CONVERTER / CHOPPER FED DC DRIVES

2.1 INTRODUCTION

Direct-current motors are extensively used in variable-speed drives and position-control systems *where good dynamic response and steady-state performance* are required. Examples are *in robotic drives, printers, machine tools, process rolling mills, paper and textile industries*, and many others. Control of a dc motor, especially of the separately excited type, is very straightforward, mainly because of the incorporation of the commutator within the motor. The commutator brush allows the motor-developed torque to be proportional to the armature current if the field current is held constant. Classical control theories are then easily applied to the design of the torque and other control loops of a drive system.

2.2 DC MOTORS AND ITS CHARACTERISTICS

When a DC supply is applied to the armature of the dc motor with its field excited by a dc supply, torque is developed in the armature due to interaction between the axial current carrying conductors on the rotor and the radial magnetic flux produced by the stator. If the voltage V is the voltage applied to the armature terminals, and E is the internally developed motional e.m.f. The resistance and inductance of the complete armature are represented by R_a and L_a in Figure 2.1(a). Under motoring conditions, the motional e.m.f. E always opposes the applied voltage V, and for this reason it is referred to as 'back e.m.f.' For current to be forced into the motor, V must be greater than E, the armature circuit voltage equation being given by

$$V = E + I_a R_a + L\frac{dI_a}{dt} \qquad (2.1)$$

The last term in equation (2.1) represents the inductive volt-drop due to the armature self-inductance. This voltage is proportional to the rate of change of current, so under steady-state conditions (when the current is constant), the term will be zero and can be ignored. Under steady-state conditions, the armature current I is constant and equation (2.1) simplifies to

$$V = E + I_a R_a \qquad (2.2)$$

2.2.1 Types of DC Motors

Based on the connections of armature and field windings DC motors classified in to three types they are

a. ***Separately excited DC motor*** [Field and armature windings are excited separately by independent sources fig2.1(a)]

b. ***Shunt excited DC motor*** [Here field winding and armature winding are connected in parallel and are excited by a common source fig 2.1 (b)]

c. ***Series excited DC motor*** [Here field winding and armature winding are connected in series and are excited by a common source fig 2.1 (c)]

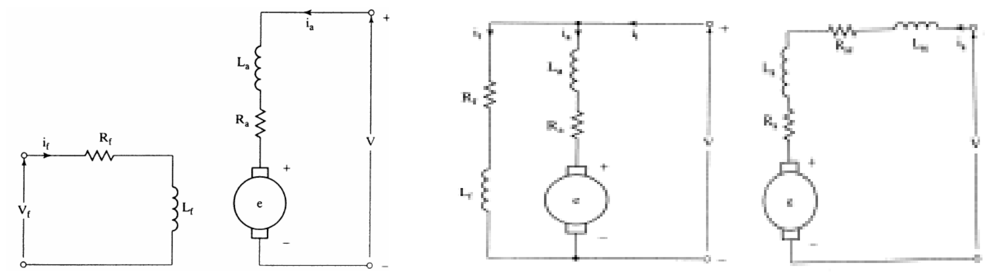


Fig 2.1 (a) Fig 2.1 (b) Fig 2.1 (c)

2.2.2 Speed -Torque and Speed -Current Relations

The motor back emf is given by

$$E_b = \frac{\phi ZN}{60}\left(\frac{P}{A}\right) Volts \qquad (2.3)$$

Where ϕ is flux per pole in Webers

Z is number of armature conductors

N is speed in rpm

A is number of parallel paths in armature

Here Z, P, A are fixed for a particular machine after wounded. Therefore for a given DC machine

$$E_b = \left(\frac{ZP}{60A}\right)\phi N \ Volts \qquad (2.4)$$

$$E_b = K_a \phi N \qquad (2.5)$$

Where N = $\frac{60}{2\pi}\omega_m$, substitute in equation (2.4)

$$E_b = \left(\frac{ZP}{60A}\right)\phi\left(\frac{60}{2\pi}\right)\omega_m \ Volts$$

$$E_b = \left(\frac{ZP}{2\pi A}\right)\phi\omega_m$$

$$E_b = K_a \phi \omega_m \qquad (2.6)$$

The torque developed by the armature is given by

$$T_a = \frac{\phi Z I_a}{2\pi}\left(\frac{P}{A}\right)N - m \qquad (2.7)$$

$$= \left(\frac{ZP}{2\pi A}\right)\phi I_a$$

$$T_a = K_a \phi I_a \qquad (2.8)$$

Where $K_a = \frac{ZP}{2\pi A}$

$$T_a = K_a \phi I_a = T \qquad (2.9)$$

<u>2.2.2.1 Separately Excited or Shunt Motor</u>

From expression (2.2) $I_a = \frac{(V - E)}{R_a}$ (2.10)

Substituting equation (2.10) in equation (2.9) we get

$$T_a = K_a \phi \left[\frac{V - E}{R_a}\right] \qquad (2.11)$$

Substituting equation (2.9) in equation (2.11) we get

$$T_a = K_a \phi \left[\frac{V - K_a \phi \omega_m}{R_a}\right] \qquad (2.12)$$

Rearranging the above equation we get,

$$\omega_m = \frac{V}{K_a \phi} - \frac{R_a}{(K_a \phi)^2} T_a \qquad (2.13)$$

The above expression gives the relationship between speed and torque for separately excited and shunt motors.

Speed –Current relationship can be obtained if $\frac{T_a}{K_a \phi}$ in the expression (2.13) is replaced with I_a (From equation 2.9) as given below

$$\omega_m = \frac{V}{K_a \phi} - \frac{R_a I_a}{(K_a \phi)} \qquad (2.14)$$

Fig 2.2 (a) and 2.2 (b) shows the speed torque characteristics and Speed current characteristics of separately excited and shunt motor when the armature and field voltages are kept constant.

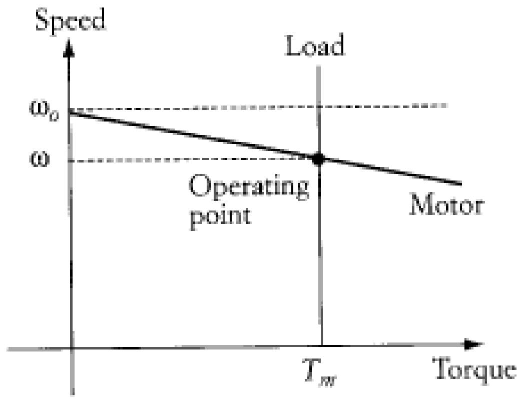


Fig 2.2(a)

Speed-Torque Characteristics

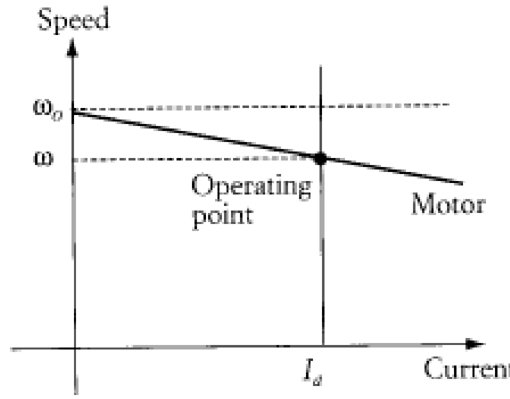


Fig 2.2 (b)

Speed-Current Characteristics

2.2.2.2 Series Motor

In the case of separately excited and shunt motor flux is almost constant when the armature voltage is fixed, but in case of series motor when the machine is loaded armature current increase which increase the flux also, because armature and field windings are in series in the case of series motor. Assuming that the motor operates in the linear region of the magnetic saturation curve we get,

$$\phi = CI_a \qquad (2.15)$$

Where C is proportionality constant. The torque developed in the armature in this case is given by, i.e equation (2.9) can be rewritten as

$$T_a = K_a \phi I_a = K_a CI_a I_a = K_a CI_a{}^2 \qquad (2.16)$$

Therefore equation (2.13) and (2.14) becomes

$$\omega_m = \frac{V}{K_a CI_a} - \frac{R_a}{(K_a CI_a)^2} T_a \qquad (2.17)$$

$$\omega_m = \frac{V}{K_a \phi} - \frac{R_a}{(K_a C)} \qquad (2.18)$$

The Speed torque characteristic of DC series motor is shown in the figure 2.3. Note that the speed of the motor is rapidly decreasing when the load is increased.

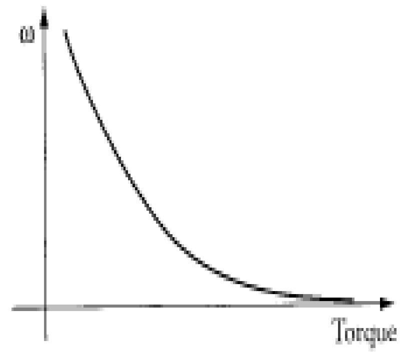


Fig 2.3

Speed Torque Characteristic of DC series motor.

2.3 Conventional Methods of Speed Control of DC Motors

2.3.1 Speed control of separately excited or DC shunt motor

As seen from in the above section 2.2, the speed torque characteristic and speed current characteristic of separately excited or shunt motor is as given below,

$$\omega_m = \frac{V}{K_a\phi} - \frac{R_a}{(K_a\phi)^2}T_a = \omega_0 - \Delta\omega \tag{2.19}$$

$$\omega_m = \frac{V}{K_a\phi} - \frac{R_a I_a}{(K_a\phi)} = \omega_0 - \Delta\omega \tag{2.20}$$

Where ω_0 the no load is speed and $\Delta\omega$ is the speed drop. The no load speed is computed when the torque and current are equal to zero. The speed drop is a function of the load torque. From the above expressions speed of the separately excited DC motor or Shunt motor can be controlled by controlling the following quantities:

a) ***Resistance in the armature circuit***: When the resistance is inserted in the armature circuit, the speed drop $\Delta\omega$ increases and the motor speed decreases.

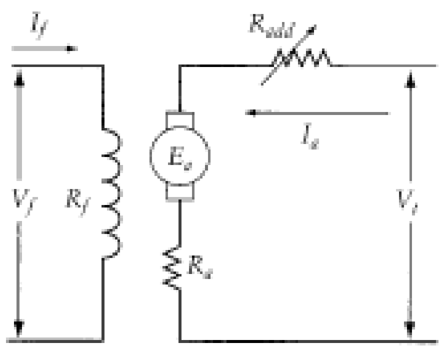
b) ***Terminal Voltage (Armature voltage):*** Reducing the armature voltage V of the motor reduces the motor speed.

c) ***Field Flux (or Field Voltage):*** Reducing Field voltage reduces the flux ϕ, and the motor speed increases.

Note: We cannot operate the electric motor with voltages higher than the rated value. Therefore we cannot control the speed by increasing the armature or field voltages beyond the rated values. *Only voltage reduction can be implemented.* Hence second method of speed control is only suitable for speed reduction (armature voltage), where third method (Field flux) is suitable for speed increase.

2.3.2 Controlling speed by adding external resistance to armature.

Figure 2.4 shows a DC motor setup with resistance added in the armature circuit. Figure 2.5 shows the corresponding speed torque characteristics. Let us assume that the load torque is unidirectional and constant. *A good example for this type of torque is elevator.* Also assume that the field and armature voltages are constant.



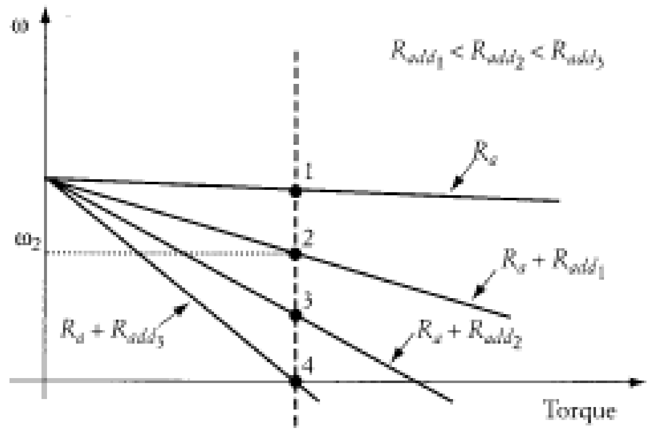


Fig 2.4

A setup for speed change by adding an armature resistance

Fig 2.5

Effect of adding an armature resistance on speed

At point 1 no external resistance is added in the armature circuit. If a resistance R_{add1} is added to the armature circuit, the motor operates at point 2, where the motor speed ω_2 is

$$\omega_2 = \frac{V}{K_a\phi} - \frac{R_a + R_{add1}}{(K\phi)^2} T_a = \omega_0 - \Delta\omega_2 \qquad (2.21)$$

or

$$\omega_2 = \frac{V}{K_a\phi} - \frac{R_a + R_{add1}}{K_a\phi} I_a = \omega_0 - \Delta\omega_2 \qquad (2.22)$$

Note that the no load speed ω_0 is unchanged regardless of the value of resistance in the armature circuit. The second term of the speed equation is the speed droop $\Delta\omega$, which increase in magnitude when R_{add} increases. Consequently, the motor speed is reduced. If the added resistance keeps increasing, the motor speed decreases until the system operates at point 4, where the speed of the motor is zero. *The operation of the drive system at point 4 is known as "holding".*

Note: *Operating a dc motor for a period of time with a resistance inserted in the armature circuit is a very inefficient method. The use of resistance is acceptable only* *when the heat produced by the resistance is utilized as a by product or when the resistance is used for a very short period of time.*

2.3.2 Controlling speed by adjusting armature voltage.

A common method of controlling speed is to adjust the armature voltage. This method is highly efficient and stable and is simple to implement. The circuit of figure 2.6 shows the basic concept of this method.

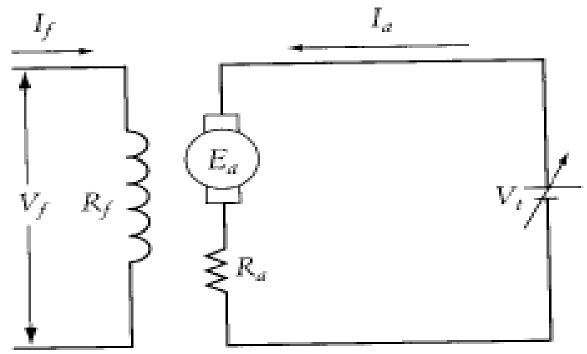


Fig 2.6

A setup for speed change by adjusting armature voltage

The only controlled variable is the armature voltage of the motor, which is represented as an adjustable voltage source. Based on equation 2.19, when armature voltage is reduced no load speed is also reduced. Moreover for the same value of load torque and field flux, the armature voltage does not affect the speed drop $\Delta\omega$. The slope of speed torque characteristic is $\frac{R_a}{(K_a\phi)^2}$, which is independent of the armature voltage. Hence the characteristics are shown as figure 2.7. Note that it is assumed that the field voltage is unchanged when the armature voltage varies.

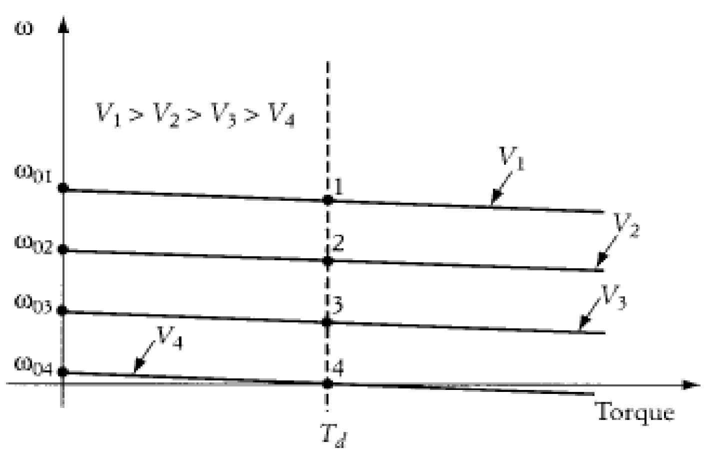


Fig 2.7

2.3.2 Controlling speed by adjusting Field voltage

Equations (2.19) and (2.20) show the dependency of motor speed on the field flux. The no load speed is inversely proportional to the flux and slope of equation (2.19) is inversely proportional to square of the flux. Therefore the speed is more sensitive to flux variations than to variations in the armature voltage.

Figure 2.8(a) shows a setup for controlling speed by adjusting the field flux. If we reduce the field voltage, the field current and consequently the flux are reduced. Figure 2.8 (b) shows a set of speed torque characteristics for three values of field voltages. When the field flux is reduced, the no load speed ω_0 is increased in inverse proportion to the flux, and the speed drop $\Delta\omega$ is also increased.

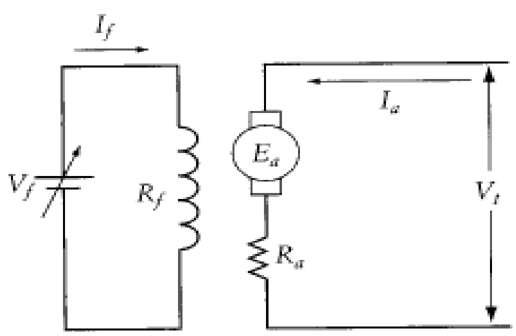


Fig 2.8 (a)

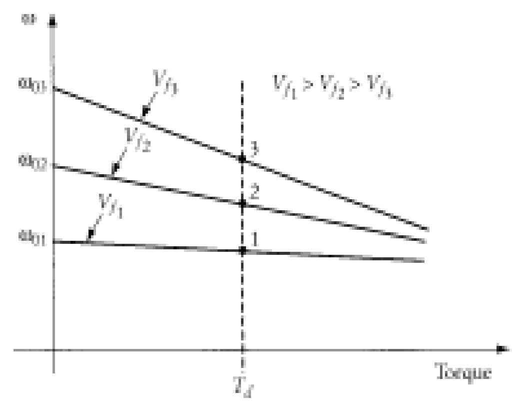


Fig 2.8(b)

The characteristics show that because of the change in speed drops, the lines are not parallel. Unless the motor is excessively loaded, the motor speed increases when the field is reduced. When motor speed is controlled by adjusting the field current, the following considerations should be kept in mind:

a. The field voltage must not exceed the absolute maximum rating

b. Since DC motors are relatively sensitive to variations in field voltage, large reductions in field current may result in excessive speed.

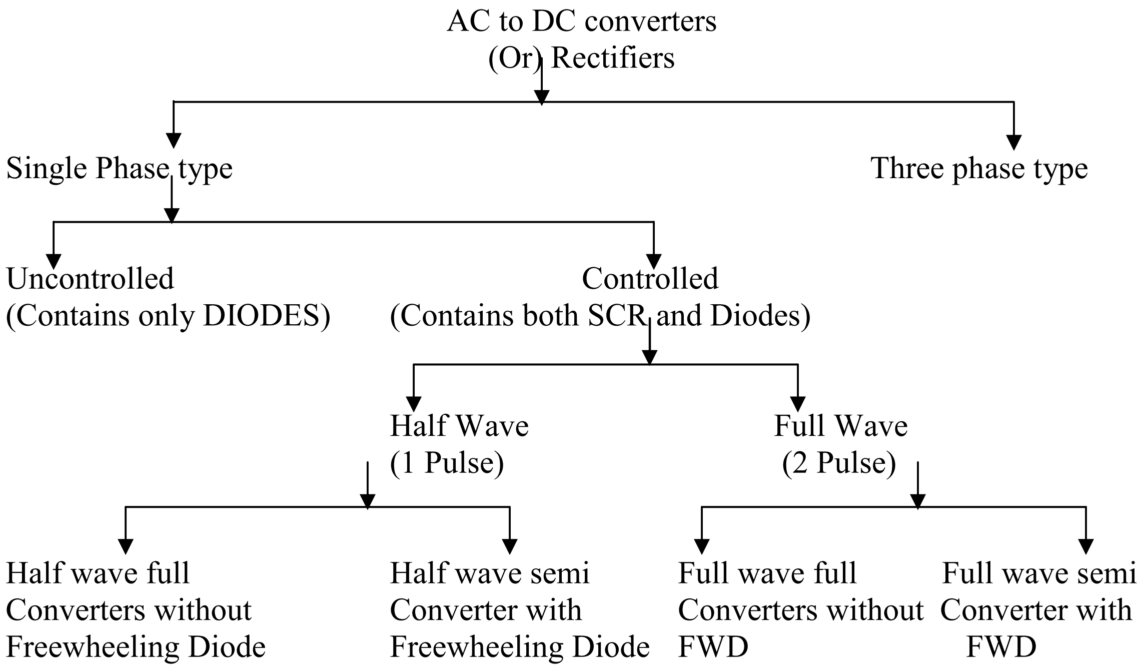
c. Because the armature current is inversely proportional to the field flux$\left(I_a = T_a / K_a\phi\right)$, reducing the field results in an increase in the armature current (Assuming that the load torque is unchanged).

Thus by combining armature and field control for speeds below and above rated speed, relatively a wide range of speed control is possible. For speeds lower than that of the rated speed, applied armature voltage is varied while the field current is kept at its rated value: to obtain speeds above the rated speed, field current is decreased while keeping the applied armature voltage constant.

2.4 Controlled Rectifier Fed DC Drives

Controlled rectifiers are used to get variable dc voltage from an ac source of fixed voltage. There are several types of converters which can be used for feeding DC motors. AS thyristors are capable of conducting current in one direction all theses rectifiers are capable of conducting current only in one direction.

2.4.1 Types of Rectifiers



2.4.2 Single Phase rectifier fed separately Excited DC motor drive

The thyristor D.C. drive remains an important speed-controlled industrial drive, especially where the higher maintenance cost associated with the D.C. motor brushes is tolerable. The controlled (thyristor) rectifier provides a low-impedance adjustable 'D.C.' voltage for the motor armature, thereby providing speed control. For motors up to a few kilowatts the armature converter can be supplied from either single-phase or three-phase mains, but for larger motors three-phase is always used. A separate thyristor or diode rectifier is used to supply the field of the motor: the power is much less than the armature power, so the supply is often single-phase. Figure 2.9 shows the setup for single phase controlled rectifier fed separately excited dc motor drive. Field circuit is also excited by a dc source, which is not shown in the figure just for simplicity.

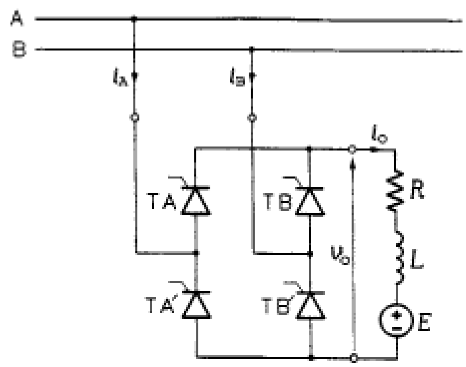


Fig 2.9

The motor terminal voltage waveform and current waveform for the dominant discontinuous and continuous conduction modes are shown in the figure 2.10(a) and 2.10 (b).Thyristors T_A and T_B are gated at $\omega t = \alpha$.The SCR's will get turned on only if $V_m \sin\alpha > E$. Thyristors T_A and T_B are given gate pulses from α *to* π and thyristors T_A' and T_B' are given gate pulses from $(\pi + \alpha)$ *to* 2π. When armature current does not flow continuously the motor is said to operate in discontinuous conduction mode. When current flows continuously, the conduction is said to be continuous. In discontinuous modes, the current starts flowing with the turn on thyristors T_A and T_B at $\omega t = \alpha$. Motor gets connected to the source and its terminal voltage equals V_s.

At some angle β known as extinction angle, load current decays to zero. Here $\beta > \pi$. As T_A and T_B are reversed biased after $\omega t = \pi$, this pair is commutated at $\omega t = \beta$ when I_a=0. From β *to* $(\pi + \alpha)$, no SCR's conducts, the terminal voltage jumps from $V_m \sin\beta$ to E .At $\omega t = \beta$ as pair T_A' and T_B' is triggered, load current starts to build up again as before and load voltage V_a follows V_s waveform as shown in the figure 2.10(a). At $(\pi + \beta)$, I_a falls to zero, V_a changes from $V_m \sin(\pi + \beta)$ to E as no SCR conducts.

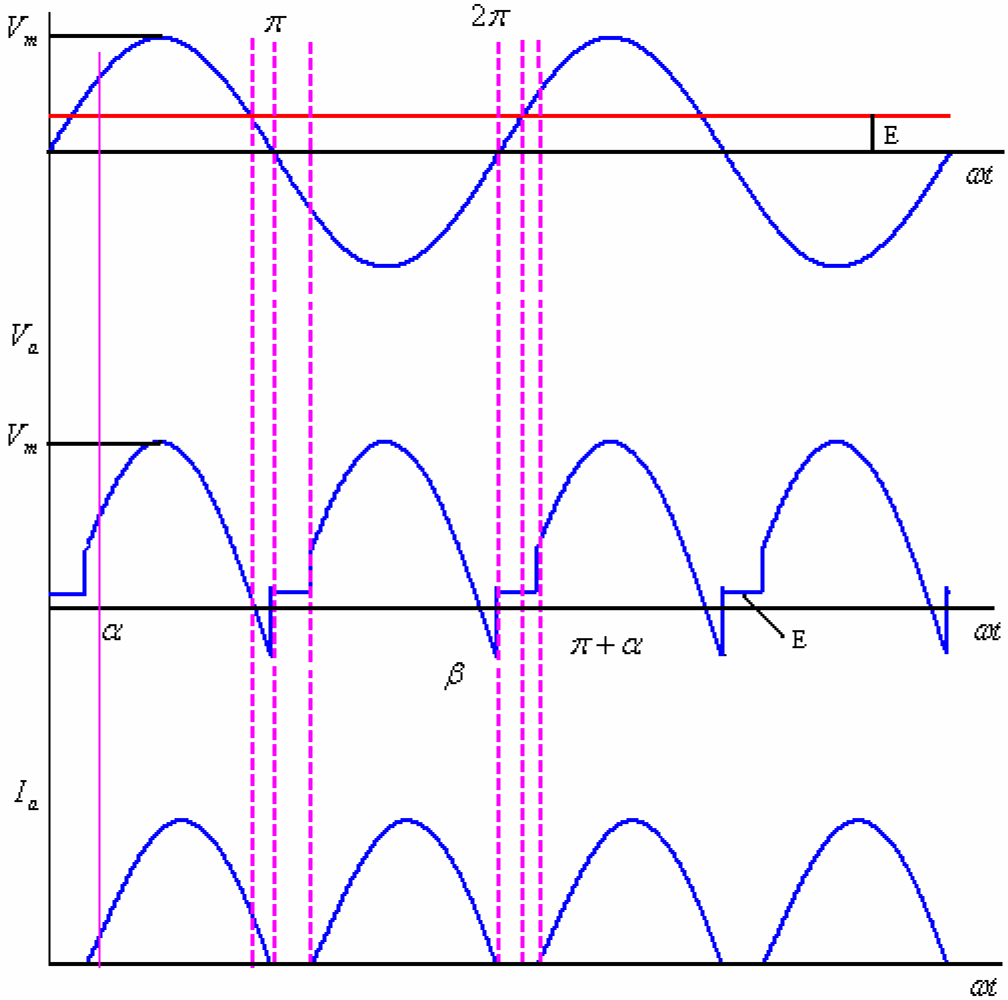


Fig 2.10 (a) Discontinuous Conduction Mode Waveforms

In continuous conduction mode, during the positive half cycle thyristiors T_A and T_B are forward biased. At $\omega t = \alpha$, T_A and T_B are turned ON. As a result, supply voltage $V_m \sin\alpha$ immediately appears across thyristors T_A' and T_B' as a reverse bias, they are turned off by natural commutation. At $\omega t = (\pi + \alpha)$ forward biased SCR's T_A' and T_B' are triggered causing turn off of T_A and T_B.

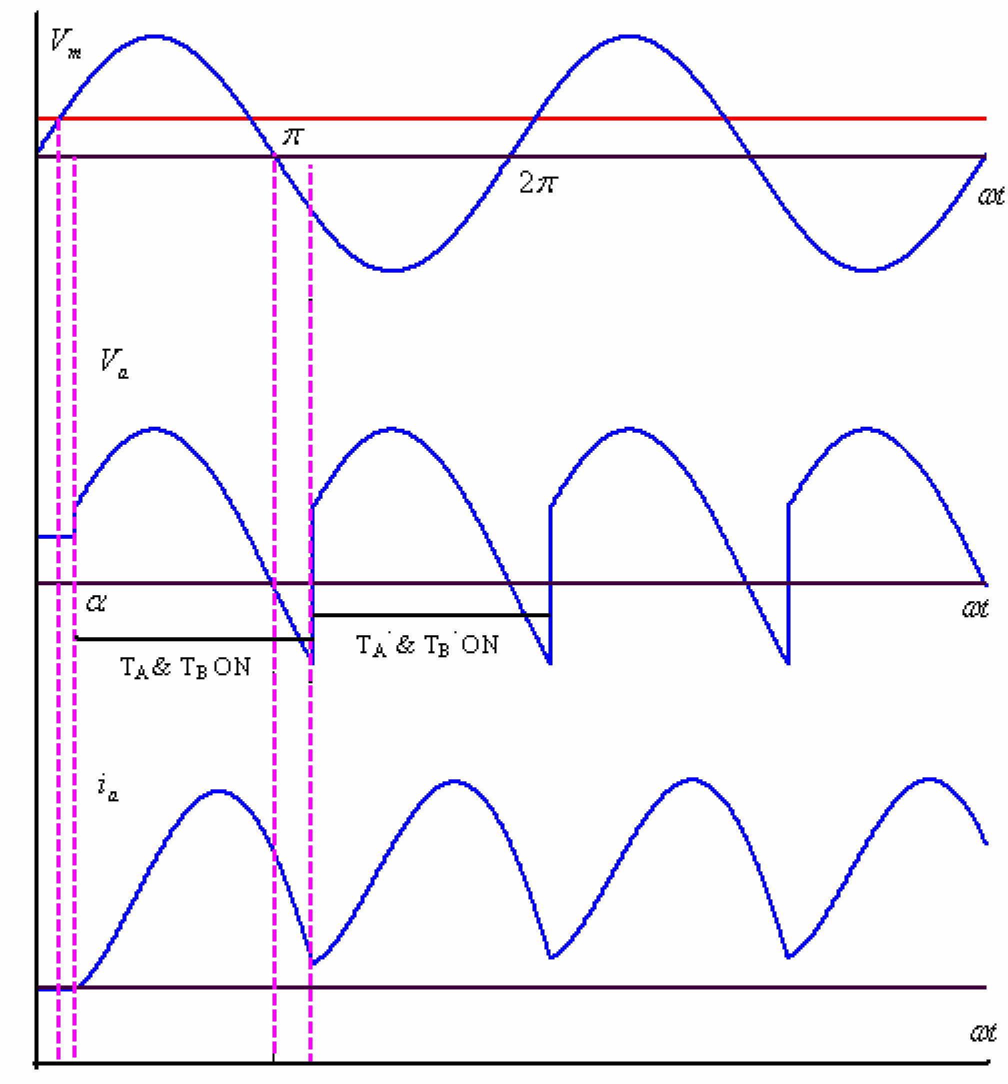


Fig 2.10 (b) Continuous Conduction Waveforms

2.4.2.1Discontinuous Conduction:

The drive operates in two intervals.

a) Conduction period $(\alpha \le \omega t \le \beta)$, T_A and T_B conduct and V_0=V_s. Also $(\pi+\alpha) \le \omega t \le (\pi+\beta)$, T_A' and T_B' conduct and V_0=V_s.

b) Idle period $\beta \le \omega t \le (\pi+\alpha)$ when I_a=0 and V_a=E.

Drive operation is described by the following equations

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E = V_m \sin\omega t \quad \text{for } \alpha \le \omega t \le \beta \tag{2.23}$$

$$V_a = E \quad \text{and } i_a = 0 \qquad \text{for } \beta \le \omega t \le (\pi + \alpha) \tag{2.24}$$

From (2.23) we get

$$\frac{di_a}{dt} = \frac{R_a}{L_a} i_a = \frac{V_m \sin\omega t - E}{L_a} \tag{2.25}$$

In order to get the speed torque characteristics for different values of α of the controlled rectifier fed separately excited DC motor, it is necessary to solve the above equation (2.25). So solving the above equation involves two mathematical steps, one is to evaluate the complementary solution of the equation (2.25) and particular integral solution of the equation (2.25).

Solution to Complementary Function:

The complementary function of equation (2.25) is

$$\frac{di_a}{dt} + \frac{R_a}{L_a} i_a = 0$$

$$\left(\frac{d}{dt} + \frac{R_a}{L_a}\right) i_a = 0 \tag{2.26}$$

(i.e.)The above equation is of the form $\left(\frac{d}{dx} + m_1\right) = 0$ where $\frac{d}{dx}$ *represents* $\frac{d}{dt}$ and m_1 represents $\frac{R_a}{L_a}$. Roots of the equation $\left(\frac{d}{dx} + m_1\right) = 0$ is D=-m_1. Therefore the roots of the equation 2.26 is m = -$\frac{R_a}{L_a}$. If the roots of the equation $\left(\frac{d}{dx} + m_1\right) = 0$ is real than the complementary function is given by, C.F = $C_1 e^{mx}$ therefore complementary function of the equation (2.26) is given by,

$$\text{C.F} = C_1 e^{-\frac{R_a}{L_a} t} \tag{2.27}$$

We know that for an RL circuit Z=R+jXL

$$\therefore \tan\phi = \frac{\omega L_a}{R_a}$$

$$\Rightarrow \cot\phi = \frac{R_a}{\omega L_a}$$

$$\Rightarrow \omega\cot\phi = \frac{R_a}{L_a}$$

Substituting the above relation in equation (2.27) we get,

$$\text{C.F.} = C_1 e^{-\omega t \cot\phi} \tag{2.28}$$

Therefore the complementary function solution of the equation (2.25) is as given in equation (2.28). The next step is to find the particular integral solution of equation (2.25). In the expression (2.25) there are two Particular Integrals they are

$$\text{P.I1}=\frac{V_m \sin\omega t}{L_a} \text{ and the other one is P.I2}=-\frac{E}{L_a}$$

Let us fine the solution of particular integrals one by one.

Solution to Particular Integral 1:

If X is sin (aX) ***or*** cos (aX) then the solution of P.I can be found out by

$$\text{PI}=\frac{1}{f(D)}\sin aX \text{ and replace } D^2 \text{ by } -a^2$$

Therefore P.I 1 can be written as,

$$PI1=\left(\frac{1}{D+\frac{R_a}{L_a}}\right)\frac{V_m}{L_a}\sin\omega t \text{ Where D = } \frac{d}{dt} \text{ and a=}\omega$$

Multiplying and dividing by the conjugate in the above expression we get,

$$PI1=\frac{\left(D-\frac{R_a}{L_a}\right)}{\left(D+\frac{R_a}{L_a}\right)\left(D-\frac{R_a}{L_a}\right)}\left(\frac{V_m}{L_a}\sin\omega t\right)$$

$$=\frac{V_m}{L_a}\left(\frac{\left(D-\frac{R_a}{L_a}\right)\sin\omega t}{D^2-\left(\frac{R_a}{L_a}\right)^2}\right)$$

replace $D^2=-\omega^2$ in the above expression we get,

$$\therefore PI1 = \frac{V_m}{L_a}\left(\frac{\left(D - \frac{R_a}{L_a}\right)\sin \omega t}{-\omega^2 - \frac{R_a^{\ 2}}{L_a^{\ 2}}}\right)$$

$$= \frac{\mathrm{V_m}}{\mathrm{L_a}}\left[\frac{D \sin \omega t - \frac{R_a}{L_a}\sin \omega t}{-\frac{\omega^2 L_a^{\ 2} - R_a^{\ 2}}{L_a^{\ 2}}}\right]$$

$$= \frac{\mathrm{V_m}}{\mathrm{L_a}}\left[\frac{\cos \omega t.\omega - \frac{R_a}{L_a}\sin \omega t}{-\frac{\omega^2 L_a^{\ 2} - R_a^{\ 2}}{L_a^{\ 2}}}\right] \text{since } \mathrm{D} = \frac{\mathrm{d}}{\mathrm{dt}}$$

$$= \frac{\mathrm{V_m}}{\mathrm{L_a}^2}\left[\frac{\cos \omega t.\omega L_a - R_a \sin \omega t}{-\frac{\omega^2 L_a^{\ 2} - R_a^{\ 2}}{L_a^{\ 2}}}\right]$$

$$= \text{-} V_{\mathrm{m}}\left[\frac{\omega L_a \cos \omega t - R_a \sin \omega t}{\left(\omega^2 L_a^{\ 2} + R_a^{\ 2}\right)}\right] \qquad (2.29)$$

Now let us consider a triangle as shown in the figure 2.11

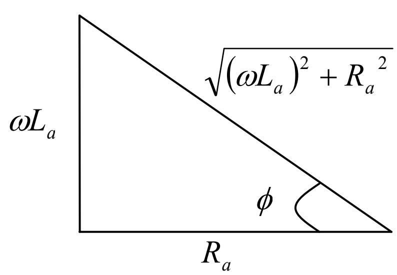


Fig 2.11

From the above figure $\cos \phi = \frac{R_a}{\sqrt{(\omega L_a)^2 + R_a^{\ 2}}}$ and $\sin \phi = \frac{\omega L_a}{\sqrt{(\omega L_a)^2 + R_a^{\ 2}}}$

Substituting the above two relation in equation (2.29) we get

$$= -\frac{V_m}{\sqrt{(\omega L_a)^2 + R_a^{\;2}}}\left[\frac{\omega L_a}{\sqrt{(\omega L_a)^2 + R_a^{\;2}}}\cos\omega t - \frac{R_a}{\sqrt{(\omega L_a)^2 + R_a^{\;2}}}\sin\omega t\right]$$

$$= -\frac{V_m}{Z}\left[\sin\phi.\cos\omega t - \cos\phi.\sin\omega t\right]$$

$$= \frac{V_m}{Z}\left[\cos\phi.\sin\omega t - \sin\phi.\cos\omega t\right]$$

$$= \frac{V_m}{Z}\left[\sin(\omega t - \phi)\right] \qquad (2.30)$$

Solution to Particular Integral 2:

$$PI2 = \frac{1}{\left(D + \frac{R_a}{L_a}\right)}\left(-\frac{E}{L_a}\right)$$

$$= \frac{1}{\left(D + \frac{R_a}{L_a}\right)}\left(-\frac{E}{L_a}e^{0i_a}\right) \because D = 0$$

$$= \frac{1}{\mathrm{R_a}/\mathrm{L_a}}\left(-\frac{E}{L_a}\right) = -\frac{E}{R_a} \qquad (2.31)$$

So combining equations (2.28), (2.30) and (2.31) gives the solution of equation (2.25)

$$i_a(\omega t) = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R_a} + C_1 e^{-\omega t\cot\phi} \qquad (2.32)$$

$$\text{Where } \mathrm{Z} = \sqrt{\mathrm{R_a}^{\;2} + (\omega L_a)^2} \qquad (2.33)$$

$$\phi = \tan^{-1}\left(\frac{\omega L_a}{R_a}\right) \qquad (2.34)$$

constant $\mathrm{C_1}$ can be evaluated by using the initial condition $\mathrm{i_a}(\alpha) = 0$ and $\omega \mathrm{t} = 0$

$$C_1 = -\left[\frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R_a}\right]e^{\alpha\cot\phi} \qquad (2.35)$$

Substituting (2.35) in (2.32) we get,

$$i_a = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R_a} - \left[\frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R_a}\right]e^{-(\omega t - \alpha)\cot\phi} \tag{2.36}$$

since $\mathrm{i_a}(\beta) = 0$ from (2.36)

$$\mathrm{i_a}(\beta) = \frac{V_m}{Z}\sin(\beta - \phi) - \frac{E}{R_a} - \left[\frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R_a}\right]e^{-(\beta - \alpha)\cot\phi} \tag{2.37}$$

β can be evaluated by iterative solution of (2.37)

$$\mathrm{V_a} = E + I_a R_a \tag{2.38}$$

From discontinuous waveforms

$$\mathrm{V_a} = \frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t \, d(\omega t) + \int_{\beta}^{\pi+\alpha} E \, d(\omega t)\right]$$

$$= \frac{\mathrm{V_m}}{\pi}[\cos\alpha - \cos\beta] + \frac{(\pi + \alpha - \beta)E}{\pi}\left[\because \int \sin\theta d\theta = \cos\theta\right] \tag{2.39}$$

From equations (2.38) and (2.39)

$$\frac{\mathrm{V_m}}{\pi}[\cos\alpha - \cos\beta] - I_a R_a = E\left(\frac{\beta - \alpha}{\pi}\right) \tag{2.40}$$

Substituting (2.6) and (2.8) in (2.40)

$$\frac{\mathrm{V_m}}{\pi}[\cos\alpha - \cos\beta] - \frac{T}{K}R_a = K\omega_m\left(\frac{\beta - \alpha}{\pi}\right)[\mathrm{Where\ K = K_a}\phi]$$

$$\omega_m\left(\frac{\beta - \alpha}{\pi}\right) = \frac{\mathrm{V_m}}{K\pi}[\cos\alpha - \cos\beta] - \frac{T}{K^2}R_a$$

$$\omega_m = \frac{\mathrm{V_m}}{K(\beta - \alpha)}[\cos\alpha - \cos\beta] - \frac{T}{K^2}.R_a.\frac{\pi}{(\beta - \alpha)}$$

$$\omega_m = \frac{\mathrm{V_m}}{K}\left[\frac{\cos\alpha - \cos\beta}{(\beta - \alpha)}\right] - \frac{R_a}{K^2}\frac{\pi}{(\beta - \alpha)}T \tag{2.41}$$

For a given α, there is a particular speed ω_{mc} when $\beta = \pi + \alpha$, indicating that at ω_{mc}, the mode of operation changes from discontinuous to continuous. ω_{mc} is called as critical speed. Substituting $\beta = \pi + \alpha$ in equation (2.37) we get,

$$\frac{V_m}{Z}\sin(\pi + \alpha - \phi) - \frac{E}{R_a} - \left[\frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R_a}\right]e^{-(\pi + \alpha - \alpha)\cot\phi} \tag{2.42}$$

$$-\frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R_a} - \frac{V_m}{Z}\sin(\alpha - \phi)e^{-\pi\cot\phi} + \frac{E}{R_a}e^{-\pi\cot\phi} = 0 \tag{2.43}$$

$$-\frac{V_m}{Z}\sin(\alpha-\phi)\left[1+e^{-\pi\cot\phi}\right]+\frac{E}{R_a}\left[e^{-\pi\cot\phi}-1\right]=0$$

$$\frac{E}{R_a}\left[e^{-\pi\cot\phi}-1\right]=\frac{V_m}{Z}\sin(\alpha-\phi)\left[1+e^{-\pi\cot\phi}\right]$$

$$E=\frac{R_a}{Z}V_m\sin(\alpha-\phi)\left[\frac{1+e^{-\pi\cot\phi}}{e^{-\pi\cot\phi}-1}\right] \tag{2.44}$$

$$E = K\omega_{mc}$$

$$\omega_{mc}=\frac{R_a}{KZ}V_m\sin(\alpha-\phi)\left[\frac{1+e^{-\pi\cot\phi}}{e^{-\pi\cot\phi}-1}\right] \tag{2.45}$$

Continuous Conduction Mode

For continuous conduction, average output voltage is given by,

$$V_a=\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}V_m\sin\omega t\,d(\omega t) \tag{2.46}$$

$$V_a=\frac{2V_m}{\pi}\cos\alpha \tag{2.47}$$

$$\omega_M=\frac{2V_m}{\pi K}\cos\alpha-\frac{R_a}{K^2}T \tag{2.48}$$

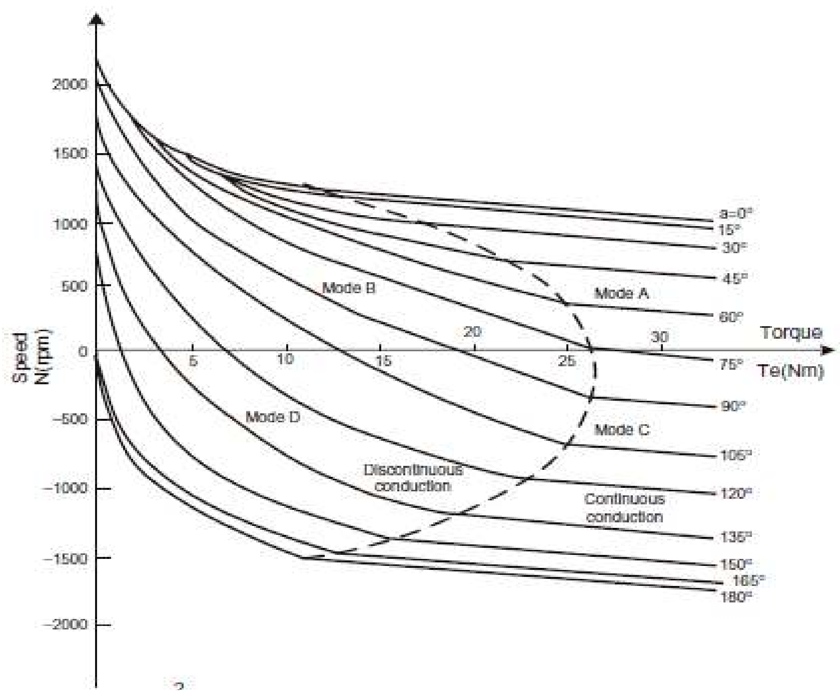


Fig 2.12

<u>2.4.3 Three Phase Fully Controlled rectifier fed separately Excited DC motor drive</u>

- ✓ Three phase controlled rectifiers are used in large power DC motor drives. Three phase controlled rectifier gives more number of voltage pulses per cycle of supply frequency. *This makes motor current continuous and filter requirement also less*

- ✓ The number of voltage pulses per cycle depends upon the number of thyristors and their connections for three phase controlled rectifiers. In three phase drives, the armature circuit is connected to the output of a three phase controlled rectifier.
- ✓ Three phase drives are used for high power applications up to mega watts power level. The ripple frequency of the armature voltage is higher than that of the single phase drives and it requires less inductance in the armature circuit to reduce the armature current ripple.
- ✓ Three phase full converters are used in industrial applications up to 1500KW drives. It is a two quadrant converter. i.e. *the average output voltage is either positive or negative but average output current is always positive.*

2.4.3.1 Principle of Operation:

Three phase full converter bridge circuit connected across the armature terminals is shown in the figure 2.13 and figure 2.14 shows the voltage and current waveforms of the converter. The circuit works as a three phase AC to DC converter for firing angle delay $0^0 < \alpha < 90^0$ and as a line commutated inverter for $90^0 < \alpha < 180^0$. A three phase full converter fed DC motor is performed where regeneration of power is required i.e. it performs two quadrant operation.

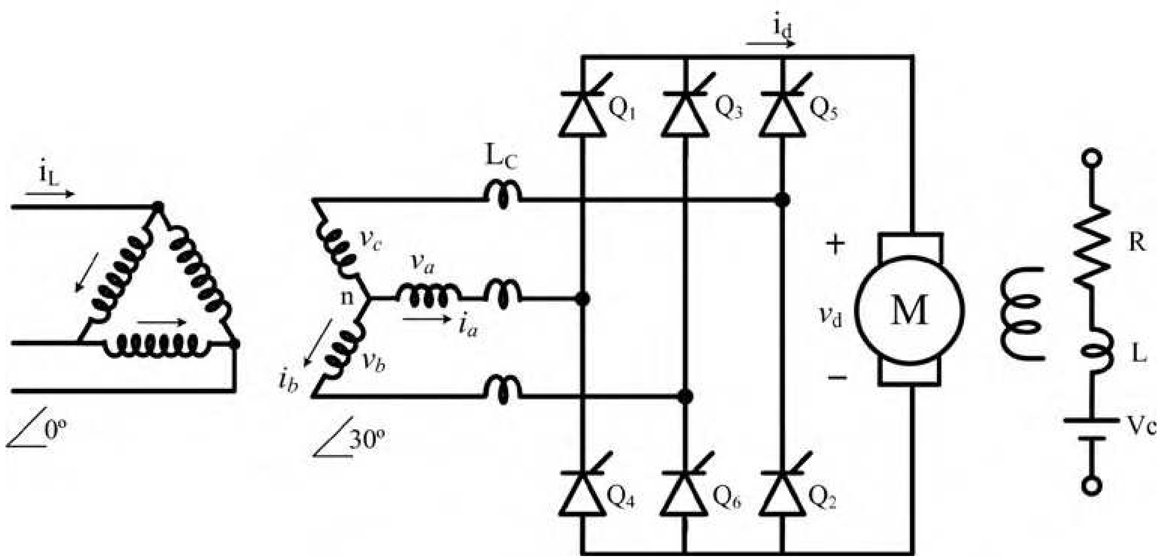


Figure 2.13

Basically, the controlled rectifier consists of six thyristors arranged in the form of three legs with two series thyristors in each leg. The center points of three legs are connected to a three-phase power supply. The transformer is not mandatory, but it provides the advantages of voltage level change, electrical isolation, and phase shift from the primary. In a three-phase bridge, one device in the positive group (Q_1 Q_3 Q_5) and another device from the negative group (Q_4 Q_6 Q_2) must conduct simultaneously to contribute load current i_d. Each thyristor is normally provided with pulse train firing for the desired conduction interval. The speed of the motor can be controlled by firing angle control of the thyristors.

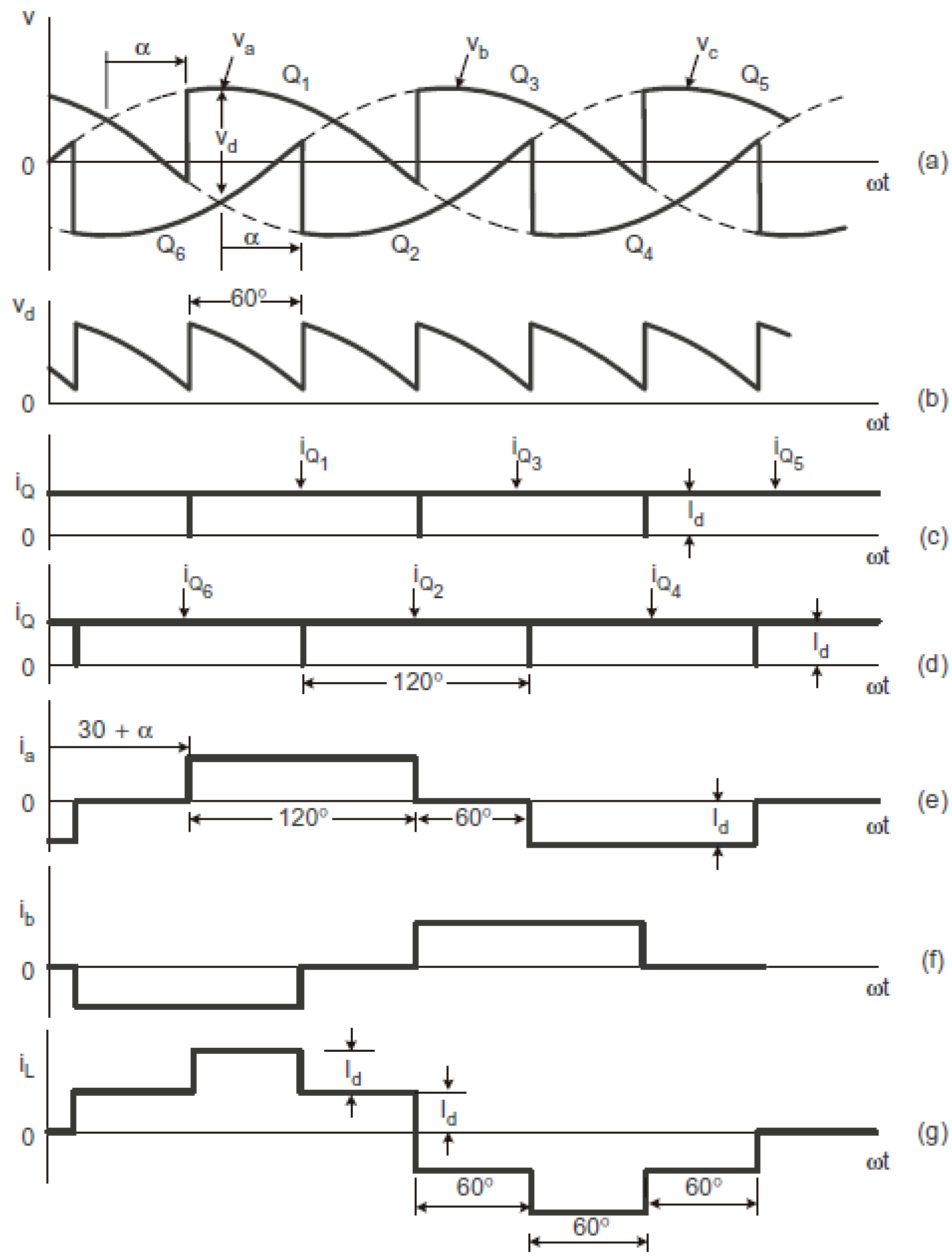


Fig 2.14 Three-phase thyristor bridge waveforms in rectification mode ($\alpha = 40°$)

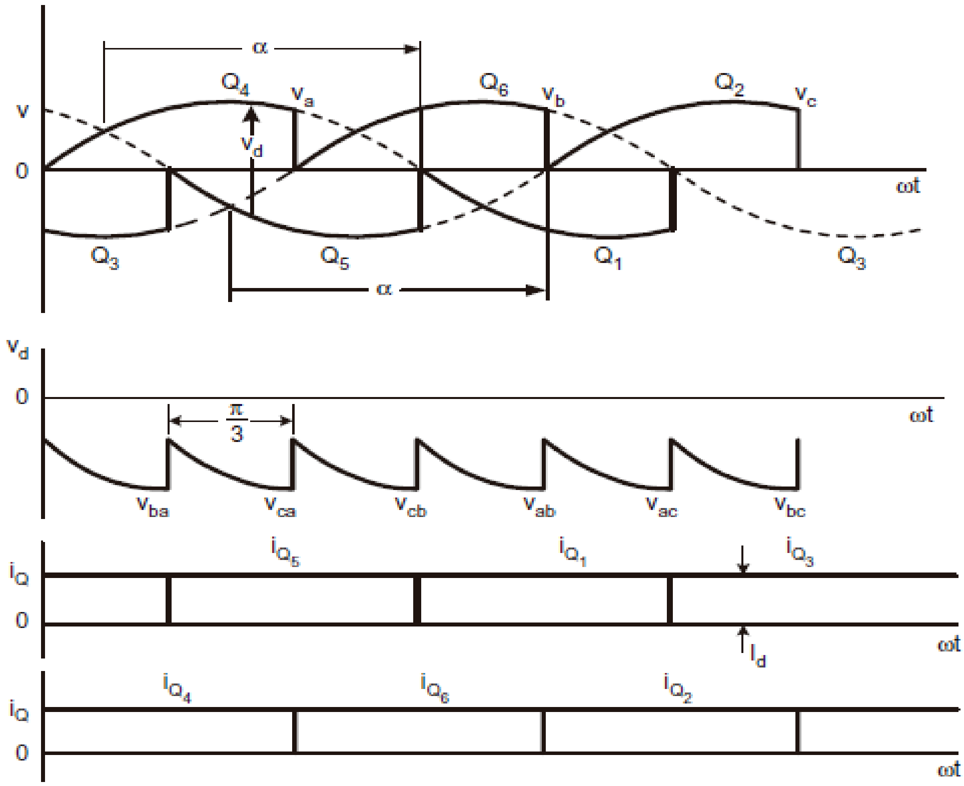


Fig 2.15 Three-phase thyristor bridge waveforms inverting mode (α = 150°)

The average motor armature voltage is given by

$$V_a = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{ab}\, d(\omega t) \tag{2.49}$$

In the above substitute $V_{ab} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)d(\omega t)$ (2.50)

We have $V_a = \frac{3\sqrt{3}}{\pi}V_m \cos\alpha$ (2.51)

<u>2.4.3.2 Speed Torque Relations:</u>

The drive speed is given by

$$V_a = E_b + I_a R_a \qquad \text{Where } E_b = K_a\phi\omega$$

$$\text{Then } V_a = K_a\phi\omega_m + I_a R_a$$

$$\omega_m = \frac{V_a - I_a R_a}{K_a\phi} \tag{2.52}$$

In separately excited DC motor $K_a\phi I_a = T$ therefore (2.52) becomes

$$\omega_m = \frac{V_a}{K_a\phi} - \frac{R_a}{(K_a\phi)^2}T \tag{2.53}$$

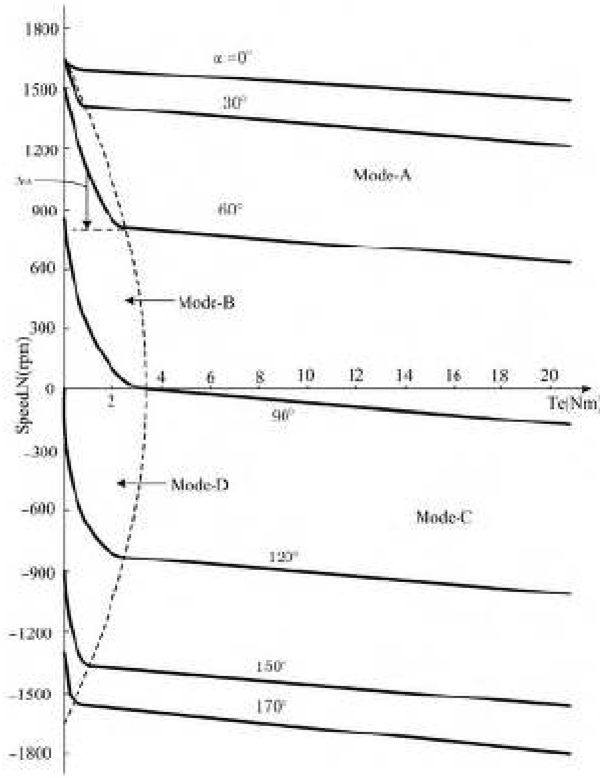


Fig 2.16

2.5 Chopper Fed DC drives

- A chopper is a static device that converts fixed DC input voltage to a variable dc output voltage directly
- A chopper is a high speed on/off semiconductor switch which connects source to load and disconnects the load from source at a fast speed.
- Choppers are used to get variable dc voltage from a dc source of fixed voltage. Self commutated devices such as MOSFET's, Power transistors, IGBT's, GTO's and IGCT's are used for building choppers because they can be commutated by a low power control signal and do not need communication circuit and can be operated at a higher frequency for the same rating.
- Chopper circuits are used to control both separately excited and Series circuits.

2.5.1 Advantages of Chopper Circuits

Chopper circuits have several advantages over phase controlled converters

1. Ripple content in the output is small. Peak/average and rms/average current ratios are small. This improves the commutation and decreases the harmonic heating of the motor.

2. The chopper is supplied from a constant dc voltage using batteries. The problem of power factor does not occur at all. The conventional phase control method suffers from a poor power factor as the angle is delayed.
3. Current drawn by the chopper is smaller than in phase controlled converters.
4. Chopper circuit is simple and can be modified to provide regeneration and the control is also simple.

2.5.2 Chopper Controlled Separately Excited DC motor

If the source of supply is D.C. (for example in a battery vehicle or a rapid transit system) a chopper-type converter is usually employed. The chopper-fed motor is, if anything, rather better than the phase-controlled, because the armature current ripple can be less if a high chopping frequency is used.

2.5.2.1 Motoring Mode of Operation

A transistor is used to chop the DC input voltage in to pieces and chopped DC voltage is given to the motor as shown in the figure 2.17. Current limit control is used in chopper. In current limit control, the load current is allowed to vary between two given limits (i.e. Upper and lower limits). The ON and OFF times of the transistor is adjusted automatically, when the current increases beyond the upper limit the chopper is turned off, the load current free wheels and starts to decrease. When the current falls below the lower limit the chopper is turned ON. The current starts increasing if the load. The load current and voltage waveforms are shown in the figure 2.18. By assuming proper limits of current, the amplitude of ripple can be controlled.

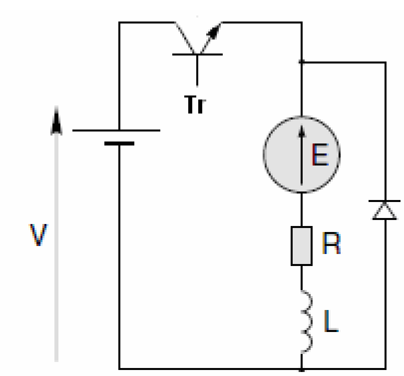


Fig 2.17

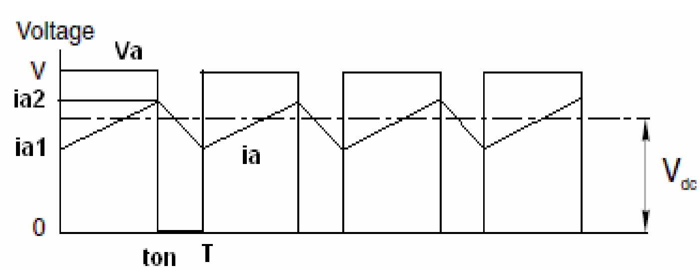


Fig 2.18

The lower the current ripple, the higher the chopper frequency. By this switching losses increase. Discontinuous conduction avoid in this case. The current limit control is superior one.

Duty Interval

During the ON period of the chopper (i.e) duty interval $0 < t < t_{ON}$, motor terminal voltage V_a is a source voltage V and armature current increases from i_{a1} to i_{a2}. The operation is describe by,

$$R_a I_a + L_a \frac{di_a}{dt} + E = V \quad 0 \le t \le t_{ON} \tag{2.54}$$

In this interval the armature current increases from I_{a1} to I_{a2} since the motor is connected to the source during this interval, it is called as ***duty cycle.***

Free Wheeling Interval

Chopper T_r is turned off at $t=t_{ON}$. Motor current free wheels through the diode D and the motor terminal voltage is zero. During interval $t_{ON} \le t \le T$. Motor operation during this interval is known as free wheeling interval and is described by

$$R_a I_a + L_a \frac{di_a}{dt} + E = 0 \quad t_{ON} \le t \le T \tag{2.55}$$

During this interval current decreases from i_{a2} to i_{a1}

Duty cycle (or) Duty Ratio:

Duty cycle is defined as the ratio of duty interval t_{ON} to chopper period T is called Duty cycle (or) Duty Ratio.

$$\delta = \frac{Duty\ Interval}{Chopper\ Period} = \frac{t_{ON}}{T} \tag{2.56}$$

From figure 2.18

$$V_a = \frac{1}{T}\int_0^{t_{ON}} V dt \tag{2.57}$$

Solving the above,

$$V_a = \frac{V}{T}\int_0^{t_{ON}} dt = \frac{V}{T}\left[t\right]_0^{t_{ON}} = V\frac{t_{ON}}{T} \tag{2.58}$$

$$V_a = \delta V \tag{2.59}$$

Then the speed of the chopper drive can be obtained as

$$V_a = E + I_a R_a$$

Substituting V_a from equation (2.59) in the above equation we get,

$$\delta V = E + I_a R_a \tag{2.60}$$

Substituting $E = K\omega_m$ we get

$$I_a = \frac{\delta V - K\omega_m}{R_a} \quad (2.61)$$

From above equation we get

$$\omega_m = \frac{\delta V}{K} - \frac{I_a R_a}{K} \quad (2.62)$$

Substituting $T = K\phi I_a$ in above equation we get

$$\omega_m = \frac{\delta V}{K} - \frac{R_a}{K^2\phi}\mathrm{T} \quad (2.63)$$

The torque speed characteristics of chopper fed separately excited DC motor is shown in the figure 2.19

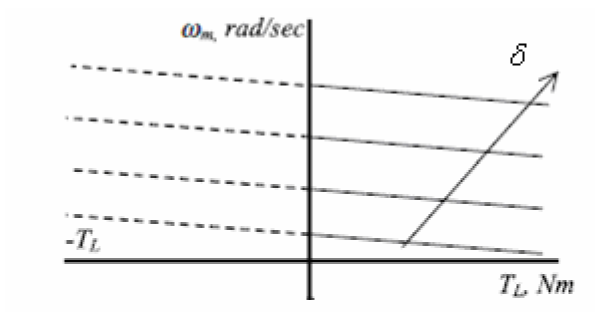
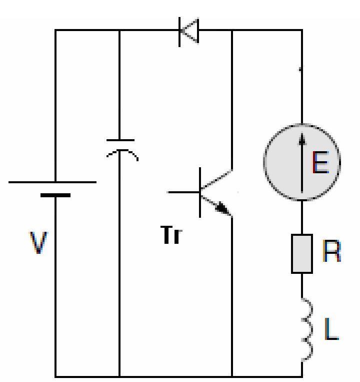


Fig 2.19

2.5.2.2 Regenerative Braking Mode

Regenerative braking operation by chopper is shown in the figure 2.20. Regenerative braking of a separately excited motor is fairly simple and can be carried out down to very low speeds.



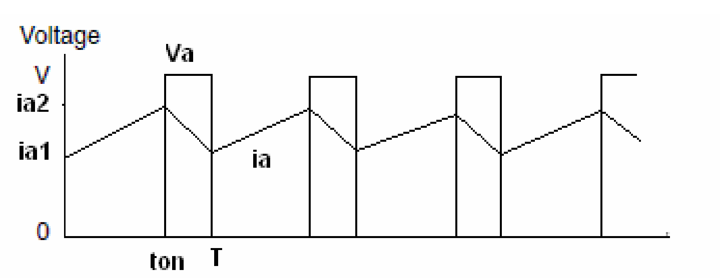


Fig 2.20 Fig 2.21

In regenerative mode, the energy of the load is fed back to the supply system. The DC motor works as a generator during this mode. As long as the chopper is ON the mechanical energy is converted in to electrical energy by the motor, now working as a generator, increases the stored magnetic energy in the armature circuit. When chopper is switched off, a large voltage appears across the motor terminals this voltage is more than that of the supply voltage V and the energy stored in the inductance and energy supplied by the machine is fed back to the supply system. When the voltage of the motor fall to V, the diodes in the line blocks the current flow preventing any short circuit of the load can be supplied to the source. Very effective braking of motor is possible up to extreme small speeds.

Energy Storage Interval

The stored energy and energy supplied by the machine is fed to the source. The interval $0 < t < t_{ON}$ is now called energy storage interval and interval $t_{ON} \le t \le T$ is the duty interval.

Here duty ratio $\delta = \dfrac{T - t_{ON}}{T}$ (2.64)

From figure 2.21

$$V_a = \frac{1}{T}\int_{t_{ON}}^{T} V dt = \frac{V}{T}\int_{t_{ON}}^{T} dt = \frac{V}{T}\left[t\right]_{t_{ON}}^{T} = \frac{V}{T}\left(T - t_{ON}\right) \quad (2.65)$$

$$V_a = V\left(\frac{T - t_{ON}}{T}\right) = V\left(1 - \frac{t_{ON}}{T}\right) \quad (2.66)$$

Therefore the speed torque relations under braking operation is given as

$$\omega_m = \frac{(1-\delta)V}{K} - \frac{R_a}{K^2\phi}T \quad (2.67)$$

2.5.3 Chopper control of DC series motor

2.5.3.1 Motoring control of series motor

The main drawback in the analysis of a chopper controlled series motor arises due to the non linear relationship between the induced voltage E and armature current Ia, because of the saturation in the magnetization characteristic. At a given motor speed ,the instantaneous back emf E changed between E_1 and E_2 as I_a changes between I_{a1} and I_{a2} as shown in figure 2.22

2.5.3.2 Regenerative Braking of DC series Motor

With chopper control, regenerative braking of series motor can also be obtained. During regenerative braking, series motor functions as a self-excited series generator. For self excitation current flowing through the winding (field) should assist residual magnetism. Therefore when changing from motoring to braking connection, when armature current reverses field current should flow in the same direction. This is achieved by reversing the field with respect to armature when changing from motoring to braking operation.

The speed of this drive ω_m can be derived from the following equation

$$E = V_a + I_a R_a \qquad \text{but } V_a = \delta V$$

$$\therefore E = \delta V + I_a R_a \qquad K_a \omega_m = \delta V + I_a R_a$$

$$\omega_m = \frac{\delta V + I_a R_a}{K_a}$$

The speed – torque characteristics gives unstable operation with most loads shown in figure 2.23.Therefore regenerative braking of series motor is difficult.

2.5.4 Four Quadrant operation of DC Drive (or) TYPE – E Four Quadrant chopper Fed Drive:Operation

The armature current I_a is either positive or negative (flow in to or away from armature) the armature voltage V_a is also either positive or negative. This is known as four quadrant chopper drive. Two type – C choppers can be combined to form a class – E chopper as shown in figure 2.24

First Quadrant – Forward motoring mode

For first quadrant operation, thyristor S_4 is kept on, thyristor S_3 is kept off and thyristor switch S_1 is operated. With S_1, S_4 ON, armature voltage $V_a = V_s$ and armature control I_a begins flow. Here both V_a and I_a are positive giving first quadrant operation, when S_1 is turned off, positive current freewheels through S_4, D2. In this manner, $V_{a,}$ I_a can be controlled in this first quadrant, and operation gives forward motoring mode.

Second Quadrant – Forward braking mode

Here thyristor S_2 is operated and S_1 , S_3 and S_4 are kept off. With S_4 on, reverse or negative current flows through L_a, S_2, D_4 and E_b. During the operation time of S_2, the armature inductance 'L_a' stores energy during the time S_2 is on. When S_2 is turned off, current is fed back to source through diodes D_1 , D_4 . Note that here (E+L(di/dt)) is more than the source voltage V_s .As the V_s is positive and I_a is negative, it is a second quadrant operation gives forward braking mode. In that power is fed back from armature to source.

Third Quadrant – Reverse motoring mode

For third quadrant operation, thyristor S_1 is kept off, S_2 is kept on and S_3 is operated, polarity of armature back emf E_b must be reversed for this quadrant operation. With thyristor S_3 is on, armature gets connected to source $V_.$ so that both V_a , I_a are negative, leading to third quadrant operation. When S_3 is turned off,negative current free wheels through S_2,D_4. In this manner only V_a and I_a can be controlled in the third quadrant.

Fourth Quadrant – Reverse Braking mode

Here thyristor S_4 is operated and other devices kept off, back emf E_b must have its polarity reversed as in third quadrant operation. With S_4 on, positive current flows through S_4,D_2,L_a and E_b (armature). Armature inductance L_a stores energy during the time S_4 is on. When S_4 is turned off, current is fed back to source through diodes D_2, D3.Here armature voltage V_a is negative, but I_a is positive, leading to the chopper drive operation in the fourth quadrant. Also power is fed back from armature to source. These four quadrant operations are clearly depicted in figure 2.25.

Solved Problems

1. A 220 volts, 1500 rpm, 10 Amps separately excited dc motor has an armature resistance of 10 **Ω**. It is fed from a single phase fully controlled bridge rectifier with an ac source voltage of 230 volts, at 50 Hz. Assuming continuous load current, compute

i. The motor speed at firing angle of 30 degrees and torque of 5 NM

ii. Developed torque at the firing angle of 45 degrees and speed of 1000 RPM

Solution:

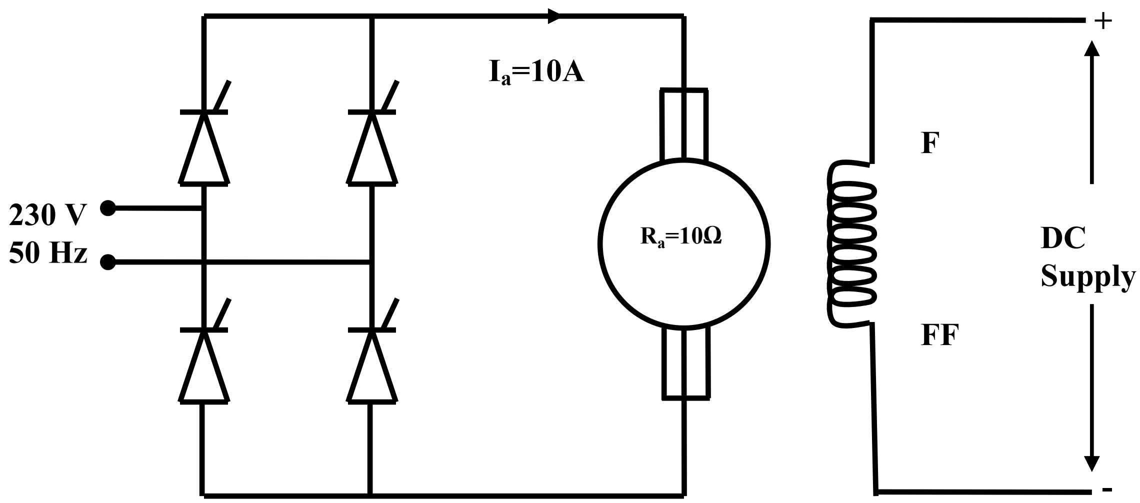
V=200 Volts

N = 1500 rpm

I_a = 10 A

R_a= 10 Ω

Source Voltage V_s=230 volts



Under operating Conditions of separately excited DC motor

$$V_a = E_b + I_a R_a$$
$$V_a = K_b \varphi \omega_m + I_a R_a$$
$$\text{Let } K_b \varphi = K_m$$

$$\therefore V_a = K_m \omega_m + I_a R_a$$
$$220 = K_m \left[\frac{2\pi \, x \, 1500}{60} \right] + (10x10)$$
$$K_m = K_b \varphi = 0.7639 \text{ Volts Seconds/Radians}$$

i. For a torque of 5 Nm, motor armature current is

$$T = K_m I_a$$
$$I_a = \frac{T}{K_m} = \frac{5}{0.7639} = 6.545A$$

The equation giving the operation of converter motor is

$$V_a = E_b + I_a R_a = \frac{2V_m}{\pi}\cos\alpha$$

$$\frac{2V_m}{\pi}\cos\alpha = K_m\omega_m + I_a R_a$$

$$\frac{2x\sqrt{2}\,x\,230}{\pi}\cos\alpha = K_m\omega_m + I_a R_a$$

$$\frac{2x\sqrt{2}\,x\,230}{\pi}\cos 30^o = 0.7639\,x\,\omega_m + 6.545\,x\,10$$

$$179.33 = 0.7639\,x\,\omega_m + 65.45$$

$$\omega_m = 149.07\,rad/\sec$$

$$\text{Motor Speed in RPM} = \text{N} = \frac{149.07\,x\,60}{2\pi}$$

N= 1423.58 rpm

ii. For $\alpha = 45^0$

$$\frac{2V_m}{\pi}\cos\alpha = K_m\omega_m + I_a R_a$$

$$\frac{2\text{ x }\sqrt{2}\text{ x }230}{\pi}\cos\alpha = 0.7639\text{ x }\frac{2\pi\text{ x }1000}{60} + \left(I_a\text{ x }10\right)$$

$$146.4 = 79.99 + 10I_a$$

I_a=6.641 Amps

Motor developed torque

$T = K_m I_a = 0.7639 \text{ x } 6.641$

T= 5.07 Nm

2. A separately excited DC motor rated at 10KW, 240 V, 1000 rpm is supplied from a fully controlled two pulse bridge converter. The converter is supplied at 250 V, 50 Hz supply. An extra inductance is connected in the load circuit to make the conduction continuous. Determine the speed, power factor and efficiency of operation for thyristors firing angles of 0 and 60 degrees assuming the armature resistance of 0.4 Ω and an efficiency of 87% at rated conditions. Assume constant torque load

Solution:

The input current to the motor at rated conditions

$$= \frac{10x10^3}{0.87} = 11.494x10^3 W$$

The supply current to the motor is

$$= \frac{11.494x10^3}{240} = 47.89A$$

Neglecting the field copper loss the armature current = 47.89A

The back EMF at the rated conditions is

$$= 240 - 47.89x0.4 = 220.843Volts$$

At $\alpha = 0$, the converter voltage is

$$V_a = \frac{2V_m}{\pi}\cos\alpha = \frac{2x\sqrt{2}x250}{\pi}\cos 0^0 = 225Volts$$

As the load torque is constant the armature current is same. Therefore the back EMF is

$$= 225 - 47.89x0.4 = 200.844Volts$$

We know that $E_b = K\omega$

$$\omega = \frac{1000x2\pi}{60} = 104.719 rad/\sec$$

$$\therefore K = \frac{E_b}{\omega} = \frac{220.844}{104.719} = 2.11Volts.\sec/rad$$

At $\alpha = 0^0$

$$\text{Speed} = \frac{E_b}{K} = \frac{200.844}{2.108} = 95.235 rad/\sec$$

$$N = \frac{95.235x60}{2\pi} = 909.43RPM$$

Displacement factor DF = $Cos\varphi = \cos 0^0 = 1$

Power Factor PF = $\frac{2x\sqrt{2}}{\pi}\cos\alpha = \frac{2x\sqrt{2}}{\pi}\cos 0^0 = 0.9$

Input = 225 x 47.89 = 10775.25 W

Output varies linearly with speed

$$\therefore output \text{ at } 909.4 \text{ rpm} = 10\text{kw}_{(\text{rated})} x \frac{909.4}{1000} = 9.094KW$$

$$\therefore \eta = \frac{O/P}{i/p} = \frac{9.094}{10.7752} = 84.4\%$$

At $\alpha = 60$, the converter voltage is

$$V_a = \frac{2V_m}{\pi}\cos\alpha = \frac{2x\sqrt{2}x250}{\pi}\cos 60^0 = 112.5Volts$$

As the load torque is constant the armature current is same. Therefore the back EMF is

$$= 112.5 - 47.89x0.4 = 93.344 Volts$$

We know that $E_b = K\omega$

$$\omega = \frac{1000x2\pi}{60} = 104.719 rad / \sec$$

$$\therefore K = \frac{E_b}{\omega} = \frac{220.844}{104.719} = 2.11 Volts.\sec/ rad$$

At $\alpha = 60^0$

$$\text{Speed} = \frac{E_b}{K} = \frac{93.344}{2.108} = 44.28 rad / \sec$$

$$\text{N}= \frac{44.28x60}{2\pi} = 422.8 RPM$$

Displacement factor DF = $Cos\varphi = \cos 60^0 = 0.5$

Power Factor PF = $\frac{2x\sqrt{2}}{\pi}\cos\alpha = \frac{2x\sqrt{2}}{\pi}\cos 60^0 = 0.45$

Input = 112.5 x 47.89 = 5.387 KW

Output varies linearly with speed

$$\therefore output \text{ at 909.4 rpm} = 10\text{kw}_{\text{(rated)}} x \frac{422.8}{1000} = 4.227 KW$$

$$\therefore \eta = \frac{O/P}{i/p} = \frac{4.227}{5.387} = 78.46\%$$

3. A 200 volts, 875 rpm, 150 A separately excited DC motor has an armature resistance of 0.06 Ω. It is fed from a single phase fully controlled rectifier with an ac source of 220 Volts, 50Hz. Assuming continuous conduction calculate

i. Firing angle for rated motor torque and 750 rpm

ii. Motor speed for α=160 degrees and rated torque

At rated Conditions

$$E_{b1} = V - I_a R_a = 200 - 150x0.06 = 191 Volts$$

$$N_1 = 875 rpm$$

$$\omega_1 = \frac{875x2\pi}{60} = 91.629 rad / \sec$$

We know that

$$E_{b1} = K\omega_1$$

$$\therefore K = \frac{191}{91.629} = 2.08 volts.\sec/rad$$

I. E_{b2} at 750 rpm

$$\omega_2 = \frac{750x2\pi}{60} = 78.54 rad/\sec$$

$$\therefore E_{b2} = 2.08x78.54 = 163.37 volts$$

$$\therefore V_a = E_{b2} + I_a R_a = 163.37 + (150x0.06) = 172.7 Volts$$

We know that

$$V_a = \frac{2V_m}{\pi}\cos\alpha$$

$$172.7 = \frac{2x\sqrt{2}x220}{\pi}\cos\alpha$$

$$\alpha = 29.3^0$$

II. At $\alpha = 160^0$ N =? at rated torque

$$V_a = \frac{2V_m}{\pi}\cos\alpha = \frac{2x\sqrt{2}x220}{\pi}\cos 160^0 = -186.12 Volts$$

We know that $V_a = E_b + I_a R_a$

$$-186.12 = E_b + (150x0.06)$$

$$E_b = -195.12 Volts$$

$$\therefore \omega = \frac{-195.12}{2.08} = -93.81$$

$$N = \frac{-93.81x60}{2\pi} = -895.79 rpm$$

4. A 220 volts, 1500 rpm, 10 Amps separately excited dc motor has an armature resistance of 0.5 Ω is fed from a three phase fully controlled rectifier. Available AC source has a line voltage of 400 volts, 50 Hz. A star-delta connected transformer is used to feed the armature so that motor terminal voltage equals rated voltage when converter firing angle is zero. Calculate transformer turns ratio. Determine the value of firing angle when

i. Motor is running at 1200 rpm and rated torque
ii. When motor is running at (-800 rpm) and twice the rated torque. Assume continuous conduction

For 3 phase controlled rectifier the average output voltage is given by

$$V_a = \frac{3V_m}{\pi}\cos\alpha$$

Given that V_a =220 Volts

$$\therefore 220 = \frac{3V_m}{\pi}\cos 0^o$$

$$\Rightarrow V_m = 230.4 Volts$$

At 1500 rpm

$$E_{b1} = V_a - I_a R_a = 220 - (10x0.5) = 215 volts$$

We know that

$$\therefore E_{b1} = K\omega_1$$

$$\omega_1 = \frac{1500x2\pi}{60} = 157.08$$

$$K = \frac{215}{157.08} = 1.37 volt.\sec/rad$$

At 1200 rpm

$$\therefore E_{b2} = K\omega_2$$

$$\omega_2 = \frac{1200x2\pi}{60} = 125.66 rad/\sec$$

$$K = 1.37 volt.\sec/rad$$

$$E_{b2} = 1.37x125.66 = 172.2 Volts$$

Average output voltage is

$$V_a = E_{b2} + I_a R_a = 172.2 + (10x0.5) = 177 Volts$$

$$\because V_a = \frac{3V_m}{\pi} Cos\alpha$$

$$\Rightarrow Cos\alpha = \frac{V_a\pi}{3V_m} = \frac{177x\pi}{3x230.4} = 0.8$$

$$\Rightarrow \alpha = 36.4^0$$

At -800 rpm, α = ? T = 2 x I_{rated}

$$\omega = \frac{-800x2\pi}{60} = -83.77 rad/\sec$$

$$\therefore E_b = 1.37x - 83.77 = 114.77 volts$$

$$Current = 2xRated = 20A$$

5. The speed of a separately excited DC motor is controlled by a chopper. The DC supply voltage is 120 V, armature circuit resistance is 0.5 Ω, armature circuit inductance is 20 mH, and back emf constant is 0.05 V/RPM. The motor drives a constant torque load requiring an average current of 20A. Assuming the motor current to be continuous, determine the range of speed control and the range of duty cycle.

Given Data:

V_s=120 volts, R_a=0.5 ohms, L_a=20mH, K=0.05 V/RPM. I_a=20A

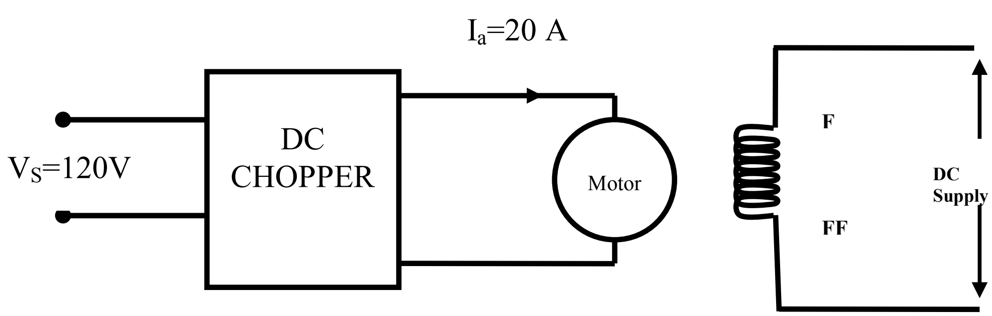
Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution



(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because \mathrm{V_a} = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 20A and minimum possible speed is **<u>ZERO</u>**

$$\alpha x120 = (0.05)x0 + (20x0.05)$$

$$120\alpha = 10$$

$$\alpha = \frac{10}{120} = 0.08$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.08 \le \alpha \le 1$$

(ii) The range of Speed

$\alpha V_s = KN + I_a R_a$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x120 = 0.05xN + (20x0.5)$$
$$120 = 0.05N + 10$$
$$N = \frac{120 - 10}{0.05} = 2200 rpm$$

The range of speed control is $0 \le N \le 2200 RPM$

6. A 230 volts, 960 rpm, 200 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a dc source of 230 volts through a chopper. Assuming continuous conduction

a) Calculate the duty ratio of chopper for monitoring operation at rated torque and 350 rpm

b) If maximum duty ratio of chopper is limited to 0.95 and maximum permissible motor speed obtainable without field weakening

Given Data

V_s=230 volts, N=960 rpm, I_a=200 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =350rpm.

(ii) If $\alpha = 0.95$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (200x0.02) = 226 volts$$
$$E \text{ at 350 rpm (ie) } E_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$220 = Kx\omega_1$$
$$\omega_1 = \frac{960x2\pi}{60} = 100.53 rad/\sec$$
$$\therefore K = \frac{226}{100.53} = 2.24 Volts.\sec/rad$$

E_2 at 350 rpm is given by

$$\omega_2 = \frac{350x2\pi}{60} = 36.651$$
$$\therefore E_2 = 36.65x2.24 = 82.1 Volts$$

Motor terminal voltage at 350 rpm is

$$V_{350rpm} = 82.1 + (200x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{350\text{ rpm}}}{V_{960\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.95\text{x}230 = 218.5\text{ Volts}$$
$$\therefore E = V_a + I_a R_a = 218.5 + (200x0.02) = 222.5 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{222.5}{2.24} = 99.330 rad/\sec$$
$$N = \frac{99.330x60}{2\pi} = 948.53 rpm$$

7. A DC series motor is fed from a 600 volts source through a chopper. The DC motor has the following parameters armature resistance is equal to 0.04 **Ω**, field resistance is equal to 0.06 **Ω**, constant $k = 4\text{ x }10^{-3} Nm/Amp^2$. The average armature current of 300 Amps is ripple free. For a chopper duty cycle of 60% determine

i. Input power drawn from the source.
ii. Motor speed and
iii. Motor torque.

Given Data

V_s=600 volts, I_a=300 amps, R_a=0.04 ohms, R_f=0.06 ohms, K=$4x10^{-3} Nm/amp^2$ $\delta = 0.6$

DC SERIES motor.

Solution

a. Power input to the motor = P = $V_a I_a$

$$V_a = \delta V_s = 0.6x600 = 360Volts$$
$$\therefore P = 360x300 = 108KW$$

b. For a DC series motor

$$E_a = K_a \phi \omega_m$$
$$= \mathrm{KI_a}\omega_m [\because \phi = I_a]$$
$$= 4x10^{-3} x300x\omega_m$$
$$\therefore V_a = E + I_a(R_a + R_s) = KI_a\omega_m + I_a(R_a + R_s)$$
$$\Rightarrow 0.6x600 = 4x10^{-3}x300x\omega_m + 300(0.04 + 0.06)$$
$$\omega_m = \frac{360 - 30}{1.2} = 27.5rad/\sec(or)2626rpm$$
$$Motor \text{ Torque T} = \mathrm{Ka}\phi \mathrm{I_a} = KI_a^2$$
$$= 4\mathrm{x}10^{-3} x300^2$$
$$= 360 \text{ N - M}$$

8. A 230 V, 1100 rpm, 220 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a chopper, which provides both motoring and braking operations. Calculate

   i. The duty ratio of chopper for motoring operation at rated torque and 400 rpm

   ii. The maximum permissible motor speed obtainable without field weakening, if the maximum duty ratio of the chopper is limited to 0.9 and the maximum permissible motor current is twice the rated current.

Given Data

V_s=230 volts, N=1100 rpm, I_a=220 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =400rpm.

(ii) If $\alpha = 0.9$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (220x0.02) = 225.6 volts$$
$$E \text{ at 400 rpm (ie) } E_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$\omega_1 = \frac{1110x2\pi}{60} = 115.192 rad/\sec$$
$$\therefore K = \frac{225.6}{115.192} = 1.95 Volts.\sec/rad$$

E_2 at 400 rpm is given by

$$\omega_2 = \frac{400x2\pi}{60} = 41.887 rad/\sec$$
$$\therefore E_2 = 41.887x1.95 = 81.68 Volts$$

Motor terminal voltage at 400 rpm is

$$V_{400rpm} = 81.68 + (220x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{400\text{ rpm}}}{V_{1100\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.9x230 = 207 \text{ Volts}$$
$$\therefore E = V_a + I_a R_a = 207 + (2x220x0.02) = 215.8 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{215.8}{1.95} = 110.667 rad/\sec$$
$$N = \frac{110.667x60}{2\pi} = 1056.78 rpm$$

9. A DC chopper is used to control the speed of a separately excited dc motor. The DC voltage is 220 V, R_a= 0.2 Ω and motor constant $K_e\phi$=0.08 V/rpm. The motor drives a constant load requiring an average armature current of 25 A. Determine
   iv. The range of speed control
   v. The range of duty cycle. Assume continuous conduction

Given Data:

V_s=220 volts, R_a=0.2 ohms, L_a=20mH, K=0.08 V/RPM. I_a=25A

Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution

(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because \mathrm{V_a} = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 25A and minimum possible speed is **ZERO**

$$\alpha x 220 = (0.08) x 0 + (25 x 0.2)$$

$$220\alpha = 10$$

$$\alpha = \frac{10}{220} = 0.04$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.04 \le \alpha \le 1$$

(ii) The range of Speed

$$\alpha V_s = KN + I_a R_a$$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1 x 220 = 0.08 x N + (25 x 0.2)$$

$$220 = 0.08N + 5$$

$$N = \frac{220 - 5}{0.08} = 2687.5 rpm$$

The range of speed control is $0 \le N \le 2687.5 RPM$