$$

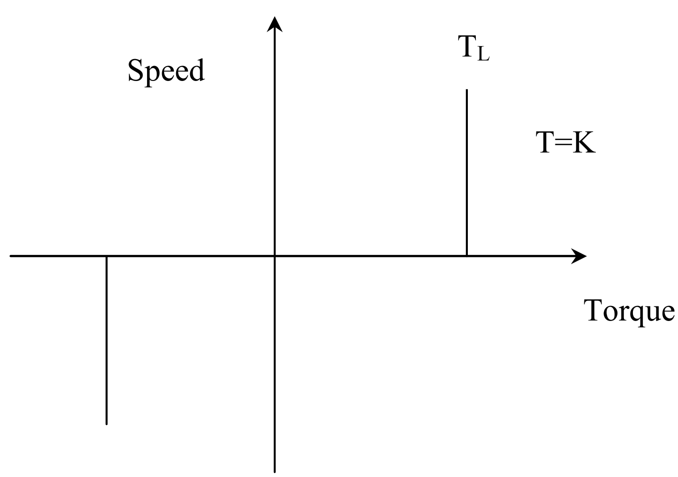
Characteristics of Different types of Loads

One of the essential requirements in the section of a particular type of motor for driving a machine is the *matching of speed-torque characteristics of the given drive unit and that of the motor*. Therefore the knowledge of how *the load torque varies with speed* of the driven machine is necessary. Different types of loads exhibit different speed torque characteristics. However, most of the industrial loads can be classified into the following four categories.

- ✓ Constant torque type load
- ✓ Torque proportional to speed (Generator Type load)
- ✓ Torque proportional to square of the speed (Fan type load)
- ✓ Torque inversely proportional to speed (Constant power type load)

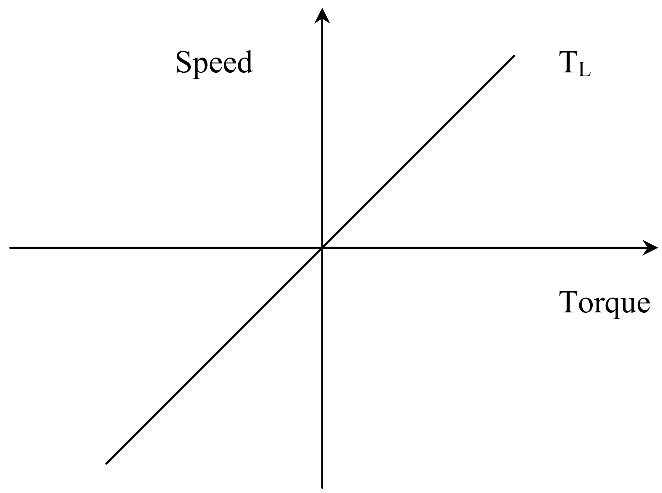
Constant Torque characteristics:

Most of the working machines that have mechanical nature of work like shaping, cutting, grinding or shearing, *require constant torque irrespective of speed*. Similarly cranes during the hoisting and conveyors handling constant weight of material per unit time also exhibit this type of characteristics.



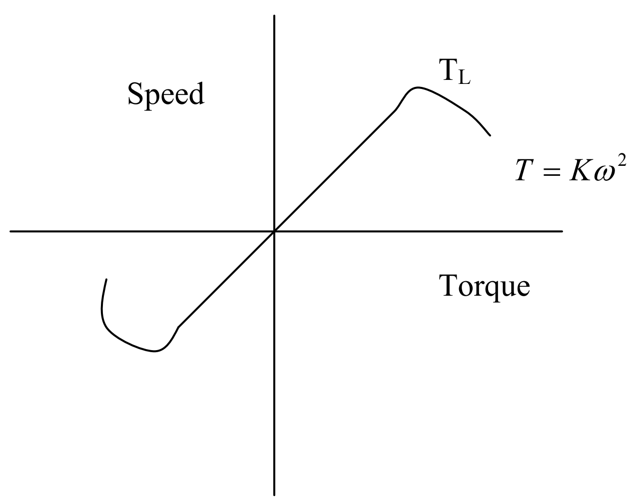
Torque Proportional to speed:

Separately excited dc generators connected to a constant resistance load, eddy current brakes have speed torque characteristics given by T=kω



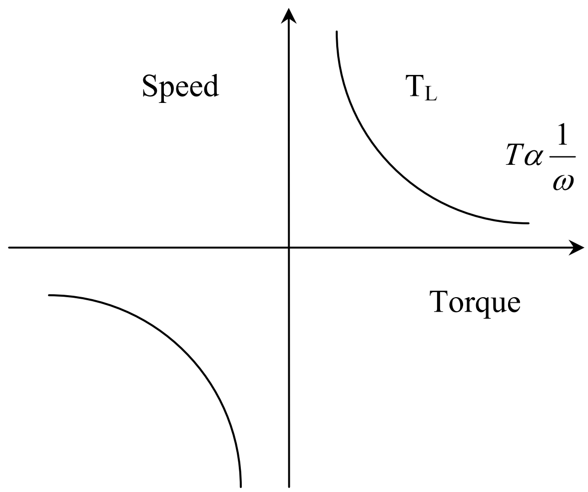
Torque proportional to square of the speed:

Another type of load met in practice is the one in which *load torque is proportional to the square of the speed*.Eg Fans rotary pumps, compressors and ship propellers.



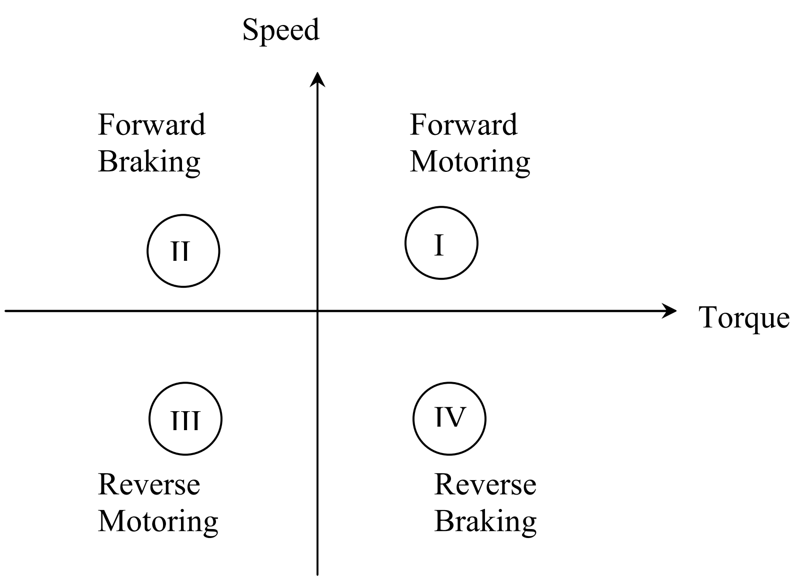
Torque Inversely proportional to speed:

Certain types of lathes, boring machines, milling machines, steel mill coiler and electric traction load exhibit hyperbolic speed-torque characteristics

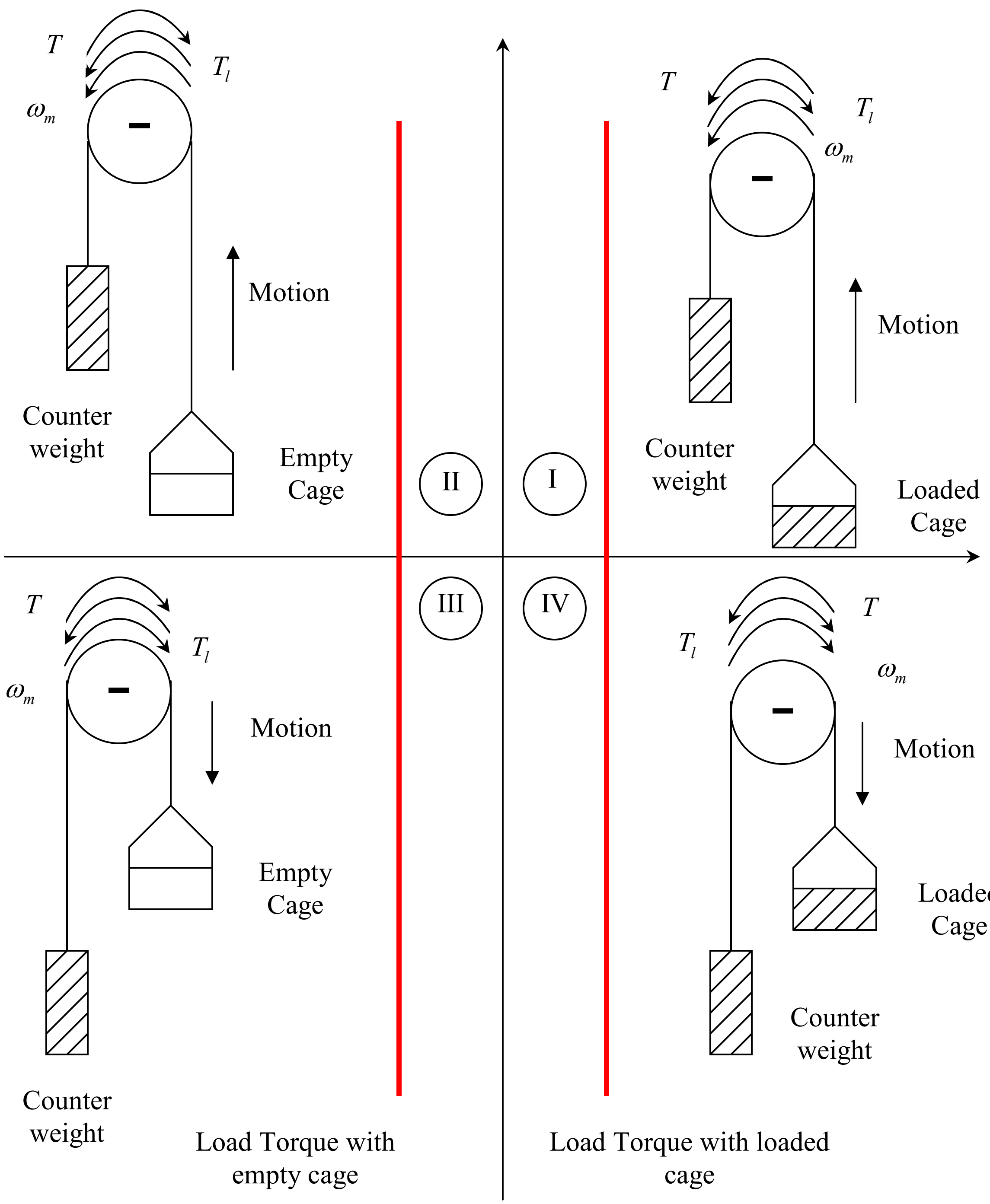


Multi quadrant Operation:

For consideration of multi quadrant operation of drives, it is useful to establish suitable conventions about the signs of torque and speed. *A motor operates in two modes – Motoring and braking.* In *motoring*, it converts *electrical energy into mechanical energy*, which *supports its motion*. In *braking it works as a generator* converting *mechanical energy into electrical energy* and thus opposes the motion. Motor can provide motoring and braking operations for *both forward and reverse directions.* Figure shows the torque and speed co-ordinates for both forward and reverse motions. Power developed by a motor is given by the product of speed and torque. *For motoring operations power developed is positive and for braking operations power developed is negative*.



In quadrant I, developed power is positive, hence machine works as a motor supplying mechanical energy. Operation in quadrant I is therefore called Forward Motoring. In quadrant II, power developed is negative. Hence, machine works under braking opposing the motion. Therefore operation in quadrant II is known as forward braking. Similarly operation in quadrant III and IV can be identified as reverse motoring and reverse braking since speed in these quadrants is negative. For better understanding of the above notations, let us consider operation of hoist in four quadrants as shown in the figure. Direction of motor and load torques and direction of speed are marked by arrows.



A hoist consists of a rope wound on a drum coupled to the motor shaft one end of the rope is tied to a cage which is used to transport man or material from one level to another level . Other end of the rope has a counter weight. *Weight of the counter weight is chosen to be higher than the weight of empty cage but lower than of a fully loaded cage*. Forward direction of motor speed will be one which gives upward motion of the cage. Load torque line in quadrants I and IV represents speed-torque characteristics of the loaded hoist. This torque is the difference of torques due to loaded hoist and counter weight.

The load torque in quadrants II and III is the speed torque characteristics for an empty hoist. This torque is the difference of torques due to counter weight and the empty hoist. Its sigh is negative because the counter weight is always higher than that of an empty cage.

The quadrant I operation of a hoist requires movement of cage upward, which corresponds to the positive motor speed which is in counter clockwise direction here. This motion will be obtained if the motor products positive torque in CCW direction equal to the magnitude of load torque T_{L1}. Since developed power is positive, this is forward motoring operation. Quadrant IV is obtained when a loaded cage is lowered. Since the weight of the loaded cage is higher than that of the counter weight .It is able to overcome due to gravity itself.

In order to limit the cage within a safe value, motor must produce a positive torque T equal to T_{L2} in anticlockwise direction. As both power and speed are negative, drive is operating in reverse braking operation. Operation in quadrant II is obtained when an empty cage is moved up. Since a counter weigh is heavier than an empty cage, its able to pull it up. In order to limit the speed within a safe value, motor must produce a braking torque equal to T_{L2} in clockwise direction. Since speed is positive and developed power is negative, it’s forward braking operation.

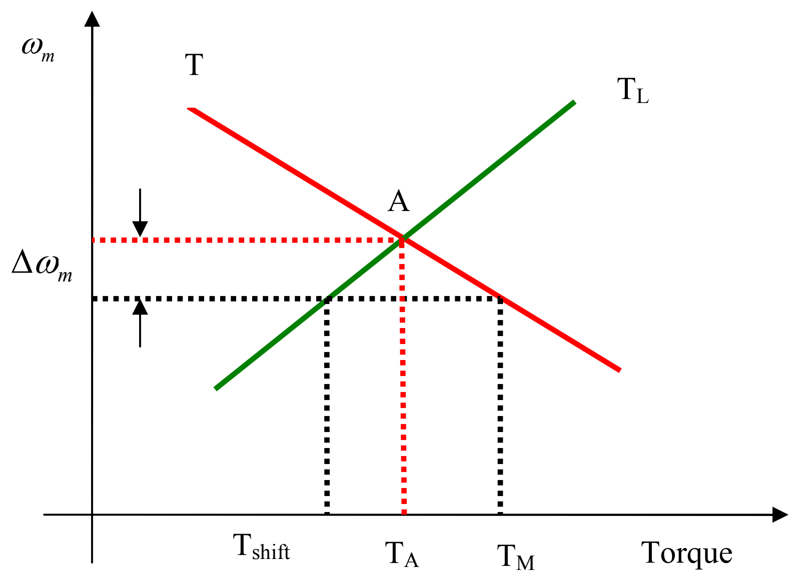
Operation in quadrant III is obtained when an empty cage is lowered. Since an empty cage has a lesser weight than a counter weight, the motor should produce a torque in CW direction. Since speed is negative and developed power is positive, this is reverse motoring operation.

Steady State Stability:

Equilibrium speed of motor-load system can be obtained when motor torque equals the load torque. Electric drive system will operate in steady state at this speed, provided it is the speed of stable state equilibrium. Concept of steady state stability has been developed to readily evaluate the stability of an equilibrium point from the steady state speed torque curves of the motor and load system.

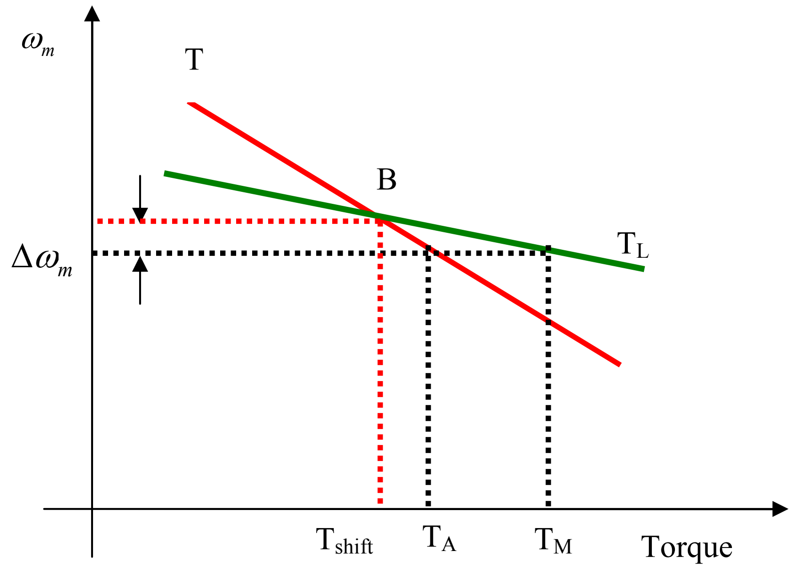
In most of the electrical drives, the electrical time constant of the motor is negligible compared with the mechanical time constant. During transient condition, electrical motor can be assumed to be in electrical equilibrium implying that steady state speed torque curves are also applicable to the transient state operation.

Now, consider the steady state equilibrium point A shown in figure below



The equilibrium point will be termed as stable state when the operation will be restored to it after a small departure from it due to disturbance in the motor or load. Due to disturbance a reduction of $\Delta\omega_m$ in speed at new speed, electrical motor torque is greater than the load torque, consequently motor will accelerate and operation will be restores to point A. similarly an increase in $\Delta\omega_m$ speed caused by a disturbance will make load torque greater than the motor torque, resulting into deceleration and restoring of operation to point A.

Now consider equilibrium point B which is obtained when the same motor drives another load as shown in the figure. A decrease in speed causes the load torque to become greater than the motor torque, electric drive decelerates and operating point moves away from point B. Similarly when working at point B and increase in speed will make motor torque greater than the load torque, which will move the operating point away from point B



From the above discussions, an equilibrium point will be stable when an increase in speed causes load-torque to exceed the motor torque. (i.e.) When at equilibrium point following conditions is satisfied.

$$\frac{dT_L}{d\omega_m} > \frac{dT}{d\omega_m} \text{-------------------} (1)$$

Inequality in the above equation can be derived by an alternative approach. Let a small perturbation in speed, $\Delta\omega_m$ results in ΔT and ΔT_l perturbation in T and T_l respectively. Therefore the general load-torque equation becomes

$$(T + \Delta T) = (T_l + \Delta T_l) + \frac{Jd(\omega_m + \Delta\omega_m)}{dt}$$

$$= \quad T + \Delta T = T_l + \Delta T_l + \frac{Jd\omega_m}{dt} + J\frac{d\Delta\omega_m}{dt} \text{----} (2)$$

The general equation is

$$T = T_l + J\frac{d\omega_m}{dt} \text{--------} (3)$$

Subtracting (3) from (2) and rearranging

$$J\frac{d\omega_m}{dt} = \Delta T - \Delta T_l \text{----------} (4)$$

From small perturbations, the speed –torque curves of the motor and load can be assumed to be straight lines, thus

$$\Delta T = \left(\frac{dT}{d\omega_m}\right)\Delta\omega_m - - - - - - - (5)$$

$$\Delta T_l = \left(\frac{dT_l}{d\omega_m}\right)\Delta\omega_m - - - - - - - (6)$$

Where $\frac{dT}{d\omega_m}$ and $\frac{dT_l}{d\omega_m}$ are respectively slopes of the steady state speed torque curves of motor and load at operating point under considerations. Substituting (5) and (6) in (4) we get,

$$J\frac{d\Delta\omega_m}{dt} + \left(\frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m}\right)\Delta\omega_m = 0 - - - - - (7)$$

This is a first order linear differential equation. If initial deviation in speed at t=0 be $(\Delta\omega_m)_0$ then the solution of equation (7) is

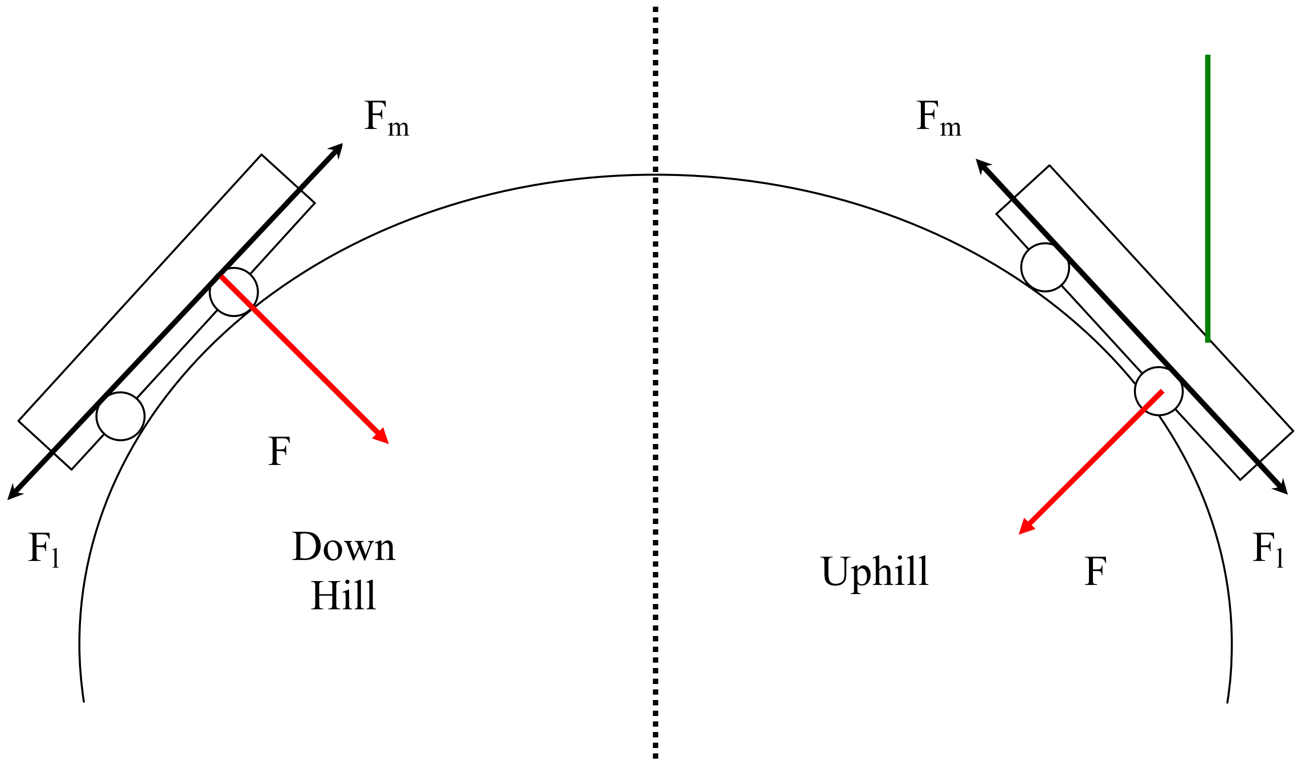
$$\Delta\omega_m = (\Delta\omega_m)_0 \exp\left\{-\frac{1}{J}\left(\frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m}\right)t\right\} - - - - - (8)$$

An operating point will be stable when $\Delta\omega_m$ approaches zero as t approaches infinity. For this to happen exponential term in equation (8) should be negative.

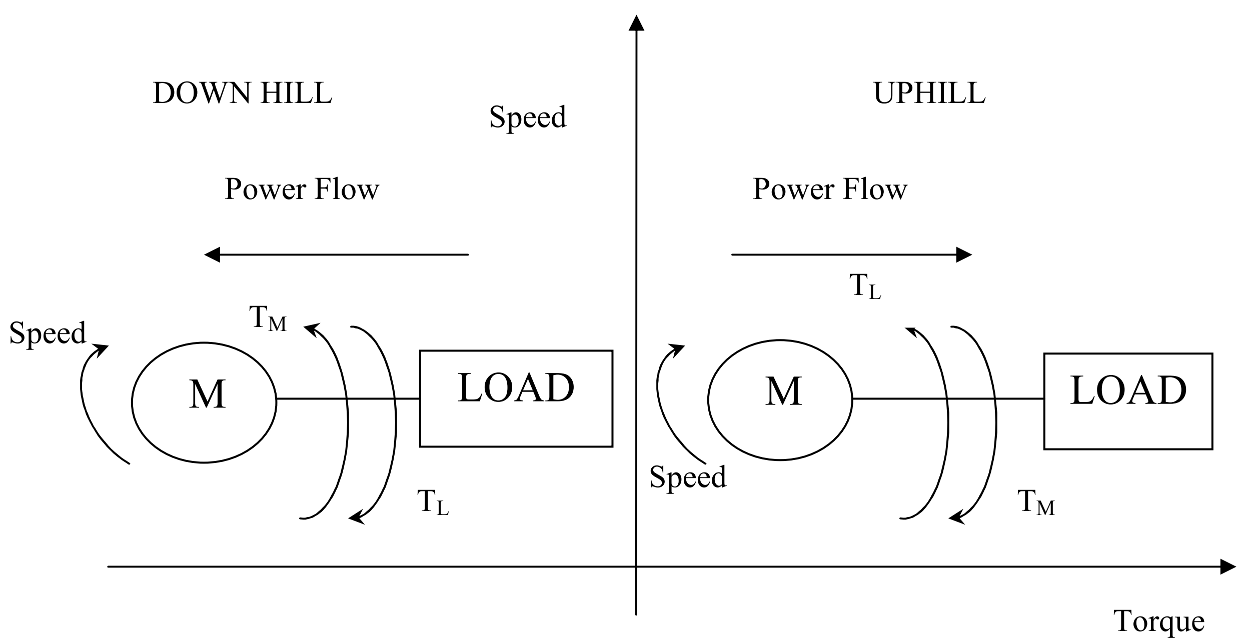
Basics of Regenerative Braking

In the regenerative braking operation, the motor operates as generator, while it is still connected to the supply. Here, the motor speed is greater than the synchronous speed. Mechanical energy is converted into electrical energy, part of which is returned to the supply and rest of the energy is last as heat in the winding and bearings of electrical machines pass smoothly from motoring region to generating region, when over driven by the load.

An example of regenerative braking is shown in the figure below. Here an electric motor is driving a trolley bus in the uphill and downhill direction. The gravity force can be resolved into two components in the uphill direction. One is perpendicular to the load surface (F) and another one is parallel to the road surface F_l. The parallel force pulls the motor towards bottom of the hill. If we neglect the rotational losses, the motor must produce force F_m opposite to F_l to move the bus in the uphill direction.



This operation is indicated as shown in the figure below in the first quadrant. Here the power flow is from the motor to load.



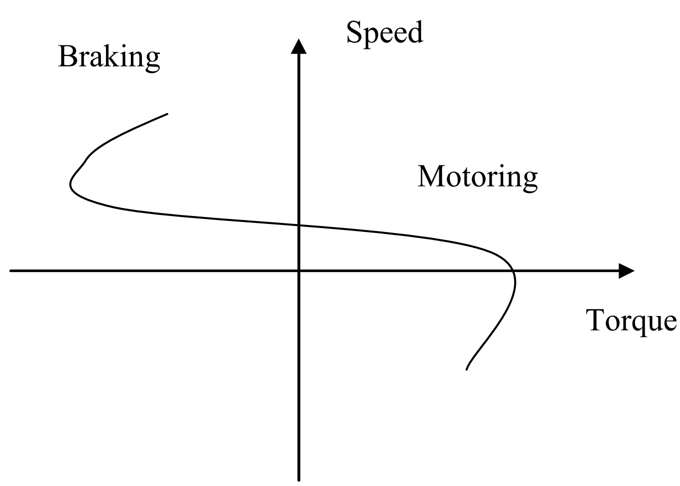
Now we consider that the same bus is traveling in down hill, the gravitational force doesn't change its direction but the load torque pushes the motor towards the bottom of the hill. The motor produces a torque in the reverse direction because the direction of the motor torque is always opposite to the direction of the load torque. Here the motor is still in the same direction on both sides of the hill. This is known as regenerative braking. The energy is exchange under regenerative braking operation is power flows from mechanical load to source. Hence, the load is driving the machine and the machine is generating electric power that is returned to the supply.

Regenerative braking of Induction motor:

An induction motor is subjected to regenerative braking, if the motor rotates in the same direction as that of the stator magnetic field, but with a speed greater than the synchronous speed. Such a state occurs during any one of the following process.

- ✓ Downward motion of a loaded hoisting mechanism
- ✓ During flux weakening mode of operation of IM.

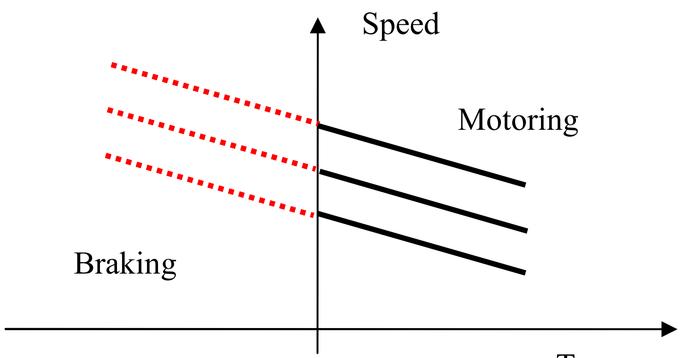
Under regenerative braking mode, the machine acts as an induction generator. The induction generator generates electric power and this power is fed back to the supply. This machine takes only the reactive power for excitation. The speed torque characteristic of the motor for regenerative braking is shown in the figure.



Regenerative Braking for DC motor:

In regenerative braking of dc motor, generated energy is supplied to the source. For this the following condition is to be satisfied.

$E > V$ and I_a should be negative



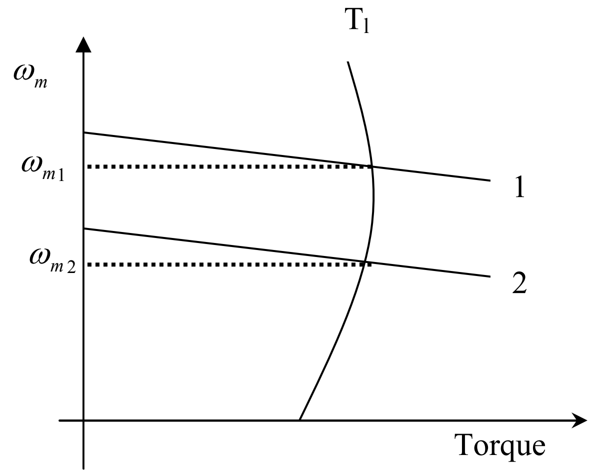
Modes of Operation:

An electrical drive operates in three modes:

- ✓ Steady state
- ✓ Acceleration including Starting
- ✓ Deceleration including Stopping

We know that $T = T_l + J\dfrac{d\omega_m}{dt}$

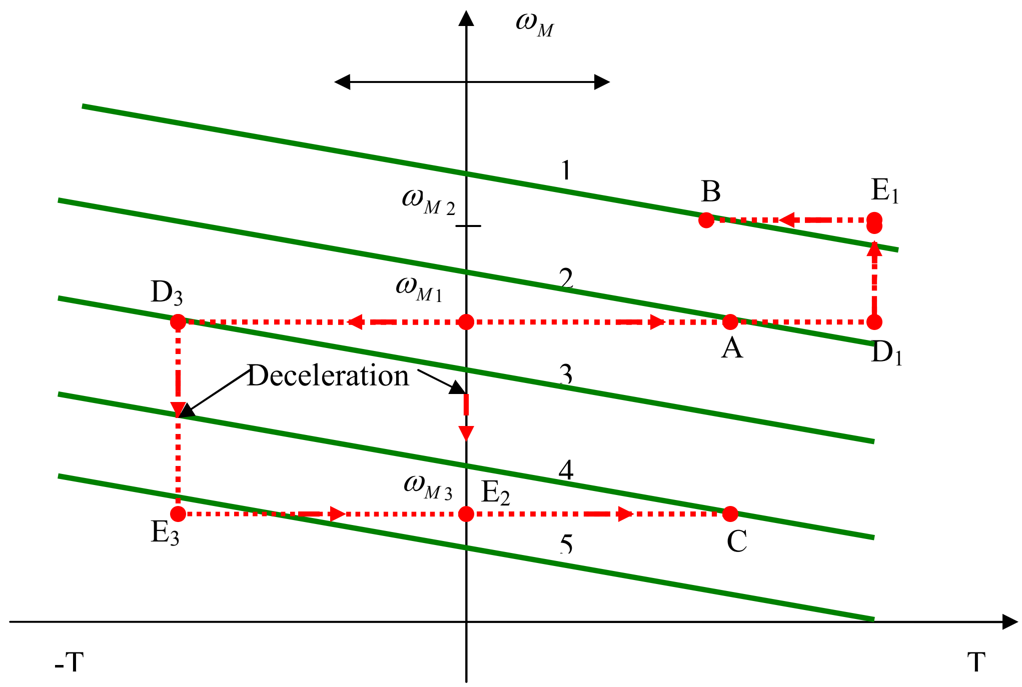
According to the above expression the steady state operation takes place when motor torque equals the load torque. The steady state operation for a given speed is realized by adjustment of steady state motor speed torque curve such that the motor and load torques are equal at this speed. Change in speed is achieved by varying the steady state motor speed torque curve so that motor torque equals the load torque at the new desired speed. In the figure shown below when the motor parameters are adjusted to provide speed torque curve 1, drive runs at the desired speed ω_{m1}. Speed is changed to ω_{m2} when the motor parameters are adjusted to provide speed torque curve 2. When load torque opposes motion, the motor works as a motor operating in quadrant I or III depending on the direction of rotation. When the load is active it can reverse its sign and act to assist the motion. Steady state operation for such a case can be obtained by adding a mechanical brake which will produce a torque in a direction to oppose the motion. The steady state operation is obtained at a speed for which braking torque equal the load torque. Drive operates in quadrant II or IV depending upon the rotation.



Acceleration and Deceleration modes are transient modes. Drive operates in acceleration mode whenever an increase in its speed is required. For this motor speed torque curve must be changed so that motor torque exceeds the load torque. Time taken for a given change in speed depends on inertia of motor load system and the amount by which motor torque exceeds the load torque.

Increase in motor torque is accompanied by an increase in motor current. Care must be taken to restrict the motor current with in a value which is safe for both motor and power modulator. In applications involving acceleration periods of long duration, current must not be allowed to exceed the rated value. When acceleration periods are of short duration a current higher than the rated value is allowed during acceleration. In closed loop drives requiring fast response, motor current may be intentionally forced to the maximum value in order to achieve high acceleration.

Figure shown below shows the transition from operating point A at speed ω_{m1} to operating point B at a higher speed ω_{m2}, when the motor torque is held constant during acceleration. The path consists of AD_1E_1B. In the figure below, 1 to 5 are motor speed torque curves. Starting is a special case of acceleration where a speed change from 0 to a desired speed takes place. All points mentioned in relation to acceleration are applicable to starting. The maximum current allowed should not only be safe for motor and power modulator but drop in source voltage caused due to it should also be in acceptable limits. In some applications the motor should accelerate smoothly, without any jerk. This is achieved when the starting torque can be increased step lessly from its zero value. Such a start is known as soft start.



Motor operation in deceleration mode is required when a decrease in its speed is required. According to the equation $T = T_l + J\frac{d\omega_m}{dt}$, deceleration occurs when load torque exceeds the motor torque. In those applications where load torque is always present with substantial magnitude, enough deceleration can be achieved by simply reducing the motor torque to zero. In those applications where load torque may not always have substantial amount or where simply reducing the motor torque to zero does not provide enough deceleration, mechanical brakes may be used to produce the required magnitude of deceleration. Alternatively, electric braking may be employed. Now both motor and the load torque oppose the motion, thus producing larger deceleration. During electric braking motor current tends to exceed the safe limit. Appropriate changes are made to ensure that the current is restricted within the safe limit.

Figure shown above shows paths followed during transition from point A at speed ω_{m1} to a point C at a lower speed ω_{m3}. When deceleration is carried out using electric braking at a constant braking torque, the operating point moves along the path AD_3E_3C. When sufficient load torque is present or when mechanical braking is used the operation takes place along the path AD_2E_2C. Stopping is a special case of deceleration where the speed of a running motor is changed to zero.

Problems:

A motor having a suitable control circuit develops a torque by the relationship $T_M = a\omega + b$, where a and b are positive constants. This motor is used to drive a load whose torque is expressed as $T_L = c\omega^2 + d$, where c and d are positive constants. The total inertia of the rotating masses is J.

a) Determine the relations amongst the constants a, b, c and d in order that the motor can start together with the load and have an equilibrium operating speed?
b) Calculate the equilibrium operating speed?
c) Will the drive be stable at this speed?
d) Determine the initial acceleration of the drive?
e) Determine the maximum acceleration of the drive?

Solution:

a) At $\omega = 0$, T_M=b and T_L=d

Hence the motor can start with the load only if b > d

T_M=T_L at equilibrium speed

$$i.e.\ a\omega + b = c\omega^2 + d$$

$$i.e.\ c\omega^2 - a\omega - (b-d) = 0$$

$$Hence\ \omega = \frac{a \pm \sqrt{a^2 + 4c(b-d)}}{2c}$$

In order that ω is finite $a^2 + 4c(b-d) > 0$, which is true

+ Sign before the radical will give a positive ω as long as

$$a^2 + 4c(b-d) > 0$$

− sign before the radial will give a positive ω only if

$$\frac{a}{2c} > \frac{\sqrt{a^2 + 4c(b-d)}}{2c}$$

$$i.e.\ a^2 > a^2 + 4c(b-d)$$

$$i.e.\ 4c(b-d) < 0$$

i.e c<0, which is not true, since c is given to be a positive constant. Hence the + sign before the radial only will give a positive finite equilibrium speed.

If $\sqrt{a^2 + 4c(b-d)} > 0$

b) Equilibrium speed $\omega = \dfrac{a + \sqrt{a^2 + 4c(b-d)}}{2c}$

c) $\frac{dT_L}{d\omega} = 2c\omega$ and $\frac{dT_M}{d\omega} = a$

If the equilibrium speed has to be stable

$\frac{dT_L}{d\omega} > \frac{dT_M}{d\omega}$ i.e. $2c\omega > a$

from the answer to (b), we have

$2c\omega = a + \sqrt{a^2 + 4c(b-d)}$ which will be always $> a$

Hence, the equilibrium operating speed determined earlier is a stable point of operation of drive.

d) Accelerating torque $J\frac{d\omega}{dt} = T_M - T_L$

Initially T_M=b and T_L=d

Therefore, initial acceleration = $\frac{b-d}{J}$

e) Accelerating torque $J\frac{d\omega}{dt} = T_M - T_L$

$$= a\omega - c\omega^2 + b - d$$

Therefore, acceleration $A = \frac{d\omega}{dt} = \frac{a\omega - c\omega^2 + b - d}{J}$

This will be maximum at a speed when

$$\frac{dA}{d\omega} = 0$$

$$\frac{a - 2c\omega}{J} = 0$$

$$\omega = \frac{a}{2c}$$

Substituting this speed at which the acceleration is maximum, in the general expression for acceleration, we get

$$A_{\max} = \frac{(a^2/2c) - \left(a^2/4c\right) + b - d}{J}$$

$$= \frac{a^2 + 4c(b-d)}{4cJ}$$